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Основы проектирования

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Notation

A = area
a = coefficient, dimension, spacing of stiffeners, opening at weld, pitch of rivets
a_k = notch toughness
B = spacing of arches
b = coefficient, dimension, root length of weld, spacing of floor beams, spacing of frames, width
C = coefficient, factor, parameter
C_a = aerodynamical coefficient
c = coefficient, dimension, distance from centre of gravity of flange to neutral axis, load combination factor
c_{sn} = snow distribution factor
c_{st} = linear stiffness ratio
c_1, c_2, c_3, c_4 = parameters used in determining conditions of column stability
D = diameter, dimension, pair of forces, quantity characterizing cable load
d = diameter, dimension
E = modulus of elasticity in tension or compression (Young's modulus)
e = arm of moment, dimension, eccentricity
F = force
F_{br} = braking force
F_h = horizontal force, thrust
F_l = longitudinal force
F_r = reaction
F_{res} = resultant force
F_{sh} = shearing force
F_t = tensile force
F' = allowable force
f = coefficient of friction, factor, safety factor
f_{sh} = shearing force per unit length
G = shear modulus of elasticity, weight
g = unit weight
H = height

h = depth of beam, girder or truss, size (leg) of weld
I = moment of inertia
i = inclination, linear stiffness, pitch, ratio
J = polar moment of inertia
k = factor, length coefficient, stress concentration factor
k_{as} = assymetry coefficient
k_d = section area distribution factor
k_{hom} = homogeneity factor
k_n = normal stress distribution factor
k_{rs} = range ratio of stress cycle
k_s = service condition factor
k_w = fillet weld area reduction factor
L = length, span
M = bending moment
m = number, ratio, relative or unit eccentricity
m_1 = equivalent eccentricity
N = number of rivets, number
n = load factor, number, ratio
P = load, axial force
p = unit dead load
Q = shear force, lifting capacity of crane
Q' = load-carrying capacity
q = unit live load, unit shearing force
R = radius of sphere, resistance or strength of material
r = radius, radius of gyration, reaction, ratio
r_a = rise of arch
r_0 = radius of curvature of a deflected beam or cable
S = a dimension, length of arch, statical moment of area
s = dimensionless quantity, ratio
T_1 = meridional force in shell
T_2 = annular force in shell
t = dimensionless quantity, temperature difference, thickness
U = energy
u = dimensionless quantity, unit energy
V = potential energy
W = section modulus, work
X = unknown (redundant) force
x = unknown distance
z = distance from centre of gravity, distance from neutral axis, length of zone of local load action

GREEK LETTER SYMBOLS

α (alpha) = angle, coefficient, factor, included angle of weld, parameter
β (beta) = angle, ratio, self-stressing factor
γ (gamma) = factor, unit weight
Δ (delta) = displacement, increment, permanent set, temperature strain, yield or pliancy
δ (delta) = deflection, unit displacement
ε (epsilon) = longitudinal or normal strain
ζ (zeta) = coefficient
η (eta) = coefficient, yield of supports
θ (theta) = angle, parameter
λ (lambda) = slenderness ratio
μ (mu) = Poisson's ratio, ratio, relative or unit eccentricity
μ_a = unit strain factor
ν (nu) = ratio of forces
ξ (xi) = relative distance of neutral axis from edge of section
π (pi) = ratio of length of circle to diameter (3.14...)
ρ (rho) = core radius of cross section
σ (sigma) = normal stress
σ' = allowable stress
τ (tau) = shearing stress
φ (phi) = an angle, buckling factor
χ (chi) = curvature of section
ψ (psi) = factor, strength factor

SUBSCRIPTS

a = actual, angle, arch, assymetry, axial, axis
act = active
al = aluminium
anch = anchor
ang = angle
b = back of angle, beam, bearer, bell, bending, bolt, bottom
bot = bottom
bp = base plate
br = bearing rope, bracing beam, braking, branch
c = combined, compression, concentrated, contour, crane
cb = collar beam
ch = chord
col = column
con = concrete, construction, controlled
cr = critical, crushing
cr·e = end crushing
cr·l = local crushing

$c \cdot r$ = compression of rollers
cw = crane wheel
cyl = cylindrical
d = design, diagonal, distribution, dome
E = Euler
e = effective, elastic, end (edge), external, external load
ec = eccentrical
emb = embedded
en = endurance
eq = equivalent
f = fillet weld, flooring, footing, form
fb = flooring beam
fd = foundation
fl = flange
g = gusset
gd = guide carrier
gr = gross
h = hinge, horizontal
hor = horizontal
hom = homogeneity
i = internal
j = joint
l = limiting, local, longitudinal
loc = local
m = main (beam), mean, minor
max = maximum
min = minimum
n = net, normal
o = opening, open-web, original, outer
opt = optimal
p = panel, plane, plastic
pl = batten plate, plate
pr = proportional
ps = prestressed
r = rated, reaction, reactive, ring, rivet, roof
red = reduced
ref = reference
rel = relative
req = required
res = resultant
s = secondary (beam), service, service conditions, span, strap, support
ser = service
sh = shank, shearing, shell
sn = snow

sph = sphere
st = steel, stiffness, strength
str = stress
suc = suction
t = tension, thread, tie, top, torsion, truss
tb = tie bar
tel = telescope
tk = tank
tr = traverse
u = ultimate, unit, unsupported
v = vertical, vibration
w = web, weld, wind
y = yield, yield point

SYMBOLS USED ON DRAWINGS

= elevation; the number adjacent to the symbol shows the elevation in metres

Ø = diameter

\# = number

L or ls = angle or angles

– = plate

I = I beam

[= channel

H = H beam

Note. All dimensions on drawings are in millimetres, unless otherwise specified.

Transliteration of the Russian Alphabet

Russian	Transliteration	Russian	Transliteration
Аа	a	Рр	r
Бб	b	Сс	s
Вв	v	Тт	t
Гг	g	Уу	u
Дд	d	Фф	f
Ее	e	Хх	kh
Жж	zh	Цц	ts
Зз	z	Чч	ch
Ии	i	Шш	sh
Йй		Щщ	shch
Кк	k	Ъъ	″
Лл	l	Ыы	y
Мм	m	Ьь	′
Нн	n	Ээ	é
Оо	o	Юю	yu
Пп	p	Яя	ya

This text sets forth methods of designing and analyzing metal engineering structures (of steel and aluminium). The first two chapters are devoted to the fundamentals of designing and the theory of analyzing metal structures and structural members with acccount of the material working not only in the elastic, but also in the elastoplastic stage. Chapters 3-5 describe various structural shapes and methods of joining together structural elements, the actual behaviour of the joints and their investigation, as well as certain industrial requirements which the design of structures must meet. In Chapters 6 8 the reader will find a detailed consideration of the principal elements of metal structures such as beams, girders, trusses, and columns, as well as information on crane girders and eccentrically loaded columns.

The design of metal structures consisting of separate structural elements is the subject matter of Chapters 9 and 10. The exposition of this material is based on examples of industrial buildings and some special large-span and high structures. The last chapter sets forth the fundamentals of designing continuous sheet-metal structures (steel shells).

All the material contained in the book conforms to the standards for designing steel structures and structures of aluminium alloys, as well as to the general building standards and regulations followed in the USSR.

CHAPTER ONE

Fundamentals of design

Metal structural members are mainly designed for taking the loads acting on a structure. The constructional shape of a structure is determined by the combination of the main constituent parts and elements such as beams, girders, trusses, columns or shells that form this structure. The best constructional shape of a structure and its elements is selected in the process of designing, which is a creative process wherein many solutions may be obtained. The duty of the designer is to find in each case the most appropriate and rational solution for his designing problems that will conform to the modern level of development of science and engineering.

1-1. FIELD OF APPLICATION AND NOMENCLATURE OF METAL STRUCTURAL MEMBERS

Steel structural members have found the widest use in the following types of structures, which can be divided into two groups.

A. *Framework* or *skeleton systems* having as their main elements beams, girders, trusses, and columns, such as:

(1) the frameworks of industrial buildings and structures (mainly the shops of iron-and-steel and machine-building works) with their internal members such as crane girders, platforms, and so forth (Fig. 1-1*a-c*);

(2) railway, highway and urban large-span bridges;

(3) civic multistorey buildings (Fig. 1-2), exhibition pavilions, various vaults, roofs, floors, domes, etc.;

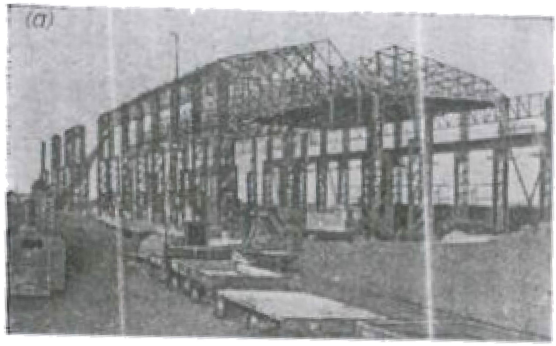
(4) special-purpose buildings such as hangars, shipbuilding slips, and so on;

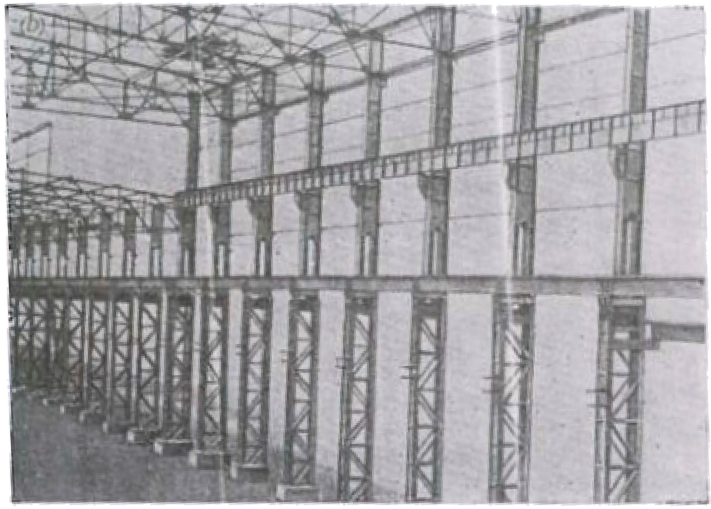
(5) special structures, for example towers and masts, headworks of mines, oil derricks, hydraulic engineering structures, trestles, cranes, etc.

B. *Shell systems* which are largely made up of plates or sheets and include, among others:

(1) gas holders and tanks for the storage and distribution of gases;

(2) tanks and reservoirs for the storage of liquids;





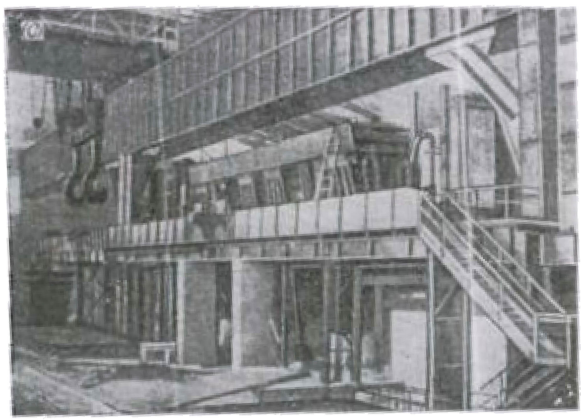


Fig. 1-1. Erection of shops of iron and steel works:
a—open-hearth shop; *b*—pipe electric welding shop;
c—platform and crane girder of open-hearth shop

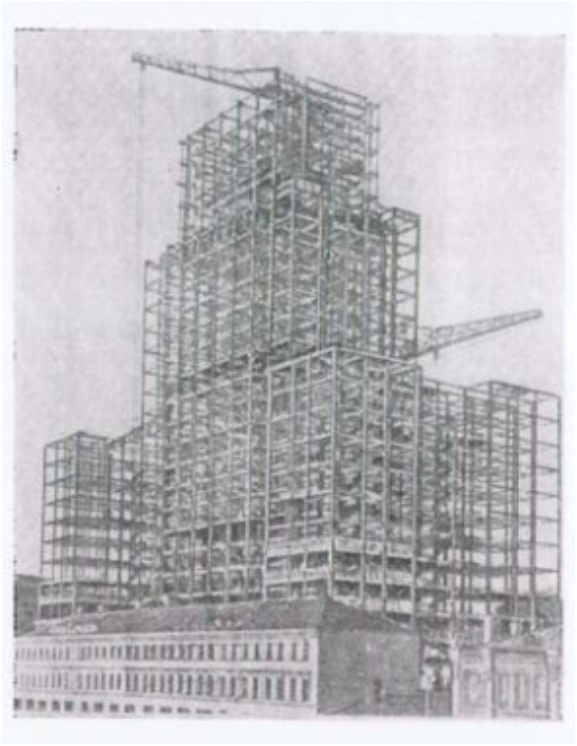


Fig. 1-2. Erection of multistorey building in Moscow

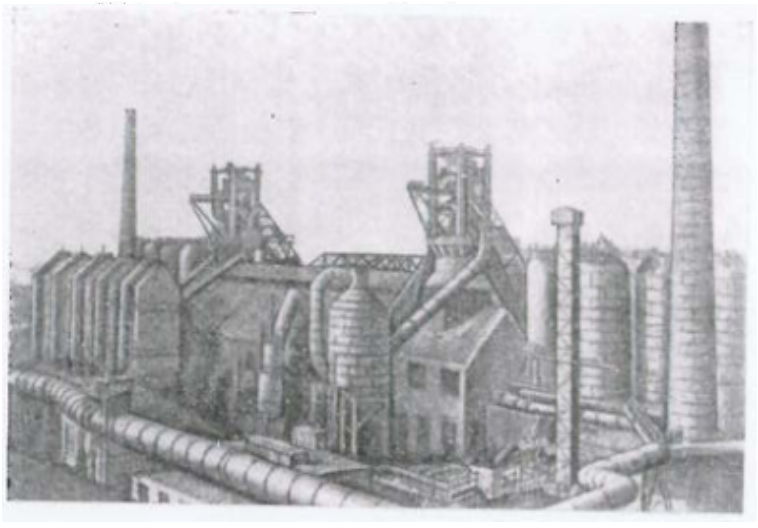


Fig. 1-3. Blast-furnace plant

(3) bins and bunkers for the storage and handling of loose materials;

(4) special structures such as blast furnaces, air heaters, gas scrubbers (Fig. 1-3);

(5) large-diameter piping employed at iron-and-steel, coke and by-product works, hydroelectric power plants, for oil and gas pipelines.

Owing to their low specific weight and high resistance to corrosion, structural members made from aluminium alloys have found use first of all in enclosing members such as roof cladding, wall panels, window sashes, etc., and also in the chemical and petroleum industries.

Aluminium members are employed in structures where the own weight of the structure is of considerable importance (transfer cranes, drawbridges, the large-span roofs of pavilions, structures erected in seismic areas and in regions difficult of access).

1-2. MERITS AND DRAWBACKS OF METAL MEMBERS

The principal merits of steel members are:

(1) the ability to resist high loads with a comparatively light weight and small size of the members, due to the high strength of steel; notwithstanding the high specific weight of steel (7.85 t/m^3) members made from this material have a lower weight in comparison with members of other construction materials. Owing to the high strength of the material, steel members are small in size, which makes them convenient for transportation;

(2) their gas- and watertightness, which is due to the high density of steel;

(3) a long service life, determined by the high and homogeneous strength and density properties of steel;

(4) the possibility of industrializing construction work, attained by the use of prefabricated members with mechanized erection thereof at the construction site;

(5) the possibility of readily disassembling or replacing steel members, which makes it easier to reinforce or replace parts of a structure.

The principal drawback of steel members and elements is their susceptibility to corrosion, which necessitates their painting or the use of other methods for their protection.

Among the merits of aluminium alloys are their low specific weight (2.7 t/m^3) with a comparatively high strength and their high resistance to corrosion.

The shortcomings of aluminium members include the relatively low value of the modulus of elasticity of alloys (710,000 kg/cm^2),

which determines the increased deformability of these members, as well as their low heat resistance (the mechanical properties of the alloy begin to deteriorate at a temperature above 100 deg C, while at a temperature over 200 deg creep begins).

1-3. BRIEF HISTORICAL REVIEW

A long time has passed since metal was first used in construction. The first metal members to be used in Russia were apparently the iron jack-rafters in the Kremlin palace (seventeenth century). The year 1725 saw the appearance of the first load-bearing cast iron members in a roof over a 12-metre span in a tower at the Nevyansk Works, after which cast iron, which has a good compressive strength, found application in arch-type members and columns.

The buildings of factories and plants at the beginning of the nineteenth century were quite primitive and consisted of brick walls and combined wood and iron roof members.

In the thirties of last century, after the appearance of the steam engine, the erection of plants with large shop spans was commenced. The trusses for them were made from wrought iron with riveted joints (1835). In the sixties, with the appearance of converter and open-hearth steel, the erection was commenced of multiple-span shops with trusses resting on steel members.

In the middle of the nineteenth century there was undertaken the erection of some large and very complicated bridges. The erection of bridges required the working out of the theory of structure design, to the development of which Russian scientists and engineers made a substantial contribution. In 1856 engineer D. Zhuravsky (1822-1891) worked out the theory of designing some types of open-web trusses and was the first to point out the existence of shear stresses in bending. Prof. F. Yasinsky (1856-1899) worked out a method of designing the compression elements of steel structures that is still partly in use at present.

The development of bridge construction is connected with the work of eminent Russian scientists and engineers, namely, Prof. N. Belelubsky (1845-1922), Prof. L. Proskuryakov (1858-1926), Academician E. Paton (1870-1953), Prof. I. Prokofyev (1877-1958), Prof. N. Streletsky, and others. They created wonderful bridges of original designs, developed the theory of bridge designing and founded a Russian school of bridge construction.

At the end of the nineteenth century industrial buildings underwent further development in connection with the appearance of electrical overhead travelling (bridge) cranes, which required special craneways—crane girders. At this period the building of all-steel shop frames with rigid craneways was commenced.

A variety of original steel structures was designed at the end of the nineteenth and the beginning of the twentieth century by Academician V. Shukhov (1853-1939). He was the author of scores of various structures. Of special interest is his work involving the creation of dimensional structures in the form of hyperboloids made from rectilinear elements (Fig. 1-4).

The years 1930-1931 saw the wide introduction of a new method of connecting steel members, viz., electric welding, invented by the engineers N. Benardos and N. Slavyanov in the eighties of last century.

Fig. 1-4. Water tower designed by V. Shukhov

The technical equipment of industry, especially in the fields of ferrous metallurgy and machine building, improved considerably. The increase in the capacity of cranes raised the requirements which the structural members of a building had to meet in service, and this led to the necessity of erecting new buildings for shops with frame-type skeletons.

During these years there appeared large bodies of design and research organizations, whose work resulted in the creation of a Soviet school of designing steel structures.

The further development of steel structures was accompanied by the introduction of low-alloy steels, the increase in the employment of new rational rolled and bent shapes, simplification of design, unification and standardization of the elements of structures and further mechanization and automation of their fabrication.

1-4. REQUIREMENTS WHICH METAL MEMBERS MUST MEET

Metal members designed to take external loads must meet a number of requirements without which their normal service will be impossible, namely, the member must correspond to its application in service, it must have the required load-carrying capacity, i.e., it must be strong and stable, and its stiffness should be adequate

The basic principles followed in designing structures are the development of structural members having a minimum weight, the lowest possible labour requirements for fabrication, and which will permit the structures to be erected in the shortest possible time.

For this reason designs of metal structural members must comply with the following requirements:

(1) General (Design) Requirements

The shape of a member and its parts should be adapted to service conditions and be convenient for protection against corrosion, the accumulation of dust, dirt, etc.

The member must have the most advantageous dimensions making possible complete utilization of its material, the tendency being to make the greatest possible use of standardized structural elements.

The structure must be well-proportioned and have a good appearance.

(2) Fabrication Requirements

It is desirable that the greatest possible number of elements used in a structure be identical. An order placed for the serial production of members with a minimum number of various types will cut the cost and labour required for the fabrication of the elements or parts of the members several times.

The design must take into account the possibilities of the fabricating plants (the use of jigs and various contrivances, the characteristics of the machine tools, the capacity of the cranes, the kind of welding equipment available, and so on).

The elements should have a simple shape and consist of the minimum possible number of parts, with a minimum of welds or rivets. The member should be easy to assemble.

(3) Erection Requirements

A member or a part thereof shipped from a plant (a so-called *shipping element*) must have dimensions within the clearance gauge of the railway or other kind of transport that will deliver it to the erection site.

The tendency should be to use the minimum number of erection elements in a structure with the greatest possible interchangeability thereof. The connections of elements should be simple and convenient for assembly. Attention should be given to the possibility of using large prefabricated members in the erection of structures.

Members that meet all the above requirements can be called high-quality and rational ones. The shapes of structural members are not fixed, they keep pace with the general development of science and engineering.

1-5. ORGANIZATION OF DESIGNING

Depending on their complicacy, structures are designed in two or three stages, viz., the preliminary project report, the detailed project report and the working project.

The *preliminary project report* has the aim of determining whether complicated structures are feasible from the technical and economical standpoints.

The *detailed project report* is a document wherein the main technical problems are solved and the cost of construction is estimated. It contains general architectural and erection drawings (top views and cross sections) indicating the material of the members, their approximate weight, a list of the building materials and quantities of work, as well as estimates. The approved detailed project report serves for drawing up the working project.

The *working project* of metal structural members consists of two stages. In the first stage the final design of the structure is determined, all the members are analyzed and the sections of all their elements are selected, the general drawings and drawings of complicated assemblies are prepared, and all the members are co-ordinated with the technological, transportation, power supply, sanitary engineering, architectural, building and other parts of the project. A specification of the metal required is also drawn up in this stage. The working drawings in the second stage, which show all the details of the members, are worked out, as a rule, by the design offices of the fabricating plants on the basis of the first stage of the working project. They are drawings of separate member elements that will be shipped ready for erection from the plants (shipping elements, standard elements), as well as erection drawings with the corresponding marking.

1-6. METHODS OF ANALYZING METAL MEMBERS

The designing of metal structures begins with the drawing up and comparison of several alternatives of structure solutions meeting the stipulated service requirements. Next the design is selected that best complies with all the requirements relating to metal members.

The analysis of members is aimed at checking the strength, stability and rigidity of the *preliminarily selected* solution of the structure, which will make it possible to determine more precisely the dimensions and ensure both reliability and the lowest possible cost.

Structures and their members and elements are analyzed on the basis of the theory of the strength of materials and structural mechanics. The main purpose of this theory is to determine the internal stresses that are induced in members under the action of applied loads.

With this aim in view there should be first drawn up what is called the *general layout of a structure* consisting of separate elements (beams, rods), temporarily disregarding the actual shape of their cross

sections. The supporting connections of the elements are endowed here with certain theoretical properties (hinged, elastically restrained or fixed connections, etc.). After having determined the stresses in the various elements according to the general layout adopted as the basis of designing, the element sections are selected, the load-carrying capacity is checked and the connection is designed to meet the stipulated conditions.

Sometimes more precise methods of determining the stresses are necessary to take into account the development of plastic deformations. The mathematical complicacy of these methods, however, often makes it necessary to use in formulas a number of coefficients and factors whose values are given in various tables.

Previously steel structures were investigated by means of the method of elastic design, based on allowable stresses. At present a more perfect method of analysis has been introduced, viz., the method of limit design, worked out by Soviet scientists headed by Prof. N. Streletsky. These principles underly the "Building Standards and Regulations" published in the USSR, Part II of which is devoted to the designing of structural members and foundations.

For a number of structures (bridges, hydraulic engineering structures, blast furnaces, etc.) special recommendations and instructions are worked out for designing that reflect the specific conditions of their service. At present certain kinds of steel members are still investigated by means of the old method of elastic design.

(1) Limit Design

In this method of design each member is considered in its limiting state. The *limiting state* of a member denotes such a state wherein it does not any longer meet the service requirements, i. e., either it loses its ability to withstand external loads, or it acquires an inadmissible deformation or local damage.

Two limiting states have been established for metal members, namely,

(a) the first limiting state, also known as the condition of non-destruction, determined by the load-carrying capacity (strength, stability or endurance). The requirements of this state must be met by all metal members;

(b) the second limiting state, determined by the development of excessive deformations (deflections and displacements). This state is used for checking members in which the magnitude of the deformations may limit the possibility of their service.

The *first limiting state* is expressed by the inequality

$$F \leqslant Q' \qquad (1\text{-}1)$$

where F = design force in a member caused by the combined action of all the design loads P in their most unfavourable combination

Q' = load-carrying capacity of the member, which is a function of the geometrical dimensions of the member and the design strength of the material R.

The maximum values of the loads tolerated in the normal service of members, which are established by the building standards and regulations, are called the *working* or *service loads* P_s (see Appendix I, Table 1). The design loads P used in the analysis of members (for the condition of non-destruction) are taken somewhat greater than the service ones. The *design load* is determined as the product of the service load and a load factor n (greater than unity), which takes into account the danger of an increase in load over its service value due to possible load changes, viz.,

$$P = P_s n \tag{1-2}$$

The values of the factor n are given in Table 1 of Appendix I.

Thus the behaviour of members is considered under the action of the design, and not the service, loads. The values of the design loads are used to determine for the members the design forces (axial force F, bending moment M, etc.), which are found in accordance with the general rules of the strength of materials and structural mechanics.

The right-hand term of the basic inequality (1-1), the load-carrying capacity of a member Q', depends upon the ultimate strength of the material, i.e., its resistance to the action of the various forces, which is characterized by the mechanical properties of the material and is called the service resistance or strength R_s, as well as upon the geometrical characteristics of the cross section (the section area A, the section modulus W, etc.).

For structural steels the service tensile, compressive and bending strength is taken equal to the minimum value of the yield point guaranteed by the corresponding USSR State Standards (GOST), i.e.,

$$R_s = \sigma_y$$

(for the most widespread grade of structural steel, ВСт. 3, the value of R_s=2,300 kg/cm^2).

In some instances, when the service of members working under tension is also possible after the metal has reached the yield point (for example, cylindrical tanks, etc.), and also for materials that do not have yielding properties (for example, high-strength wire, iron castings, etc.) the service tensile strength of the steel is taken equal to the minimum guaranteed value of the ultimate tensile strength, i.e.,

$$R_s = \sigma_u$$

For example, for steel of grade BCт. 3 (the designations of the various grades of steel used in USSR State Standards and followed in this book are described in the next Chapter) the service tensile strength R_s=3,800 kg/cm^2.

The service tensile, compressive and bending strength of aluminium alloys is taken equal to the smaller of the following two quantities:

1. Seven-tenths of the minimum guaranteed value of the ultimate tensile strength, i.e., 0.7 σ_u.

2. The arbitrary yield point, also known as the *offset stress* $\sigma_{y\,0.2}$, corresponding to the stress producing a unit permanent elongation of 0.2%.

The design strength R is obtained by multiplying the service strength by a *homogeneity factor* k_{hom} (less than unity) that takes into account the danger of a reduction in the strength of the material in comparison with its service strength due to possible variations in the mechanical properties of the material:

$$R = k_{hom} R_s^{1)} \tag{1-3}$$

For ordinary carbon steels (when $R_s=\sigma_y$) the factor k_{hom}=0.9, while for high-quality (low-alloy) steels and for aluminium alloys k_{hom}=0.85.

Thus the *design strength* R is a stress equal to the lowest possible value of the yield point (or the ultimate tensile strength) of a material, which is taken as the limiting strength of the member.

To ensure safety of a structure it is necessary to take account of possible deviations from the normal service conditions, as well as special features of work of the member (for example, the particularly strenuous working conditions of girders supporting certain types of overhead cranes; a corroding medium causing an increased rate of metal corrosion; constant work of the members at the limit loads with only slight variation thereof; and where there is a high probability that the stresses produced by these loads coincide with the lowest value of the strength of the material).

To take account of all these circumstances, there is introduced a safety factor known as the *factor of service conditions* of structural members and their elements k_s, which reduces the value of the design strength when necessary, and thus we have

$$R = k_{hom} k_s R_s \tag{1-4}$$

Thus, for example, when designing crane girders for overhead cranes with strenuous service conditions, such as are encountered in

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[1]) The values of the service strengths and the homogeneity factors are contained in Chapter II-A.10-62 of the Building Standards and Regulations, headed "Structural Members and Foundations. Basic Prescriptions for Designing". This manual is in general use in the USSR.

the shops of iron and steel works, a service condition factor of $k_s=0.9$ is introduced, while when designing a tank that works under conditions of a practically constant load produced by the material filling it (with a small load factor) the value assigned to the service condition factor is usually 0.72 to 0.80.

The values of the service condition factor for structural members and their elements are given in Appendix I, Table 2.

In all instances when the design strength is determined in accordance with the ultimate tensile strength (and not the yield point) the design strength is additionally corrected by a factor taking into account the service conditions of the material in the member (less than unity), that allows for uneven distribution of the stresses when the material is working beyond the yield point.

Finally, the values of the design strengths for riveted and bolted connections are corrected by a factor of connection service conditions that takes into account the kind and nature of behaviour of the connection, as well as the quality of the holes.

The values of the last two factors mentioned above are contained in the Building Standards and Regulations.

Thus the first limiting condition (1-1) for checking the strength can now be expressed as

$$F \leqslant AR \quad \text{or} \quad M \leqslant WR \tag{1-5}$$

where F and M are the design axial forces and bending moments caused by the design loads (with regard to the load factors n).

When investigating a contemplated member first the section of an element is selected and then the stress caused by the design forces is checked. This must never exceed the design strength (corrected by any or all of the safety factors indicated above that are essential for ensuring reliable service of the structure).

The basic formulas for checking the strength of members being designed will thereupon take the following form

$$\sigma = \frac{F}{A_n} \leqslant R \quad \text{or} \quad \sigma = \frac{M}{W_n} \leqslant R \tag{1-6}$$

where σ = design stresses in element (caused by design loads)
A_n = net area of section (holes subtracted)
W_n = net section modulus
R = design strength of material, taken from the Building Standards and Regulations.

Several categories of stresses are distinguished, viz., principal, local, additional and internal stresses.

The *principal stresses* are stresses that develop inside a body as the result of balancing the action of external loads. They are taken into account in analysis.

Uneven distribution of the force field over a cross section caused by a sharp change in the section or by the presence of a hole therein leads to a *local concentration of stresses.* In plastic materials, which include structural steel, these local excessive stresses are equalized upon increasing of the load and are therefore not considered in investigations.

The *additional stresses* are those appearing in the elements of a member upon the application of external loads by additional restraints that are not required for maintaining the general equilibrium of the system (for example, rigid connection of the elements at the joints

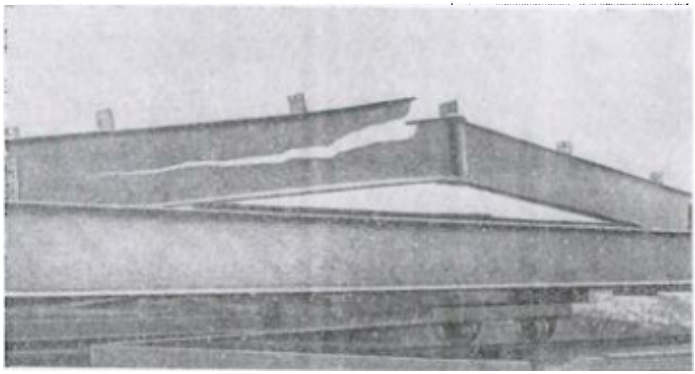

Fig. 1-5. Failure of I beam caused by improperly applied weld

of a truss). These stresses do not affect the equilibrium of the system as a whole, and in members of plastic materials are not generally taken into account in investigations.

The *internal stresses* arise not as the result of the application of external loads, but owing to the action of heat (thermal effect) on the metal under restricted conditions of deformation. Depending on the cause originating them, they are often also called thermal (temperature or heat), shrinkage, welding, residual, inherent, initial stresses, etc.

An example of internal stresses are the internal initial stresses in rolled I beams appearing as the result of their uneven cooling after being rolled. Internal stresses do not reduce the margin of safety of a member, since they are mutually balanced therein. They must be taken into consideration, however, when as the result of some external cause internal equilibrium is violated; thus, for example, Fig. 1-5 depicts the failure of an I beam after the improper application of a weld at a fillet of the beam, which disturbed the equilibrium of the internal stresses.

The *second limiting state* of a member characterized by the appearance of excess deformations (deflections) requires that the member have sufficient stiffness. Under conditions of normal service the magnitude of the unit strain $\left(\frac{\delta}{L} = \frac{\text{deflection}}{\text{span}} \text{ or } \frac{\delta}{h} = \frac{\text{deflection}}{\text{depth}}\right)$ must not exceed the value of the tolerated unit strain $(\frac{1}{r_0})$ established by the standards for various members (see Appendix I, Tables 3 and 4).

Thus the condition should be observed that

$$\frac{\sigma}{L} \leqslant \frac{1}{r_0} \tag{1-7}$$

When determining the deformations (deflections) the service load, and not the design one, is used, i.e., no attention is paid to the load factors.

(2) Allowable Stress Design

When designing is based on allowable stresses, the member is considered in its working state under the action of the loads tolerated in normal service of the structure, i. e., of the working or service loads.

To ensure adequate strength of a member, the stresses therein induced by the service loads must not exceed the values of the allowable stresses established by the Building Standards and Regulations. The allowable stresses are equal to a certain fraction of the ultimate stress of the material, which for structural steel is taken equal to the yield point σ_y. The main formulas for checking the strength of structural members are

$$\sigma = \frac{F_s}{A_n} \leqslant \sigma' = \frac{\sigma_y}{f} \quad \text{or} \quad \sigma = \frac{M_s}{W_n} \leqslant \sigma' = \frac{\sigma_y}{f} \tag{1-8}$$

where F_s and M_s = axial forces and bending moments induced by the service loads

σ' = allowable stress established by the corresponding standards and regulations

σ_y = yield point of steel

f = safety factor.

The safety factor has been introduced here since there may be deviations of the actual load from the theoretical one, and of the actual design of the member from the scheme used in analysis. It also covers variations in the mechanical properties of the material. These possible deviations are allowed for by the safety factor (which is equal on the average to about 1.5), which relates the design assumptions to the actual behaviour of the member in service.

In comparing the new method of limit design with the old one of allowable stress design [expressions (1-6) and (1-8)] it can be seen that the analysis procedure is identical; in the first case, however, the stresses in the member are obtained from the design loads (with account of the load factors), and these stresses are compared with the design strength, while in the second case the stresses are computed from the service loads and compared with the allowable stresses.

The main distinguishing feature of the new method of analysis consists in the member being considered not in its working state, but in a limit one. This required more precise formulation of the limiting conditions leading to failure of the member, as well as investigations of the nature of the safety factor, and it was this reason that led to the replacement of a single general safety factor f with three differentiated ones, viz., n, k_{hom} and k_s. Members designed on the basis of allowable stresses with a single safety factor regardless of the different service conditions and the different effect of the loads (their changeability) actually had different margins of safety.

The introduction of three factors, of which the most important one is the load factor n, makes it possible to appraise not only the absolute values of the loads, but also the influence of their ratio. This results in the appearance of structures with all members having equal strength in resisting service loads. The new procedure unifies the methods of analysis and design, it is based on physical parameters, and makes it possible to consider in a more simple way not only the elastic, but also the elastoplastic phase of work of the material. The load and the homogeneity factors obey the statistical laws of distribution and can be controlled. Thus scientific grounds are created for the method of designing structural members.

1-7. LOADS

The main purpose of any steel structure is to withstand various service (useful) loads. These loads include overhead and other cranes, equipment, etc., in industrial buildings; occupancy, etc., in civil buildings; vehicles on bridges; water in hydraulic engineering structures; loose materials for bins and bunkers, and so on. Besides the service loads, structures are also subjected to other loads such as snow, wind, thermal action, etc., as well as the own weight of the members.

In respect to the duration of their action loads are divided into:

1. *Permanent* or *dead loads*—for example the own weight of the structural members, the weight of the floors, roofs, walls, the weight and pressure of soils, etc.

2. *Temporary* or *live loads* acting for a long period, known as *movable loads*—for example the weight of stationary equipment, the loads on the floors of stores and warehouses, libraries, theatres; the pressure of gases, liquids and loose materials in tanks and reservoirs; the continuous thermal action of equipment.

3. *Live loads* acting for a short time, called *moving loads*—for example cranes and other mechanical handling equipment, the load of the occupants of buildings, wind loads, temperature (climatic) action, erection and other loads.

4. *Special loads*—for example loads caused by earthquakes, accidents, the settlement of foundations, etc.

As a rule not one, but various combinations of loads act on a structure. The probability of the simultaneous action of the maximum loads of all kinds on a structure is very small, and a structure designed for such a combination of loads would have an excessive margin of safety. The values of the most frequently encountered service loads and load factors, as well as the combinations of loads that should be taken into account when designing structures, are established by the Building Standards and Regulations, which provide for the following three categories of load combinations:

1. *main combinations*, consisting of the dead loads, movable loads and one, most important, moving load;

2. *additional combinations*, consisting of the dead loads, movable loads and all the moving loads;

3. *special combinations*, consisting of the dead loads, movable loads, possible moving loads and one of the special loads.

When these combinations are considered the vertical and horizontal loads induced by overhead cranes are treated as one moving load.

In exceptional cases the main combination also takes into consideration the combined action of a snow load and one or two overhead cranes (except cranes with light and medium duties).

When designing members with regard to the additional load combinations, the values of the design live loads (or of the stresses in the members corresponding to them) should be multiplied by a load combination factor $c=0.9$, while when taking account of special combinations a factor $c=0.8$ should be employed.

For special structures (bridges, hydraulic engineering structures, special cranes, etc.) the loads and their combinations are established by special documents.

The own weight of the steel structural members is usually only a small fraction of the total load and can be preliminarily established on the basis of an approximate estimate, or from similar existing structures (see, for example, Table 9-3).

The service live loads are given in various building standards and codes. The wind load is considered not only as the result of active pressure on the windward wall, but also as being produced by the suction acting on the roof and the inner side of the opposite wall.

With dynamic loads that create impacts and cause vibrations of a structure (cranes, trains) the same service loads are used in analysis, but after being multiplied, besides the usual load factor, by a special dynamic load factor.

CHAPTER TWO

Materials and their behaviour in members

2-1. MATERIALS USED IN METAL MEMBERS

(1) Carbon and Low-Alloy Steels

Depending upon its carbon content, carbon steel is classified as:

(a) low-carbon soft or mild steel containing from 0.09 to 0.22% carbon (used mainly in construction);

(b) medium-carbon or medium steel containing 0.25-0.5% carbon (employed chiefly in machine-building);

(c) high-carbon or hard steel, with a carbon content varying from 0.6 to 1.2%, used for making tools.

Steel always contains admixtures of manganese, silicon, phosphorus and sulphur whose total content does not generally exceed one per cent. Phosphorus and sulphur are harmful impurities, but they cannot be completely eliminated in the process of manufacturing steel. A phosphorus content of over 0.045% renders the steel brittle at low temperatures (cold short). A sulphur content exceeding 0.055% causes the steel to be red short or brittle when heated, i.e., facilitates the formation of cracks in a hot state.

The greater part of structural steel is made by the open-hearth process. Until recently converter (Bessemer and Thomas) steels ordinarily contained a somewhat larger amount of harmful impurities than their open-hearth counterparts. They sometimes contained dissolved gases (nitrogen, oxygen, etc.) which are also harmful admixtures.

At present, owing to the improvements in the process of producing converter steel (the top blowing of oxygen), its widespread use in construction has become possible.

By adding alloying elements such as manganese, nickel, chromium and others, it is possible to produce a high-strength alloy steel. In ordinary construction such steels are seldom used owing to their high cost. In many instances low-alloy steel is employed in the construction of large and critical structures.

The main structural steel is low-carbon steel, obtained by the open-hearth or the new converter process, rimming, killed or semi-killed. Its distinguishing properties are poor hardening, high plasticity and good weldability.

Killed steel advantageously differs from rimming steel in that the solidification of the steel in the ingot moulds takes place quietly, without violent evolution of the gases contained in the steel in large quantities. This is attained by introducing deoxidizing agents such as silicon, aluminium and others that bind the gases ("kill" the steel), forming slag. There is less possibility of gas bubbles forming in the steel, around which there may concentrate nonmetallic inclusions, for example various sulphur compounds, that have a harmful effect on steel in rolling (exfoliation) and welding, especially in thick plates. Killed steel costs more than rimming steel, and for this reason it should be used only for certain kinds of critical structures, especially welded ones.

There also exists semi-killed steel that has not been completely deoxidized, obtainable in special ingot moulds.

These three kinds of steel are used in welded members as follows:

1. Killed steel—in members designed for a service temperature below —30 deg C, and also (regardless of the service temperature) in members working in especially strenuous conditions—under dynamic and vibrating loads.
2. Semi-killed steel (designated "пс")—in the main load-carrying members of roofings and ceilings (trusses, frame collar-beams, beams).
3. Rimming steel (designated "кп")—in the remaining instances.

Somewhat more relaxed conditions for selecting the kind of steel to be used are possible for structural members having no welded connections.

Iron and steel works give information on the chemical composition of each melt of steel, which is entered in a special certificate. The maximum contents of chemical elements in steel are established in the USSR by the State Standard covering the delivery of ordinary quality carbon steel (GOST 380-60). There are various grades of steel in use, the grade depending upon the contents of various elements. The grades of structural steel are listed in Table 2-1.

The main structural steel is grade Ст. 3. This steel, owing to the relatively low content of carbon (as a rule less than 0.22%) and silicon (less than 0.3%) has good welding properties.

Steel of grade Ст. 0 is not manufactured specially, but is obtained as the result of rejection of other grades of steel both for chemical and mechanical reasons, and therefore it may be used only in the elements of members that are not analyzed.

Open-hearth steel is delivered according to GOST 380-60, while converter steel—to GOST 9543-60, with division into 3 groups, group A having guaranteed mechanical properties, group Б—guaranteed chemical composition, and group B with both mechanical properties and chemical composition guaranteed.

Seeing that the same guaranteed mechanical properties and design strength are established, as a rule, for definite groups of carbon steel grades, regardless of the method of manufacture and the conditions of delivery thereof, these groups for purposes of simplification in the following exposition will be named "steel 3", "steel 4" and "steel 5", covering all the grades of the respective steels.

TABLE 2-1 **Chemical Composition of Structural Steels (Open-Hearth) (According to GOST 380-60)**

Grade of steel	Content of elements, %				
	Carbon	Silicon	Manganese	Phosphorus (not over)	Sulphur (not over)
Ст.0	Not over 0.23	—	—	0.07	0.06
Ст.2 кп	0.09-0.15	Not over 0.07	0.25-0.50	0.045	0.055
Ст.3 кп	0.14-0.22	Not over 0.07	0.30-0.60	0.045	0.055
Ст.3 пс	0.14-0.22	Not over 0.17	0.40-0.65	0.045	0.055
Ст.3	0.14-0.22	0.12-0.30	0.40-0.65	0.045	0.055
Ст.4 кп	0.18-0.27	Not over 0.07	0.40-0.70	0.045	0.055
Ст.4	0.18-0.27	0.12-0.30	0.40-0.70	0.045	0.055
Ст.5	0.28-0.37	0.15-0.35	0.50-0.80	0.045	0.055

Since structural members require both definite guarantees in respect to mechanical properties, and compliance with limitations in respect to chemical composition, then steel of subgroup B will be the main kind used for them.

Low-alloy steels are delivered according to GOST 5058-57*[1] and to special specifications and are used in the main in heavy members, as well as in members that see service at low temperatures.

The designation of low-alloy steel grades, consisting of letters and numbers, mainly characterizes the chemical composition of the steel. The left-hand figures indicate the average carbon content in hundredths of a per cent, the letters after these figures denote the constituents, viz., Г for manganese, С for silicon, Х for chromium, Н for nickel, Д for copper, Т for titanium, М for molybdenum, etc. The numbers following these letters indicate the approximate content of the element in per cent (in whole numbers) exceeding one per cent. If the quantity of the constituents is less than 0.3% they are not indicated in the grade designation.

[1] An asterisk following the designation of a USSR State Standard (GOST) indicates a revised edition of the original standard.

Low-alloy steels of grades 15ХСНД and 10ХСНД (naturally-alloyed, from the Orsk-Khalilov deposits) are used in construction.

Of the cheaper steels, containing no nickel, grades 14Г2, 15ГС, 10Г2С, 10Г2СД, 09Г2Т, 09Г2ДТ(М) and others are employed. The use of low-alloy steels leads to a reduction in the weight of the members of about 15%. The cost of the members, however, remains practically the same. A considerably greater economy of steel in structural members can be obtained by using heat-treated low-alloy or low-carbon steels, in which the yield point and the ultimate strength are considerably higher with a very slight reduction in plasticity. Heat treatment consists in heating the steel to a temperature of 900-930 deg C (above the upper critical point) and hardening (quenching) in water without or with following tempering.

There will be soon available heat-treated steels of grades T30, T45, T60 and T75 (here the figure indicates the lowest value of the yield point of the steel). GOST 9458-60 provides meanwhile for the delivery of only grade T30 steel obtained on the basis of low-carbon rimming steel 3. The introduction in the nearest future of the remaining types of heat-treated steels, which will lead to a great reduction in the weight of members and a general saving in costs of about 25-30%, will be of very great significance.

(2) Aluminium Alloys

Aluminium alloys containing various elements such as magnesium, manganese, silicon, copper and others are used in structural members. For brevity we shall call members made from aluminium alloys aluminium ones.

Aluminium alloys are divided into *casting* ones, used in machine building, and *malleable* or *forging* ones (worked under pressure—in presses, by extrusion, rolling, stamping, etc.) used in construction.

The alloys are given the required strength not only by adding the appropriate components, but also by mechanical action consisting in cold deformation of the billets—cold working (cold hardening, drawing), while for some of the alloys heat treatment (hardening, ageing, etc.) is employed.

The alloys are designated in accordance with the alloying elements as follows:

АМг denotes aluminium-magnesium alloys (in the grade designation АМг6 the number 6 indicates that the alloy contains about six per cent of magnesium);

АМц stands for an aluminium-manganese alloy;

АВ (Avial) and АД are alloys of aluminium with magnesium and silicon;

Д 1, Д16, etc. designate duralumins (the figure indicates the number of the alloy), the basic components of these alloys being aluminium, magnesium and copper;

B means high-strength alloys (B65, B92 and others) consisting mainly of aluminium, magnesium, copper and zinc. These alloys are more costly.

Alloys designated by the letters АД correspond to aluminium (A) malleable (Д) alloys of an international standard. The figures following the letters show the number of the alloy (АД31, АД33).

The state in which the material is delivered is also designated by letters: M denotes annealed (soft) material, T—hardened and naturally aged (T1—artificially aged), H—cold worked, П—semicold worked.

The strength of the alloys can be increased by heat treatment from 1.3 to 1.5 times, but this will be accompanied by a reduction in the unit elongation. Duralumins, Avials and high-strength alloys are strengthened by heat treatment. Grade Д16-T duralumin is a strong alloy recommended for riveted structural members, but it is difficult to weld, since annealing of the weld zone takes place; this sometimes leads to the formation of cracks. Duralumin has a lower resistance to corrosion.

The heat-treated alloys AB and АД can be used for welded members provided that welding is followed by heat treatment; this is necessary for increasing the strength of the weld, since after welding this strength will be about 60% of that of the basic metal.

The alloys AMr and АМц are not strengthened by heat treatment. The alloys AMr6 and АД33, owing to their good weldability and relatively good mechanical properties, have found the widest use in welded members; they have a comparatively high resistance to corrosion. The alloy АМц has a low strength, and a high resistance to corrosion, it can be welded and is comparatively cheap. It can be employed for enclosures and guards.

2-2. PRINCIPAL MECHANICAL PROPERTIES OF STEEL. BEHAVIOUR OF STEEL UNDER TENSION. BRITTLENESS

The most important mechanical properties of steel are its strength, elasticity and plasticity, characterized by stresses and elongations, as well as its tendency to brittle destruction, which in an indirect way is determined by the toughness or impact strength of the material. *Strength* is determined by the resistance of the material to external loads and forces. *Elasticity* is the property of a material to restore its initial shape after removal of the external loads. *Plasticity* is the reverse of elasticity, i.e., the property of a material not to return to its initial dimensions after removal of the external loads or, in

other words, the property of obtaining permanent sets. *Brittleness* is characterized by destruction of the material upon slight deformation.

The relation between stresses and strains for various materials is established experimentally. The most simple and reliable test is the tension test of specimens, which is used to determine the strength of steel and also its elastic and plastic properties.

Other characteristics of steel are its *notch toughness* (see page 50), as well as the angle when testing for bending in a cold state, which characterizes the state of the surface and is a simple field test of the plasticity and ductility of the material.

(1) Behaviour of Steel Under Tension

Should a specimen be subjected to tension by gradually increasing the load P, and should the resulting elongations ΔL be measured, the results can be used to plot an experimental tension diagram of elongation versus load. For convenience in comparison with similar charts such a diagram is usually plotted in stress against unit elongation (also called normal or longitudinal strain)

$$\sigma = \frac{P}{A} \text{kg/cm}^2; \qquad \varepsilon = \frac{\Delta L}{L_o} 100^0/_0 \tag{2-1}$$

where σ = normal stress
A = original cross-sectional area of the specimen
ε = longitudinal strain in per cent
L_o = original length of specimen.

The magnitude of the longitudinal strain depends upon the length and the cross-sectional area of the specimen.

Two types of specimens, namely, long and short ones, are in use (GOST 1497-61).

An experimentally obtained tension diagram for carbon steel 3 is illustrated in Fig. 2-1*a*.

First the relation between the stresses and the strains follows the law of a straight line, i.e., they are proportional to each other. This is expressed by a linear equation (Hooke's law)

$$\sigma = E\varepsilon \text{ kg/cm}^2 \tag{2-2}$$

Here E is a constant factor of proportionality, known as the *modulus of elasticity in tension*, the *modulus of elasticity for axial loading* or *Young's modulus*. For steel $E = 2{,}100{,}000$ kg/cm^2.

The highest stress in a material, after which the relation between stress and strain no longer remains linear, is called the *proportional limit* σ_{pr} (Fig. 2-1*a*). Somewhat higher than this point is the *elastic*

limit σ_e corresponding to the maximum stress which can be developed in a material without causing a permanent set.

When low-carbon steels are loaded above the proportional limit, the curve in the stress-strain diagram for tension moves away from the straight line and, after rising smoothly, makes a jump (forming at high loading speeds a characteristic "tooth") and then with insignificant fluctuations runs parallel to the x-axis (Fig. 2-1*a*). The specimen grows in length without any increase in load, the material flows or yields. That normal, practically constant stress which causes an increase of strain without an increase in load, is called the *yield point* σ_y.

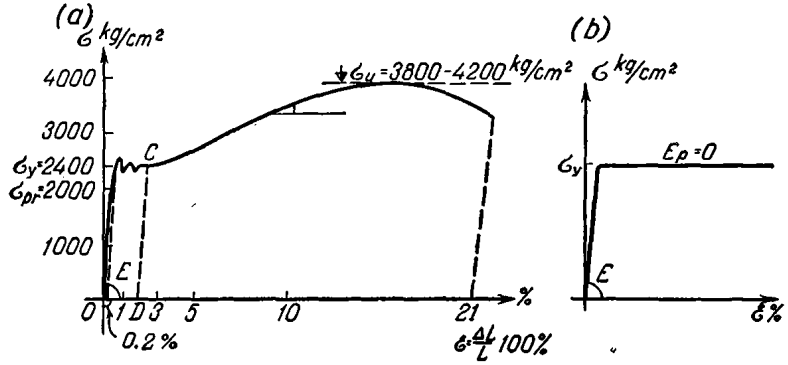

Fig. 2-1. Stress-strain diagram for steel 3

The horizontal section of the diagram, called the *yield area*, is obtained in low-carbon steels for strains varying from 0.2 to 2.5%. The existence of a yield area in the diagram of a material is a positive factor in the behaviour of steel structural members.

In other steels, not low-carbon ones, the transition to the plastic stage takes place gradually, and thus the elastic limit and the yield point in these steels do not in essence differ from each other. In such steels the yield point, which in this instance is called the *offset stress* or *yield strength*, is taken by convention equal to the stress corresponding to a permanent set of 0.2%.

Upon removal of the load from the specimen after it has received plastic deformation, the unloading diagram follows the straight line *C-D* parallel to the straight line of elastic loading (Fig. 2-1*a*).

When the strain reaches a certain value (about 2.5% for steel 3) the material stops yielding and again becomes capable of resisting loads. It behaves as if it has acquired new strength. Here the stress-strain relation will follow a curvilinear law, witha rapid increase of the strains. A neck forms in the specimen and, finally, it completely ruptures.

The maximum stress a material can withstand, which characterizes its strength, is determined by the maximum stress in the process of destruction (referred to the initial cross-sectional area of the specimen). This stress is known as the *ultimate stress* σ_u (*ultimate tensile strength*).

The total strain, measured after destruction of the specimen, is a measure of the plasticity of the steel.

Since the modulus of plastic deformations E_p is small (Fig. 2-1*a*), it is often taken equal to zero in theoretical investigations when considering the elastoplastic behaviour of steel, an idealized tension diagram of an ideal elastoplastic material being adopted for all kinds of steel (Fig. 2-1*b*, Prandtl's Diagram).

Thus the most important mechanical properties characterizing the behaviour of steel are the yield point, the ultimate tensile strength and the longitudinal strain. These properties together with the chemical composition are indicated in the certificates that are supplied with each shipment of metal.

State Standards of the USSR and special specifications for the delivery of structural steels guarantee their mechanical properties as set forth in Table 2-2.

(2) Structure of Steel and Phenomenon of Flow

Low-carbon steel is a homogeneous crystalline material consisting of fine crystals of ferrite, forming grains, and pearlite (a mixture of cementite Fe_3C and ferrite) mainly located along the joints of the ferrite grains and forming a sort of a "grid" or disseminations between the grains (Fig. 2-2, the dark lines). Pearlite is considerably harder than ferrite and is more brittle. In the course of elastic strain under the action of externally applied loads, the forces of interaction between the atoms of the crystals change, and as a result the shape of the crystals is distorted. After removal of the loads, the initial shape is restored. In the plastic strain of low-carbon steel tension specimens there can be noticed the appearance of characteristic lines known as *flow lines* (Chernov and Lüders lines) that are directed at an angle of 45 deg to the line of action of the tensile forces (Fig. 2-3). These lines, which are noticeable to the naked eye, are traces of plastic displacements of the layers of metal that remain after removal of the load. Their direction corresponds in the main to that of the maximum shear stresses.

There exists a hypothesis assuming that since in low-carbon steels the pearlite, which is located at the boundaries of the grains in a comparatively thin layer, is considerably harder than the ferrite, it will first hold back the plastic strain of the latter. However, at the spots subjected to the greatest pressure of the ferrite crystals when

they move and turn under loads exceeding the elastic limit, local destruction of the brittle pearlite is possible. When this happens, the energy accumulated by the plastic ferrite will lead to its increased displacement. Upon displacement of a large number of the grains noticeable plastic displacements will be formed. This explains the appearance of a "tooth" and the presence of the yield area in the tension diagram.

Thus the yield area is the consequence of the energy being released that had been accumulated in the grains of ferrite and restrained by the pearlite grid.

Owing to the presence of a large number of variously oriented crystals, steel may be considered as a homogeneous material, notwithstanding the heterogeneity of its microstructure.

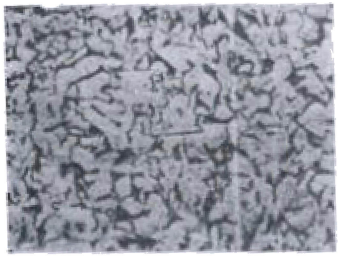

Fig. 2-2. Structure of steel 3

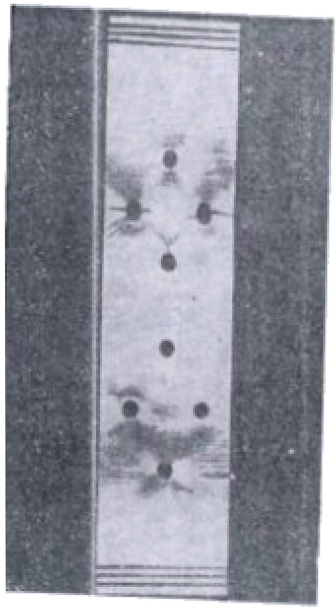

Fig. 2-3. Flow lines in stretched strip

Upon loading a specimen above its yield point, when the entire yield area is used up (i.e., when the energy accumulated in the ferrite grains is exhausted) the material acquires the ability of further load resistance, and the tension diagram becomes curved, reflecting the uniform development of plastic strains in the whole mass of the metal up to the moment of destruction.

Failure occurs as a result of the accumulation of large plastic shear strains and the development of local shearing stresses that disrupt the atomic bonds. A fine-grain crystal structure can be observed in the fracture.

In case a macrocrystalline structure is formed (for instance when the metal is heated to a certain temperature) the yield area is reduced and the yield point lowers, which is explained by the reduction in the

TABLE 2-2 **Mechanical Properties of Structural Steels. GOST 380-60 and GOST 5058-57***

Kind of steel	Grade of steel	Yield point by rolled stock thickness groups, kg/mm², at least			Ultimate tensile strength, kg/mm²	Longitudinal strain, per cent, at least		Test for bending through 180° in cold state (t = thickness of specimen, d = diameter of mandrel)
		1	2	3		long specimen	short specimen	
Carbon	Ст.0	—	—	—	At least 32	18	22	$d=2t$
	ВСт.2кп	22	20	19	34-42	26	31	$d=0$
	ВСт.3	24	23	22	38-40	23	27	$d=0.5t$
					41-43	22	26	
	ВСт.3кп	24	23	21	44-47	21	25	
	ВСт.4				42-44	21	25	
	ВСт.4кп	26	25	24	45-48	20	24	$d=2t$
					49-52	19	23	
	ВСт.5	28	27	26	50-53	17	21	
					54-57	16	20	$d=3t$
					58-62	15	19	
Low-alloy		Thickness of rolled stock	Yield point					
	14Г2	4-10	34		48	18	—	$d=2t$
		11-20	33		47	18	—	
	15ГС	4-10	35		50	18	—	$d=2t$
		11-20	34		48	18	—	
	10Г2С 10Г2СД (МК)	4-32	35		50	18	—	$d=2t$

NOTE. The rolled stock thickness groups are established as follows:

1st group—section steel (angles, rods, bars)—up to 40 mm inc usive; steel shapes (I beams, channels)—up to 15 mm inclusive; in web thickness; sheets and wide strips—from 4 to 20 mm inclusive;

2nd group—section steel—over 40 to 100 mm; shapes over 15 to 20 mm, sheets and wide strips—over 20 to 40 mm;

3rd group—section steel—over 100 to 250 mm; shapes—over 20 mm, sheets and wide strips—over 40 to 60 mm.

TABLE 2-2 *(Concluded)*

Kind of steel	Grade of steel	Thickness of rolled stock, mm	Yield point, kg/mm²	Ultimate tensile strength, kg/mm²	Longitudinal strain, per cent, at least		Test for bending through 180° in cold state (t=thickness of specimen, d=diameter of mandrel)
					long specimen	short specimen	
Low-alloy	15ХСНД (НЛ2)	4-32	35	52	18	—	$d=2t$
	10ХСНД	4-32	40	54	16	—	$d=2t$
		33-40	37	51	15	—	$d=2t$

total length of contact along the boundaries of the grains with respect to that observed in a fine-grain structure.

In alloy steels and in steels containing over 0.3% of carbon the pearlite inclusions have larger dimensions and do not form a grid or network surrounding the ferrite grains. For this reason deformation takes place more smoothly, there is no "tooth" in the tension diagram, and the yield area is reduced or completely disappears; the steel becomes harder. Thus a large yield area is characteristic of low-carbon steels with a carbon content varying from 0.1 to 0.3%.

(3) Behaviour of Steel Subjected to Combined Stresses

Upon performance of a tensile test, the yield point σ_y is determined for the specimen when it is stressed only in one direction. If a combination of stresses is involved (for example, the combined action of normal and shear stresses in bending) the transition to the plastic state, in accordance with the maximum energy theory of strength, is characterized by the ultimate value of the unit work of deformation of a body (when only the shape, and not the volume, of the body changes under the influence of crystal displacement).

This transition to a plastic state is usually expressed through a reduced or equivalent stress by taking it equal to the yield point found in simple tension.

The deduction of the condition of plasticity in the maximum energy theory is based on the assumption that under a combined stress the potential strain energy per unit of volume should not exceed the potential strain energy in uniaxial tension.

The unit strain energy or work in uniaxial tension can be represented as the ratio between the elastic strain energy in tension (equal to half the product of the final value of the force and the final elongation) and the volume of the body

$$u = \frac{U}{AL} = \frac{0.5P\,\Delta L}{AL} = \frac{0.5P\,\dfrac{PL}{EA}}{AL} = \frac{P^2}{2A^2E} = \frac{\sigma^2}{2E} \qquad (2\text{-}3)$$

The unit energy in a combined stressed state of an elementary parallelepiped with dimensions equal to unity in a body, having no shear stresses in its principal planes, and with the principal normal stresses σ_1, σ_2 and σ_3 applied to its sides, will be expressed as follows:

$$u_c = \frac{1}{2}\sigma_1\varepsilon_1 + \frac{1}{2}\sigma_2\varepsilon_2 + \frac{1}{2}\sigma_3\varepsilon_3$$

in which ε_1, ε_2 and ε_3 are the respective strains.

Since $\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E}$, where μ is the factor of lateral strain (Poisson's ratio) taken equal to 0.3 for steel (ε_2 and ε_3 are expressed in a similar manner), then substitution yields

$$u_c = \frac{1}{2E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)\right] \qquad (2\text{-}4)$$

Let us separate the part of the energy that corresponds only to a change in the form of the body characteristic of plastic strain (plastic flow). As is known, for this end a value of $\mu = 0.5$ should be taken. The unit energy in a combined stressed state spent for changing only the form of the body ($u_{c.f}$) will be found from expression (2-4) with 0.5 substituted for μ.

The condition of plasticity in a combined stressed state, according to the maximum energy theory, will be obtained by equating the specific energies, viz., $u = u_{c.f}$. This condition can be expressed in the form of a reduced stress if a square root is extracted from both parts of expressions (2-3) and (2-4) with 0.5 substituted for μ:

$$\sigma_{red} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)} = \sigma_y \qquad (2\text{-}5)$$

Thus the reduced stress is the stress in such a uniaxial stressed state, which in respect to the condition of transition to a plastic state corresponds to the given combined stressed state; in other words, the reduced stress is a quantity equivalent to the yield point in a uniaxial stressed state.

For a plane or biaxial stressed state, for example in a cut out element of a shell, where the normal stresses σ_1 and σ_2 develop in two mutually perpendicular directions, the limiting state will be expressed

through the following reduced stress ($\sigma_3=0$)

$$\sigma_{red}=\sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2}\leqslant R \tag{2-6}$$

In simple bending, when there are two components of the stressed state, namely, normal and shear stresses, for example in a simple beam having the stresses $\sigma_x=\frac{M_x}{W_x}$ and $\tau_{xy}=\frac{QS_x}{I_x t}$, the main stresses σ_1 and σ_2, as is known, will be

$$\sigma_1=\frac{\sigma_x}{2}+\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \quad \text{and} \quad \sigma_2=\frac{\sigma_x}{2}-\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \tag{2-7}$$

Substitution of these expressions in formula (2-6) gives the reduced stress for bending

$$\sigma_{red}=\sqrt{\sigma_x^2+3\tau_{xy}^2}\leqslant R \tag{2-8}$$

From this expression it is possible to obtain the condition of yielding for the maximum possible values of the shear stresses in pure shear, i.e., when $\sigma_x=0$ (assuming that $R=\sigma_y$):

$$\tau_y=\frac{\sigma_y}{\sqrt{3}}=0.577\sigma_y\cong 0.6\,\sigma_y \tag{2-9}$$

The shear modulus of elasticity (or simply the shear modulus) G is taken equal to 840,000 kg/cm^2 for steel.

For a biaxial stressed state when three components of this stressed state are present (σ_x, σ_y and τ_{xy}), the reduced stress will be

$$\sigma_{red}=\sqrt{\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2}\leqslant R \tag{2-10}$$

If in a biaxial stressed state the stress in a plate in one direction reaches the tensile yield point (for example, $\sigma_1=+\sigma_y$), then in the other direction the material will practically not be capable of withstanding either compressive or tensile stresses ($\sigma_2=0$). To permit the material to work ($\sigma_2\neq 0$) a reduction of the stress σ_1 is necessary, i.e., σ_1 should be less than σ_y.

Should the stress in a plate in one direction reach the compressive yield point ($\sigma_1=-\sigma_y$), then the plate can still take considerable compressive stresses in the other direction ($-\sigma_2$), but is practically unable to withstand tensile forces ($+\sigma_2=0$).

(4) Brittleness in Steels

Brittleness is characterized by failure of the material upon small deformations within the limits of elastic behaviour. In this case failure takes place by breaking off of the material, while the tension diagram has the form illustrated in Fig. 2-4*a*, without a yield area and a plastic stage.

Brittleness may appear in steels not only as a result of different qualities thereof, depending upon their chemical composition and heat or mechanical treatment, but also as a result of certain conditions

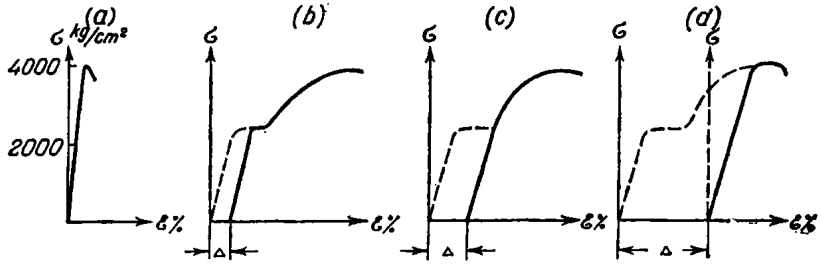

Fig. 2-4. Tension diagrams (Δ=permanent set)

of service characterized by the distribution of the force field. The latter induces a combined stressed state which depends on the shape of the body (Fig. 2-5*d*). The conditions facilitating the appearance of brittleness include phenomena leading to strain hardening, ageing,

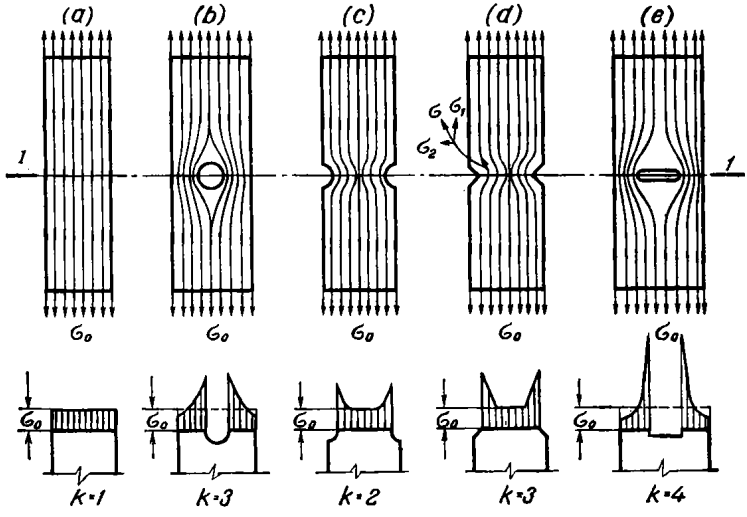

Fig. 2-5. Lines of constant stresses (k = stress concentration factor)

uneven distribution of the stresses, and also various temperature changes. The brittle state of a material often facilitates the appearance of cracks and leads to collapse of the member. For this reason designers should avoid creating such conditions that may lead to brittle failure of structural members.

Strain Hardening. If steel is subjected to tension until a plastic state is reached and is then unloaded, a permanent set will appear (Fig. 2-4*b*). Upon repeated loading of the specimen after a certain "rest"[1] of the material the steel will again behave elastically, following the unloading curve, and then it will follow the usual path of a tension diagram obtained in single loading. The same will occur if unloading is commenced after the whole yield area has been used up (Fig. 2-4*c*). It is obvious that upon repeated loading the material will not have any yield area. Figure 2-4*d* shows a diagram for an instance when a preliminarily tensioned specimen obtained a large permanent set of the same sign, and its rupture took place almost within the limits of elastic work with small strains. Such an increase of the elastic range as a result of preceding plastic deformation is called *work-hardening*, or *strain hardening*. The effect of strain hardening is to increase brittleness; this effect cannot be considered as a positive property of steel structural members.

In some cases, when the reduction in the elongation is not of great significance, strain hardening is used to increase the yield point and strength (for example in the thin reinforcing wire employed for prestressed members). Strain hardening takes place when elements are bent, along the edges of holes being driven in members, when cutting in shears, etc.

Ageing. Ageing denotes a change in the properties of a material in time without any noticeable alteration of its microstructure.

In the process of ageing, hardening of the material takes place, accompanied by a reduction in plasticity and ductility, and an increase of brittleness. Two kinds of ageing are distinguished, namely, *thermal* (*dispersion* or *precipitation hardening*) and *strain ageing*. Thermal ageing takes place after the material has been heated to comparatively low temperatures (*artificial ageing*, also called *temper hardening*) or (in alloys) goes on at the ambient temperature (*natural ageing*). Strain ageing occurs after plastic deformation at temperatures below the temperature of recrystallization.

The phenomenon taking place in ageing consists in the gradual tendency to pass from a fixed, less stable structure of a supersaturated solid solution, to a more stable state.

With time the excessive component will be liberated from the supersaturated solid solution in the form of the most minute particles (in a dispersed state). For example, in low-carbon steels there will be

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[1] With a rapidly changing variable load beyond elastic limits, without "resting" of the material, unloading and loading will form small loops in the diagram instead of following one straight line.

liberated inclusions of cementite and nitrides, i. e., compounds of iron with nitrogen that prevent displacement of the ferrite crystals.

As a result the yield area decreases and, although the elastic properties of the steel improve, its elongation is considerably reduced, and thus the brittleness grows.

The process of ageing depends upon the composition of the steel and the technology followed in its production.

Thermal ageing is employed for hardening aluminium alloys.

Uneven Distribution of Stresses. Attention was devoted above to the behaviour of smooth specimens of a regular shape, wherein the stresses in all sections removed from the place of load application are distributed uniformly.

Upon drawing lines of constant stresses or forces, we shall obtain rectilinear forcelines inside the specimen (Fig. 2-5*a*) that determine a linear uniaxial stressed state.

If a hole is made in a flat specimen or its sides are notched (Fig. 2-5*b-e*) the forcelines will change their direction and flow around the new boundaries. Concentration of the forcelines characterizes a growth of the stresses, while deviation of these lines from a straight direction is an indication of the existence of stresses acting in two directions and bending the path of the lines (Fig. 2-5*d*). In this instance there will be observed a plane (biaxial) stressed state that appears exclusively as a result of changes in the design of the specimen boundaries. As is known from the course in the strength of materials, in a plane stressed state the maximum shearing stress is equal to half the difference between the principal stresses

$$\tau = \frac{\sigma_1 - \sigma_2}{2}$$

When the difference between the two principal stresses is small or equal to zero (when $\sigma_1 = \sigma_2$), the shearing stress will also be small or equal to zero.

Yielding of a material is to a considerable extent the result of displacements under the influence of shearing stresses. This is why with small values of τ and high values of σ_1 and σ_2 the yield point of the material sharply increases, while for $\tau = 0$ it disappears completely, and failure takes place by rupture of the material within the limits of elastic strains, i.e., in the form of brittle fracture. Thus, with an increase in the sharpness of the notch (slot), the plasticity will be sharply reduced (as well as the elongation) and the ultimate strength will grow (Fig. 2-6).

A similar result is obtained upon extending a flat specimen by means of identical forces acting in two directions (an element of a boiler,

a gas holder or an element inside the closed contour of a weld tensioned as the result of shrinkage thereof upon cooling).

In analyzing the elements of structural members the local stresses are not usually determined, since the point of primary interest is the ultimate strength of the element as a whole, determined by the mean, uniformly distributed stress. When designing parts, however, especially in welded members, the local stresses must be taken into account. Local stresses relating to the category of principal ones may be omitted in analysis only when plastic steels are used, wherein equalization of the stresses is ensured.

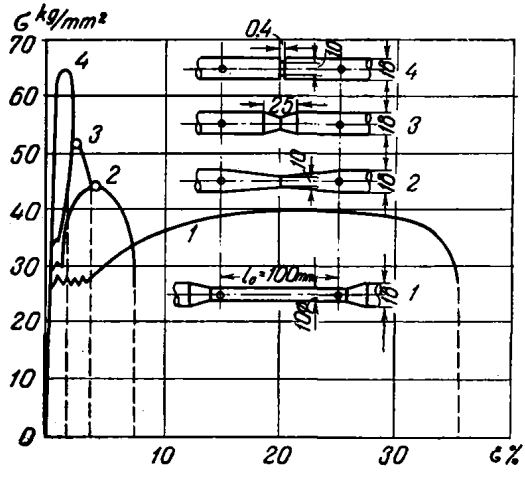


Fig. 2-6. Diagram showing behaviour of notched specimens

Influence of Temperature. The mechanical properties of low-carbon steel when it is heated to a temperature of 200-250 deg C change comparatively slightly, but already at a temperature of 300-330 deg C the steel in a fracture obtains a coarse-grain structure and becomes more brittle (blue brittleness). At this temperature it is bad practice to deform the steel or subject it to blows. Upon a further increase in temperature this property disappears, but rapid decreasing of the yield point and ultimate strength values will begin. Thus, for example, for steel 3 at a temperature of about 500 deg C we have $\sigma_y = 1{,}400$ kg/cm^2 and $\sigma_u = 2{,}500$ kg/cm^2, while at a temperature of about 600 deg the corresponding values will be 400 and 1,500 kg/cm^2, i.e., the yield point rapidly approaches zero. At a temperature of about 600-650 deg C thermal plasticity sets in. Heating for a long time at temperatures above 700 deg (a cherry-red colour) facilitates the growth of crystals and the formation of a coarse-grain structure. This phenomenon is called *overheating* and is the cause of poorer mechanical properties of steel.

Upon heating for a long time in air to a temperature approaching the melting point (a bright yellow-white colour) overburning of the metal is possible. Overburned metal is a cause for the rejection of members.

Temperatures below freezing point improve somewhat the strength of steel, but this is accompanied by a growth of brittleness. At tem-

peratures below —10 deg C the plasticity of steel begins to decline at a noticeable rate; at temperatures below —45 deg (for steel 3) and —60 deg (for low-alloy steel) the steel becomes brittle.

Fatigue of Metal. By the fatigue of metal is meant the phenomenon of its failure under the action of repeated (several million times) reversed or simply alternating loading at values of the stresses below the ultimate strength (for example, the rupture of wire when it is bent back and forth several times). The ability of metal to resist such destruction is called its *endurance*, while the stress at which the metal

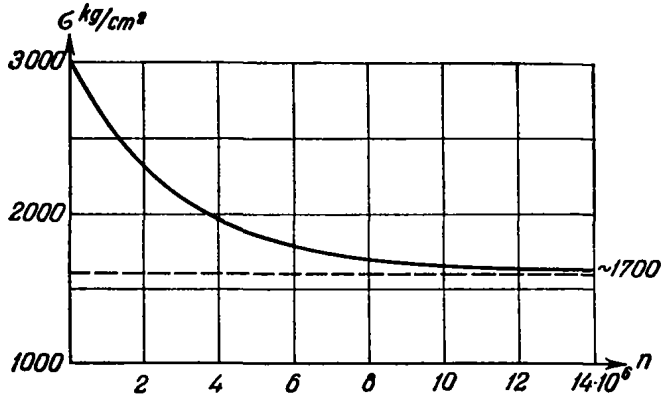


Fig. 2-7. Vibration strength (fatigue) curve

fails is known as its *fatigue* or *vibration strength* σ_v. Figure 2-7 shows a curve of the vibration strength for grade Ст. 3 steel with completely reversed loading cycles against the number of cycles n. With a great number of cycles the vibration strength curve asymptotically approaches a certain limit known as the *endurance* (*fatigue*) *limit*. Its value equals approximately four tenths of the ultimate tensile strength

$$\sigma_{en} \approx 0.4\sigma_u \qquad (2\text{-}11)$$

The vibration strength of a material is reduced by the presence of notches, sharp changes in section and other stress concentrators that create a combined stress state. Upon the failure of steel as a result of fatigue a brittle fracture is generally obtained, and it may be presumed that destruction commences with the appearance of some sort of microcracks.

Building standards and regulations prescribe a reduction of the design strengths for members and connections that are subjected to alternating or vibrating loads by introducing a factor (γ) less than unity [see expression (2-41)], and also draw special attention to the

necessity of employing designs that will not produce considerable concentrations of stresses.

Testing Steel for Impact Strength. In view of the harmful influence of brittleness in steels, which facilitates the appearance of cracks, it

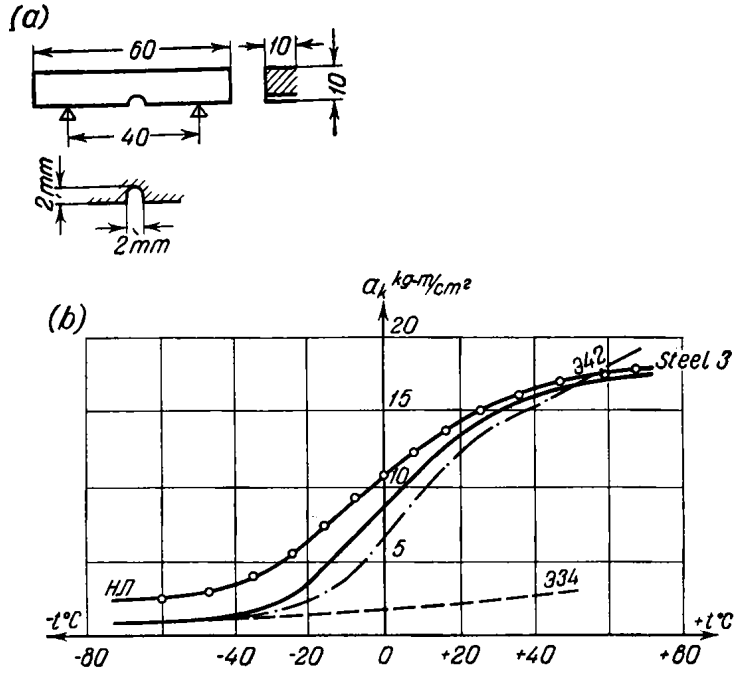


Fig. 2-8. Testing of steel for notch toughness:
—— steel 3; — . — . — metal deposited using electrodes Э42; — — — — ditto, electrodes Э34; —o—o— steel НЛ (low-alloy)

is necessary to have the possibility of making a comparative appraisal to permit designers to judge how the steel withstands the transition to a brittle state.

For this purpose use can be made of the resistance of steel to impacts. The measure of the strength of steel under impact loads is the magnitude of the work required to destroy the specimen, and not the destructing force or stress. The unit work, i.e., the work spent for the impact destruction of a standard notched specimen (Fig. 2-8*a*) per unit of cross-sectional area (in kilogram-metres per sq cm) is known as the *notch toughness.* In plastic steels *ductility* characterizes the energy absorbed by a unit of volume of the material in plastic deformation. Notch toughness is a comparative characteristic.

Low-carbon steels at a normal temperature have a notch toughness of 7 to 10 kg-m/cm^2. The notch toughness is shown versus temperature in Fig. 2-8*b*.

2-3. PRINCIPAL MECHANICAL PROPERTIES OF ALUMINIUM ALLOYS

The mechanical properties of aluminium alloys depend not only on their chemical composition, but also on the conditions of their machining and working. The principal difference between the behaviour of aluminium alloys and that of steel consists in their being more liable to deformation, i.e., less rigid. In aluminium alloys the modulus of elasticity in tension $E=710{,}000$ kg/cm^2, while the shear modulus of elasticity $G=270{,}000$ kg/cm^2, which is about one-third of the corresponding values for steel. For this reason, under equal stresses the deflections of aluminium members are three times greater. Poisson's ratio $\mu=0.3$ for aluminium. In tension diagrams of aluminium alloys there are no yield areas (Fig. 2-9). The yield strength is by convention taken equal to the stress producing a strain of $\varepsilon=0.2\%$. At temperatures above 100 deg C a certain reduction of the strength characteristics is observed, while beginning from about 200 deg C creep appears. The coefficient of thermal expansion of aluminium is 0.000023, which is twice that of steel. At reduced temperatures all the mechanical properties of aluminium alloys improve. The notch toughness of the alloys at normal temperature is lower than that of steel (about 3 kg-m/cm^2) and decreases very negligibly at temperatures below freezing point. Table 2-3 contains the mechanical properties of certain aluminium alloys.

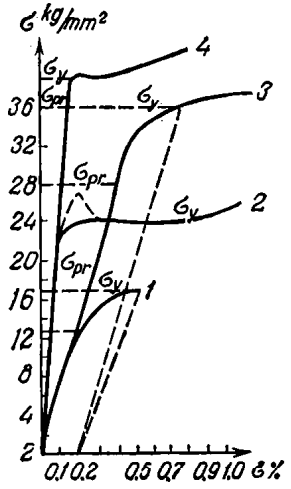


Fig. 2-9. Tension diagram of aluminium alloys in comparison with steels:
1—alloy АМг 6; *2*—steel 3; *3*—alloy Д16-Т; *4*—steel 15ХСНД

The merits of aluminium alloys include:

(1) a comparatively high strength with a low weight of the material itself, which has a great importance for large-span, mobile and enclosing members;

(2) a high workability when subjected to pressing, rolling or forging, which makes it possible to manufacture articles of a complicated configuration;

(3) a high resistance to corrosion;

TABLE 2-3 **Mechanical Properties of Aluminium Alloys**

Grade of alloy	State of delivery	Ultimate strength, kg/mm²	Yield point, kg/mm²	Longitudinal strain, %	Ultimate shearing strength, kg/mm²	Endurance limit, kg/mm²	Brinell hardness number Bhn, kg/mm²	USSR Standard or specification
Д1	Д1-Т	36	22	12	—	—	—	GOST 4977-52
Д16	Д16-Т	40-49	30	10	28		105	GOST
	Д16-М	21	11	18	13		42	4977-52
АВ	АВ-Т1	33	28	10	20	9.8	95	АМТУ 258-55
	АВ-М	15	—	20	—	4.5	30	АМТУ 252-57
В92	В92-Т	36	20	20	—	—	—	АМТУ 258-55 АМТУ 252-57
АМг6	АМг 6-М	32	16	15	—	—	70	АМТУ 443-59
АМц	АМц-М	13	5	20		5.5	30	АМТУ
	АМц-П	16	13	16		6.5	40	252-57

(4) the retaining of its high mechanical characteristics at temperatures below freezing point;

(5) the absence of sparks when struck with various articles.

The drawbacks of aluminium alloys are:

(1) a relatively low modulus of elasticity;

(2) a higher coefficient of thermal expansion;

(3) the relative complicacy of making connections;

(4) the short supply and, as yet, high cost;

(5) the comparatively low refractoriness.

2-4. BEHAVIOUR OF MATERIAL IN COMPRESSION. PROBLEM OF STABILITY

(1) Behaviour of Steel in Compression

Steel when subjected to compression in short elements behaves the same as in tension. The magnitudes of the yield point, the yield area and the modulus of elasticity are equal to those obtained in tension.

It is not possible, however, to destruct by compression short specimens made from plastic steel and thus determine the ultimate strength of the material, since the specimen flattens out.

An absolutely different picture will be observed in long compressed elements whose length is several times greater than the width of their cross section (slender elements). In this case the element may lose its load-carrying capacity, i.e., the ability to withstand external forces, not as a result of failure of the material, but owing to the loss of stability (buckling).

(2) Problem of Stability

In steel structural members the problem of stability is of a very great significance. Underestimation of this factor may lead to disastrous results.

If a straight rod is compressed by an axially applied force P, then the rod will initially remain straight, and this state of equilibrium will be stable. The stable state of equilibrium of an elastic rod is characterized by the rod returning to its original position after performing insignificant damped oscillations if it had been loaded and received an insignificant possible deflection due to any cause, after which the action of this cause stopped. This takes place because the external compressive force cannot overcome the resistance of the rod when it has been subjected to insignificant bending (deflection of its axis due to a small disturbing transverse force); the internal elastic work of deformation involved in the bending of the rod, resulting from deflection of its axis (the potential energy of bending ΔV), is greater than the external work (ΔW) done by the compressive force when the ends of the rod approach each other in bending: $\Delta V > \Delta W$ (Fig. 2-10*a*). Upon a further increase in the compressive force it may reach such a value that its work will be equal to the work of deformation in bending induced by any small disturbing factor. In this instance we have $\Delta V = \Delta W$ and the compressive force reaches its critical value P_{cr}. Thus a straight rod when loaded with a force up to the critical state has a linear shape in the stable state of equilibrium. When the force reaches its critical value, the linear shape of equilibrium stops being stable, the rod may buckle in the plane of least stiffness and it will now have a new curvilinear shape of stable equilibrium.

That magnitude of the force which causes the original stable form of equilibrium of the rod to become unstable is known as the *critical force* (or *load*).

If there is a slight initial curvature of the rod (or a slight eccentricity of the point of compressive force application) the rod upon an increase in the load will immediately deflect from a straight position.

This deflection is first small, however, and only when the compressive force approaches its critical value (differing from it by not more than one per cent) do the deflections become considerable. This indicates a transition to an unstable state. Thus the unstable state of equilibrium is characterized by great displacements taking place even with a

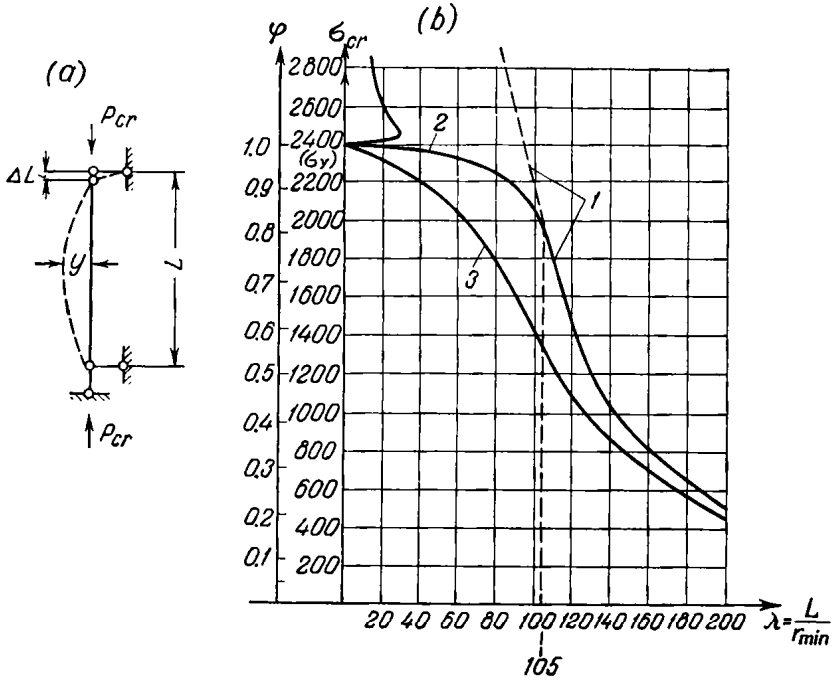


Fig. 2-10. Buckling of axially compressed rods:
a—basic case; *b*—curves of critical stresses and buckling factor for steel 3; *1*—Euler's curve; *2*—curve of critical stresses with account taken of plastic behaviour of material; *3*—curve of factor φ

small increase of the load. A further increase of the compressive force above the critical value calls forth ever growing deflections, and the rod will lose its load-carrying capacity. It should be noted that different values of the critical load correspond to different end conditions of the rod. For the axially compressed rod shown in Fig. 2-10*a*, having hinged end constraints (the basic case), the critical load was determined by the renowned mathematician L. Euler in 1744 as follows

$$P_{cr} = \frac{\pi^2}{L^2} E I_{min} \qquad (2\text{-}12)$$

The stress that is induced in the rod by the critical force is known as the *critical stress*

$$\sigma_{cr} = \frac{P_{cr}}{A_{gr}} = \frac{\pi^2 E I_{min}}{L^2 A_{gr}} = \frac{\pi^2 E r^2_{min}}{L^2} = \frac{\pi^2 E}{\left(\frac{L}{r_{min}}\right)} = \frac{\pi^2 E}{\lambda^2} \qquad (2\text{-}13)$$

where $r_{min} = \sqrt{\frac{I_{min}}{A_{gr}}}$ =minimum radius of gyration

A_{gr} = gross cross-sectional area of the rod

$\lambda = \frac{L}{r_{min}}$ = slenderness ratio of the rod, equal to the ratio between the length of the rod and the minimum radius of gyration of its cross section.

A glance at expression (2-13) will show that the critical stress depends upon the slenderness ratio of the rod (since the numerator $\pi^2 E$ is a constant quantity), while the slenderness ratio is a quantity that depends only on the geometrical dimensions of the rod. Hence the possibility of increasing the value of the critical stress by changing the slenderness ratio of the rod (mainly by increasing the radius of gyration of the cross section) is in the hands of the designer and he must use this possibility rationally.

Thus the critical stress may also be considered as a parameter that shows whether the section of an element designed for service under compression has been advantageously selected.

The graph of Euler's formula (2-13) is a hyperbola (Fig. 2-10*b*, Curve *1*).

The critical stresses determined from Euler's formula are true only for a constant modulus of elasticity E, i.e., within elastic limits (more exactly, within proportional limits), and this can take place only with high slenderness ratios (exceeding 105), which follows from expression (2-13)

$$\lambda = \sqrt{\frac{\pi^2 E}{\sigma_{pr}}} = 3.14 \sqrt{\frac{2{,}100{,}000}{2{,}000}} = 105$$

Here $\sigma_{pr} = 2{,}000$ kg/cm^2 is the proportional limit for steel 3.

The critical stresses for rods (columns, struts) with a small (below 30) and medium (from 30 to 100) slenderness ratios are obtained above the proportional limit but, of course, below the yield point. The theoretical determination of the critical stresses for such rods is rather complicated owing to buckling taking place upon the partial development of plastic deformations and with a variable modulus of elasticity.

As a result of numerous experiments that have confirmed the correctness of the theoretical conclusions, critical stresses have been established for rods with small and medium slenderness ratios, their values being shown in the form of a curve in Fig. 2-10*b* (Section *2*).

The load-carrying capacity of a compressed member is also noticeably affected by the local stability of its elements, which depends on the slenderness ratio of the flanges, webs or other elements forming the cross section of the member. The slenderness ratios of these elements are determined by the ratio between their characteristical dimension (width of flanges or depth of section web) and their thickness, viz., $\frac{b}{t}$ or $\frac{h}{t}$. Figure 2-11 shows the loss of load-carrying capacity of an axially loaded column as the result of a loss of local stability of its flanges and web.[1)]

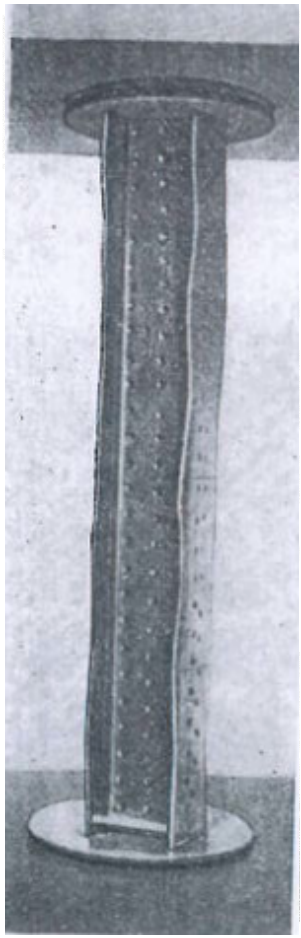


Fig. 2-11. Testing model of column in compression. Loss of local stability of flanges and web

Thus, the load-carrying capacity of a compressed element may be exhausted owing to two causes:

(1) as the result of the stress in the member reaching the yield point (loss of strength);

(2) owing to the stress in the member reaching its critical value (loss of stability).

These two causes, which are absolutely different in their nature, should never be confused.

The limiting states of compressed rods in respect to strength and stability are expressed by the condition that the stresses in the member should be

$$(1)\ \sigma \leqslant \sigma_y; \qquad (2)\ \sigma \leqslant \sigma_{cr}$$

in which $\sigma =$ stresses in member caused by design loads.

If the ratio between the two limiting stresses is denoted

$$\varphi = \frac{\sigma_{cr}}{\sigma_y}\ (\text{whence } \sigma_{cr} = \varphi\sigma_y)$$

then the second check for stability may be written as follows (the design strength R being taken as the lowest yield point)

$$\sigma \leqslant \varphi R$$

[1)] The loss of local stability in the elements of sections due to the action of normal or shearing stresses will be considered in Sections 6-4 and 6-7

or, in order to simplify the calculations and the comparison of their results, in the form of the working formula

$$\sigma = \frac{F}{\varphi A_{gr}} \leqslant R \tag{2-14}$$

The factor φ, which reduces the design strength to values ensuring stable equilibrium, is known as the *buckling factor*. The building standards and regulations establish the values of the buckling factor with regard to the influence of chance eccentricities

$$\varphi = \frac{\sigma_{cr}}{\sigma_y} \times \frac{\sigma_{cr \cdot e}}{\sigma_{cr}} = \frac{\sigma_{cr \cdot c}}{\sigma_y} \tag{2-15}$$

In the above expression $\sigma_{cr \cdot e}$ is the critical stress of a rod compressed by a force that is applied with a possible chance eccentricity of e.

The buckling factor is of interest to designers only when it is less than unity, seeing that otherwise $\sigma_{cr} > \sigma_y$, and the case of the loss of load-carrying capacity due to the lack of strength now becomes dangerous. Since it characterizes the critical stresses, the buckling factor φ is a function of the slenderness ratio of a rod. Curve *3* in Fig. 2-10*b* illustrates the buckling factor. The values of this factor for steel 3 and low-alloy steel are contained in Table 1 of Appendix II.

Analysis of aluminium alloy members in compression is similar to that of their steel counterparts. The values of the buckling factor φ for aluminium alloys are set forth in Table 2 of Appendix II.

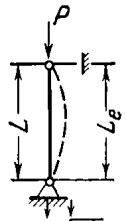
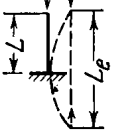
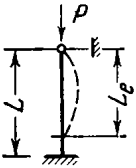
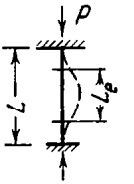
The values of the critical stresses and, therefore, of the buckling factor, depend to a very great extent upon the end conditions of the compressed member. The values of the buckling factor that are tabulated are determined for the basic end conditions of rods, viz., for rods with both ends pinned (such ends are also called pivoted, hinged or round ends). For other end conditions, the shape of the buckling curve changes, but it can be reduced to the basic case by substituting for the actual length L the *effective*, or *equivalent length* L_e, for which purpose the length of the rod should be multiplied by a length coefficient k. Hence the slenderness ratio of a rod with any end conditions will be determined from the expression

$$\lambda = \frac{L_e}{r} = \frac{kL}{r} \tag{2-16}$$

This method of analyzing buckling by using the effective lengths was proposed by prof. F. Yasinsky (1894) who devoted much time to the problems of buckling.

The values of the length coefficient k for some end conditions are given in Table 2-4.

TABLE 2-4 **Effective Lengths of Comp ession Rods**

Type of end conditions	Sketches showing actual and effective lengths	Length coeffi-cient k	Effective length of rod $L_e=kL$	Remarks
1. Both ends pinned		1	$L_e=L$	Rotation is possible at both ends, but horizontal displacements of the rod ends are impossible
2. Lower end fixed, upper end free		2	$L_e=2L$	Horizontal displacements and rotation of upper end are possible
3. Lower end fixed, upper end pinned		0.7	$L_e=0.7L$	Rotation is possible at upper end, horizontal displacements of both ends are impossible
4. Both ends fixed		0.5	$L_e=0.5L$	No rotation or horizontal displacement is possible at either end

2-5. BEHAVIOUR OF MATERIAL IN BENDING AND TORSION

(1) Limiting State of Steel Beams in Bending for Strength Analysis

When a beam is subjected to bending (Fig. 2-12*a*) within elastic limits, a triangular diagram of normal stresses (Fig. 2-12*b*) will be obtained in the cross sections of the beam. The maximum value of these stresses in the extreme fibres will be determined from the expression

$$\sigma_x = \pm \frac{M}{W_x} \tag{2-17}$$

Upon an increase in the load the extreme fibres will be the first ones to reach the yield point, after which the growth of the stresses in them will cease (with continuing increase in strains). Yielding of

the material spreads along the depth of the cross section, an elastic core remaining in the middle part of the section (Fig. 2-12*c*) and the latter will be in an elastoplastic state. An increase in the load brings up to the yield point all the fibres in the heaviest loaded cross section, characterized by a rectangular stress diagram (Fig. 2-12*d*), and at the neutral axis, at the spot where the greatest moment is acting, a so-called *plastic hinge* will be formed. The spreading of yielding along the length of the beam is shown by the shaded area in Fig. 2-12*e*. Under the influence of such action a large increase in the strains will take place at the location of the plastic hinge, the beam will sag, but

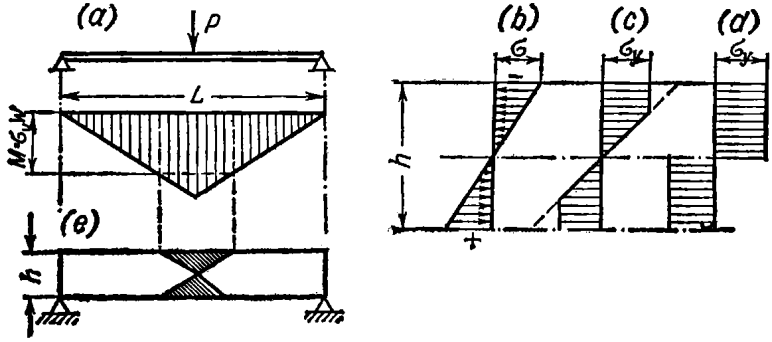


Fig. 2-12. Sequence of stress development upon plastic behaviour of beam in bending

will not fail. In this instance the beam usually loses either its total stability, or local stability of certain of its parts[1]. The appearance of a plastic hinge transforms a simple beam into a changeable system. The maximum moment corresponding to the plastic hinge can be determined from the expression

$$M_p = \sigma_y \int_A y dA = \sigma_y 2S = \sigma_y W_p \qquad (2\text{-}18)$$

where W_p is the plastic section modulus, equal to the double statical moment of half the cross-sectional area relative to the axis passing through the centre of gravity.

The plastic section modulus W_p is somewhat greater than the ordinary (elastic) section modulus W_e. Thus, for a rectangular section $W_p = \frac{bh^2}{4}$ and for rolled sections $W_p = cW_e$ (the values of c are given in Table 2-5).

---

[1] The loss of total and local stability in beams subjected to bending will be discussed in Section 6-4.

TABLE 2-5 **Values of Coefficient c for Rolled Sections**

With bending in plane of	I beams (GOST 8239-56*)	Channels (GOST 8240-56*)
Web	1.12	1.13
Flange	1.5	1.8

The Building Standards and Regulations allow account to be taken of the development of plastic strains for simple rolled beams (I beams and channels) constrained to prevent buckling and carrying a statical load. The values of the plastic section moduli in this instance are taken equal to $W_p=1.12\ W_e$ for bending in the plane of the web and $W_p=1.2W_e$ for bending parallel to the flanges (and not $1.5W_e$ or $1.8W_e$, as indicated in Table 2-5) in order to reduce the edge strains.

The Building Standards and Regulations also permit account to be taken of the development of plastic strains for constant-section welded beams with a ratio between the width of the compressed flange overhang and the flange thickness of $\frac{b}{t}\leqslant 10$, and a ratio between the depth of the web and its thickness of $\frac{h}{t}\leqslant 80$ (for steel 3).

At spots with the greatest bending moments large shearing stresses are not tolerated. They should comply with the inequality $\tau\leqslant 0.3R$.

When a long zone of pure bending is present, the corresponding section modulus, in order to avoid excessive strains, is taken equal to half the sum of the elastic and plastic moduli, viz., 0.5 (W_e+W_p).

In continuous beams the formation of plastic hinges is taken as the limiting state, but on condition that the system remains unchanged.

In the analysis of continuous beams (rolled and welded) that comply with all the limitations indicated above for simple beams the Building Standards and Regulations permit the design bending moments to be determined on the basis of equalization of the support and the span moments (provided that adjacent spans differ by not more than 20%).

In this instance the values of the design moment are taken equal to:

1. In continuous beams with freely supported ends—the greater of the two values

$$M=\frac{M_{01}}{1+D/L}\quad\text{or}\quad M=\frac{1}{2}M_{02}$$

in which M_{01} and M_{02} = maximum bending moments in extreme and intermediate spans respectively, computed as for a freely supported simple beam

D = distance to the extreme support from the section corresponding to the moment M_{01}

L = extreme span.

2. In simple and continuous beams with fixed ends

$$M = 0.5M_0$$

where M_0 is the greatest value of the moments calculated as for a beam with pinned supports.

In all cases when the design moments are determined on the assumption of plastic strains developing (equalization of the moments) the beam strength should be checked in accordance with formula (2-17) using the elastic section modulus. In analyzing beams made from aluminium alloys the development of plastic strains is not taken into consideration.

(2) Limiting State Strength Analysis of Steel Beams in Bending Under Simultaneous Action of Moments and Shear Forces

With simultaneous action in a beam subjected to bending of normal and shearing stresses σ and τ, the transition of the beam to a plastic state is expressed, as indicated in Sec. 2-2, (3), through the *equivalent* or *reduced stress*, which establishes the relation between σ and τ at the moment when yielding appears at an extreme point, or any other point of the cross section.

Until a plastic hinge is obtained, however, the beam being bent can still be additionally loaded. In this instance if large shearing stresses ($\tau > 0.4R$) are present yielding begins to spread over the cross section more rapidly than if only normal stresses were acting. Obviously, if the shearing stress τ grows, the normal stress σ should decrease, inasmuch as the equivalent stress must not be greater than the design strength R. The relation between σ and τ, or between the corresponding values of M and Q acting together, at which a plastic hinge is obtained, has not been precisely determined for various cross sections. The boundary line between the areas of the elastoplastic and the plastic states in the coordinate plane σ-τ may be taken as a convex curve (or surface if there are several components of the stressed state). Every point of this line or surface is a point where a plastic hinge is formed and a maximum point of the elastoplastic state, and for this reason a tangent to the line will lie outside it. For convenience of notation, let us express the coordinates of the

boundary line (surface) of transition to a plastic state by dimensionless quantities

$$s=\frac{M^Q}{M_p}<1 \text{ and } t=\frac{Q^M}{Q_p}<1$$

where M^Q = maximum moment observed upon the formation of a plastic hinge with the simultaneous action of a shear force Q

M_p = maximum moment observed upon the formation of a plastic hinge with action only of the moment

Q^M = maximum shear force observed upon the formation of a plastic hinge with the simultaneous action of a moment M

Q_p = maximum shear force observed upon the formation of a plastic hinge with action only of the shear force.

It is obvious that with common action of M and Q a plastic hinge will be formed with lower values of the forces than when they act separately.

Investigations conducted by B. Broude have resulted in the following equation for the boundary curve

$$s^2+t^2+as^2t^2=1 \tag{2-19}$$

in which a is a factor varying from -0.8 to -0.9 for I sections.

Assuming that $M^Q=\sigma W$ and that the shear force is mainly taken by the web of the beam, we shall obtain for I beams

$$s=\frac{M^Q}{M_p}=\frac{\sigma_M}{\sigma_y W_p}=0.89\,\frac{\sigma}{\sigma_y}$$

$$t=\frac{Q^M}{Q_p}=\frac{\tau_m A_w}{\frac{\sigma_y}{\sqrt{3}}A_w}=\frac{\tau_m\sqrt{3}}{\sigma_y}$$

Substitution of these values for s and t in equation (2-19) yields

$$\frac{\sigma^2}{\sigma_y^2}(0.89)^2+\frac{3\tau_m^2}{\sigma_y^2}-a\,\frac{3\tau_m^2\sigma^2(0.89)^2}{\sigma_y^4}=1$$

whence, assuming that $\sigma_y=R$, we shall determine, with certain approximations to increase the margin of safety, the ultimate equivalent or reduced stress in a beam corresponding to the moment when a plastic hinge is formed

$$\sigma_{h\cdot red}=\sqrt{\frac{3}{4}\sigma^2+3\tau^2\left(1-\frac{1}{2}\,\frac{\sigma^2}{R^2}\right)}\leqslant R \tag{2-20}$$

Here σ is the maximum absolute value of the boundary stress in the web of the beam in elastic behaviour of the material, and

$\tau = \frac{Q}{h_w t}$ is the mean shearing stress in the web of the beam.

Checking of the reduced stress in the webs of beams with the aid of expression (2-20) is required when τ is greater than $0.4\ R$.

(3) Behaviour of Material in Torsion

Torsion may appear in metal structural members when a load is applied that bends an element outside the plane passing through the axis of symmetry of the section. The torsional strength of the separate elements of metal members is very small, and it is therefore essential to avoid designs permitting torsion.

Assume that we have a beam which is not symmetrical with respect to its vertical axis, and that has a channel section which a load is applied to, directed at right angles to the centre line of the beam with an eccentricity to the left or the right from the web, as shown in Fig. 2-13*a*. The beam will be twisted to the left or the right. Hence, upon shifting the plane in which the load is acting to the left we shall find such a position thereof in which only bending (without torsion) will be observed. This plane of action of a force perpendicular to the centre line of a beam intersects the axis of symmetry in sections that are symmetrical with respect to their horizontal centre line at a point known as the *centre of bending* (Fig. 2-13*b*).

The stresses and strains when an element is in torsion, as distinct from tension, depend upon the shape of its cross section. It is well known that in a round solid body, which a twisting moment or torque is applied to, only shearing stresses develop that are distributed over the section in a triangular diagram (Fig. 2-13*c*). Here two cross sections, while turning with respect to each other, remain plane, perpendicular to the axis. The maximum shearing stresses in this instance will be

$$\tau_{max} = \frac{M_t}{W_p} = \frac{M_t r}{J}$$

where M_t = twisting moment
$W_p \approx 0.2d^3$ = polar section modulus
$J \approx 0.1d^4$ = polar moment of inertia
r and d = radius and diameter of round bar, respectively.

In a tube or pipe the shearing stresses can be approximately taken as uniformly distributed ones (Fig. 2-13*d*). For thin-walled pipes in which the diameter to wall thickness ratio exceeds five, i.e., $\frac{d}{t} > 5$, it

can be assumed, with an error of less than five per cent, that $W = 2\pi r^2 t$ and $I = 2\pi r^3 t$.

In a rectangular strip with a solid section (Fig. 2-13*f*), when $\frac{b}{t} > 8\text{-}10$, the shearing stresses in torsion will be distributed unevenly,

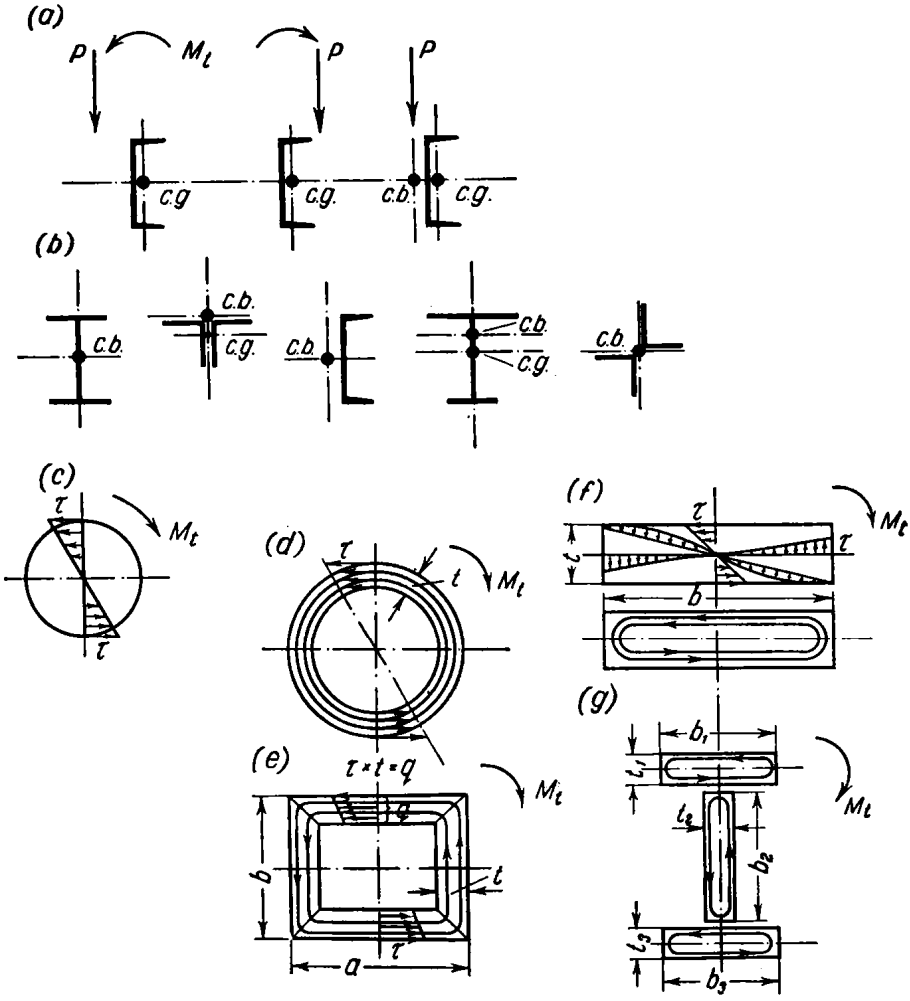


Fig. 2-13. Torsional behaviour of sections

their maximum value being found on the edges at the middle of the long side of the section. This value will be

$$\tau_{max} = \frac{M_t t}{I_t} \qquad (2\text{-}21)$$

In this expression $I_t = \frac{bt^3}{3}$ is the moment of inertia of the section in torsion. In this instance after twisting the cross sections do not remain plane.

In a rectangular box section made up of plates, the shearing stresses, the same as in a round pipe or tube, will be distributed along the periphery of the section uniformly over the thickness of the plate, having their centre at the middle of the contour, and not at the middle of each plate (Fig. 2-13*e*). Such a section with a closed contour has a high twisting strength.

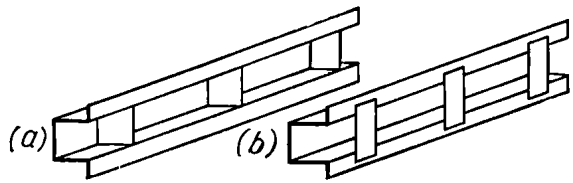


Fig. 2-14. Bracing of open sections with transverse membranes or batten plates

Immediately this contour is cut open, however, its twisting strength will be equal to that of the unfolded plate. I sections can be considered as such open sections.

The stresses in such sections are determined from expression (2-21), the moment of inertia in torsion being taken equal to

$$I_t = \alpha \sum \frac{bt^3}{3} \tag{2-22}$$

in which for an I section $\alpha = 1.3$, for channels $\alpha = 1.12$, for angles $\alpha = 1$, for welded beams with stiffening ribs welded to the flanges $\alpha = 1.5$, for riveted beams $\alpha = 0.5$.

Thus, for example, for an I section (Fig. 2-13*g*) we obtain

$$I_t = \alpha \sum \frac{bt^3}{3} = \frac{1.3}{3}(b_1 t_1^3 + b_2 t_2^3 + b_3 t_3^3)$$

The installation of transverse membranes (Fig. 2-14*a*) and especially of transverse batten plates (Fig. 2-14*b*) which close the open section at some spots increases the torsional strength of the element many times.

In a closed thin-walled section the unit shearing force induced by the flow of shearing stresses appearing in torsion will be constant over the whole section

$$q_{sh} = \tau t \text{ kg/cm} \tag{2-23}$$

The external twisting moment is balanced by an internal one equal to the sum of the products of the forces per unit of length $L\Delta$ and the

arm r over the whole contour

$$M_t = q_{sh} \sum r\Delta L = q_{sh} A_c \tag{2-24}$$

in which $A_c = \sum r\Delta L$ = double area of section contour.

Hence

$$q_{sh} = \frac{M_t}{A_c} \quad \text{and} \quad \tau = \frac{q_{sh}}{t} = \frac{M_t}{A_c t} \tag{2-25}$$

For rectangular and round closed sections (Fig. 2-13*e* and *d*) we shall have, respectively

$$\tau = \frac{M_t}{2_{abt}} \text{ kg/cm}^2 \quad \text{and} \quad \tau = \frac{M_t}{2\pi r^2 t} \text{ kg/cm}^2$$

2-6. BEHAVIOUR OF MATERIAL IN ECCENTRIC TENSION AND COMPRESSION

(1) Eccentric Tension

If an axial tensile force F and a bending moment M act simultaneously on an element, or if it is placed in tension by a force applied eccentrically with respect to its axis, the strength in the elastic stage is checked by means of the expression

$$\sigma = \frac{F}{A_n} + \frac{M_x}{W_{n.x}} = \frac{F}{A_n}\left(1 + \frac{e_x}{\rho_x}\right) = \sigma_a (1 + m_x) \leqslant R \tag{2-26}$$

where A_n = net cross section, cm^2

$W_{n.x}$ = net section modulus, cm^3

$e_x = \frac{M_x}{F}$ = eccentricity of force application, cm

$\rho_x = \frac{W_x}{A}$ = core (kern) radius of cross section, cm

$\sigma_a = \frac{F}{A_n}$ = axial stress induced by longitudinal force

$m_x = \frac{e_x}{\rho_x} = \frac{M_x}{F}\frac{A}{W_x}$ = relative or unit eccentricity.

When account is taken of the behaviour of a material in the plastic stage (which is tolerated by the Building Standards and Regulations for welded and rolled elements of steel members that are not directly subjected to dynamic loads), the load-carrying capacity of the element is higher than when only its elastic behaviour is taken into consideration. The limiting state for an eccentrically tensioned element which a moment M and a force F are applied to can be determined in the same way as that for a beam in bending when the moment M and the force Q are applied to it [see Sec. 2-5, (2)]. For this purpose it is necessary to determine the relation between M and F at which a plastic hinge will be obtained, and thus find the boundary lines separating the elastic, elastoplastic and plastic areas of behaviour of

the material in a member. If we designate by $F_y = A\sigma_y$ the force at which a yield stress is reached in a section (in the absence of a moment, Fig. 2-15*a*) and by $M_y = W_p\sigma_y$ the moment corresponding to the formation of a plastic hinge (in the absence of a longitudinal force, Fig. 2-15*b*), then with the simultaneous action of F and M their limiting values, obviously, will be

$$F_1^M < F_y \text{ and } M_1^F < M_y, \text{ i. e.,}$$

$$\nu = \frac{F_1^M}{F_y} = \frac{F_1^M}{A\sigma_y} < 1 \text{ and } \mu = \frac{M_1^F}{M_y} = \frac{M_1^F}{W_p\sigma_y} < 1 \qquad (2\text{-}27)$$

With a gradual increase of the eccentrically applied force P (Fig. 2-15*c*) the stress diagrams in the section, after having passed the stages shown in Fig. 2-15*d* and *e*, will reach the limiting diagram with a plastic hinge that is offset from the axis of the element (Fig. 2-15*f*). It may be considered here that the moment is taken by an internal pair of forces resulting from stresses in equal areas of the compressed and part of the tensioned zones of the stress diagrams, whereas the

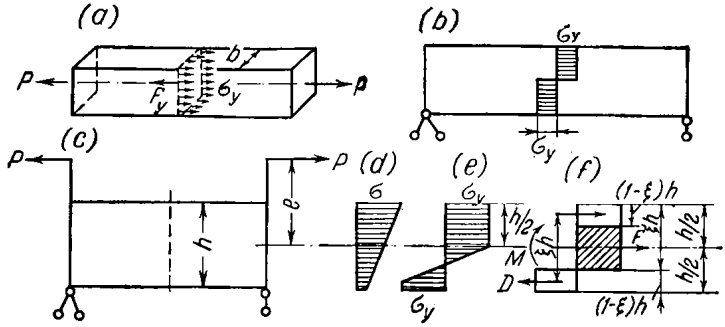


Fig. 2-15. Limiting stress diagrams of eccentrically tensioned element

longitudinal force is taken by the middle symmetrical part of the stress diagram shaded in Fig. 2-15*f*.

Thus, for a rectangular section (with a thickness b and a depth h) we obtain

$$M_1^F = D\xi h = \sigma_y b(1-\xi)h\,\xi h = \sigma_y bh^2\xi(1-\xi)$$

$$F_1^M = \sigma_y bh(2\xi - 1)$$

and

$$\nu = \frac{F_1^M}{A\sigma_y} = 2\xi - 1; \qquad \mu = \frac{M_1^F}{W_p\sigma_y} = \frac{4M_1^F}{bn^2\sigma_y} = 4\xi(1-\xi) \qquad (2\text{-}28)$$

in which the quantity $\xi<1$ designates the relative distance of the neutral line from the edge of the section. Upon excluding the quantity ξ from equations (2-28) we shall obtain the sought for equation of the boundary curve in dimensionless quantities, in the form of a parabola (Fig. 2-16)

$$\nu^2+\mu=1 \quad \text{or} \quad \left(\frac{F_1^M}{F_y}\right)+\frac{M_1^F}{M_y}=1 \tag{2-29}$$

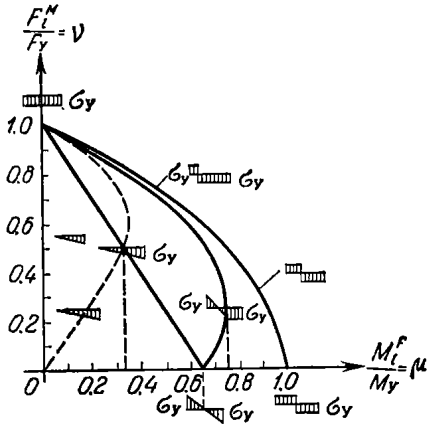


Fig. 2-16. Boundary curves of stressed state areas with simultaneous action of bending moments M and longitudinal forces F

For I sections the boundary lines (surfaces) are nearer to a straight line (plane); for this reason the Building Standards and Regulations, to ensure a margin of safety, propose to compute the strength of solid-web eccentrically tensioned elements that are not subjected to the direct action of dynamic loads from the formula

$$\left(\frac{F}{A_n R}\right)^{3/2}+\frac{M_x}{W_{x\cdot p}R}\leqslant 1 \tag{2-30}$$

(2) Eccentrically Compressed and Compressed and Bent Elements

Upon the application of a compressive force with a certain eccentricity an element is subjected to eccentric compression. Bending also appears together with this compression and, correspondingly, a deflection (Fig. 2-17*a*). When a longitudinal axial force and a lateral load that induces bending are applied together, the element will be both compressed and bent (Fig. 1-17*b*). The behaviour of elements of these two types differ somewhat from each other, mainly in the limiting state at low slenderness ratios. Nevertheless (with an increase of the margin of safety) compressed and bent elements, when considering their critical states, are equalled to eccentrically compressed ones with an eccentricity of $e=\frac{M}{F}$. Eccentrical compression, notwithstanding the existence of a deflection from the very beginning of load application, also relates to the problem of stability, and not only to that of strength; moreover, axial compression is in essence a particular case of eccentrical compression with a small chance eccentricity. Indeed, the load-carrying capacity of each element (it should not be

forgotten that a loss of stability is always connected with a loss of load-carrying capacity) is characterized by only one value of the strain corresponding to each value of the load applied to the element. In the given instance with a growth of the force P (Fig. 2-17*a*) the external moment will also increase owing to a greater deflection, viz., $M_e = P(e+\delta)$. As soon as the stress in the extreme fibre of the bent element at one side of the section passes the yield point, then in accordance with the condition of equilibrium in elastoplastic bodies, upon an increase of the moment (and the deflection) the normal force

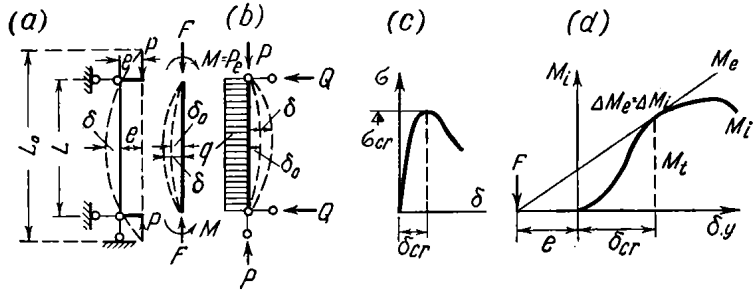


Fig. 2-17. Behaviour of eccentrically compressed element

[see equation (2-29)] should decrease, i.e., the value of the force as a function of the deflection will pass through its maximum and begin to diminish.

The stress-strain curve shown in Fig. 2-17*c* has a rising line of stable equilibrium until the critical value (critical stress) is reached, after which the strains begin to grow without any increase of the load. Further retaining of equilibrium is possible only upon reduction of the load, and it is this that characterizes the loss of stable equilibrium

$$\frac{d\sigma}{d\delta} = 0$$

Upon the loss of stability, equilibrium is disturbed between the external force factors and the internal forces which develop inside the section of the element, the latter having a certain possible curvature.

Generally with small and medium slenderness ratios of an element, the limiting state of equilibrium is reached in elastoplastic behaviour of the material. The main condition for the critical state, in which the load-carrying capacity of an eccentrically compressed element is exhausted, i.e., limiting equilibrium is reached, will be equality of the increments of the external moment ΔM_e and the internal moment ΔM_i

$$\Delta M_e = \Delta M_i$$

This condition for a critical state makes it possible to determine the critical forces with the aid of the graphoanalytical method, by plotting a curve of M_i against the deflection δ and drawing a straight line $M_e=F(e+\delta)$. The eccentricity e is laid off to the left of the origin of coordinates (Fig. 2-17*d*). Here is shown a curve of M_i plotted against the strain δ for a certain definite value of σ equal to σ_0. This curve can be considered as a characteristic of the section. It does not depend on the external load, but is determined only by the shape of the section, its moment of inertia, slenderness ratio and other geometrical characteristics connected with possible curvature of the element or its deflection. The curve of M_i in the region of elastic behaviour is concave (a slow growth of the deflection) and in the plastic region convex, and it has a maximum corresponding to the formation of a plastic hinge.

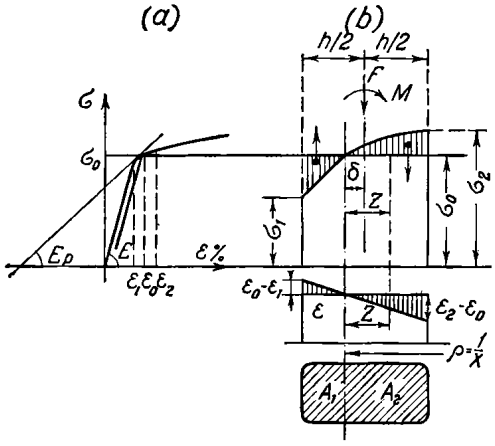


Fig. 2-18. Distribution of stresses upon loss of stability in eccentrically compressed element

The M_i curve is plotted with a number of assumptions facilitating this plotting. The difficulties consist in determining the position of the neutral line in the section of the element during its elastoplastic behaviour. This position depends on the tension diagram of the material and, accordingly, on the value of the modulus of elasticity in compression or tension. It also depends on the shape of the bent centre line and on that of the element cross section.

For each given value of the mean stress $\sigma_0=\frac{F}{A}$ beyond the proportional limit, the tension diagram can be used to find the modulus of plastic strains E_p by drawing a tangent to point $(\sigma_0,\ \varepsilon_0)$ (Fig. 2-18*a*). On the basis of the hypothesis of plane sections, the longitudinal strains ε_1 and ε_2 are directly proportional to the distance z from the neutral line, the curvature χ related by a differential expression to the deflection y being the proportionality coefficient

$$\varepsilon=\chi z;\ \chi=\frac{1}{\rho}=y''=\frac{P_y}{EI}$$

With definite curvature of the element and shape of its section the stresses σ_1 and σ_2 (Fig. 2-18*b*) can be found, while from the equality

of the stress diagrams in bending ($\int_{A_1}\sigma_1\,dA=\int_{A_2}\sigma_2\,dA$) the distance to the neutral line z will be determined. Hence the internal moment due to the bending which the curvature of the element corresponds to can be found from the expression

$$M_i=\int_{A_1}\sigma_1\,zdA+\int_{A_2}\sigma_2\,zdA=E\int_{A_1}\varepsilon_1 zdA+\int_{A_2}E_p\varepsilon_2 zdA$$

This internal moment characterizes the section. Let us lay off the eccentricity of the force F along the x-axis to the left of the origin of coordinates (Fig. 2-17*d*), and draw a line tangent to the curve M_i. This straight line will represent the external moment $M_e=F\,(e+\delta)$. At the meeting point the condition $\Delta M_e=\Delta M_i$ will be complied with and thus the values of M_{cr} and δ_{cr} will be determined that correspond to the respective M_i. Hence, the critical force will be found equal to

$$F_{cr}=\frac{M_{i\cdot cr}}{e+\delta_{cr}} \tag{2-33}$$

or, correspondingly, the critical stress

$$\sigma_{cr}=\frac{F_{cr}}{A} \tag{2-34}$$

It is obvious that the critical stresses of an eccentrically compressed element are less than those of an axially compressed one.

Indeed, by continuing the curve of the elastic line of the element (Fig. 2-17*a*) up to the line connecting the longitudinal forces, we shall obtain an imaginary axially compressed element with a length and a slenderness ratio greater than those of the eccentrically compressed element and, therefore, $\sigma_{ec.cr}<\sigma_{0cr}$.

A glance at Fig. 2-17*d* will show that the meeting point of the straight line M_e always lies to the left of the maximum point of the curve M_i which characterizes the plastic hinge. Thus it has been established that in the critical state of an eccentrically compressed pinned-end (statically determinate) element there will always be present an elastic core, which is mainly the part that resists bending. In statically indeterminate systems having elastic constraints the formation of a plastic hinge is possible. Redistribution of the forces or a change in the adopted design scheme of the structure will then take place.

Thus the critical stress of an eccentrically compressed element, while depending on δ_{cr}, does not depend on the external load, as shown by equation (2-33). It can be expressed as a function of the slenderness ratio of the element and the relative eccentricity con-

nected with bending

$$m = \frac{e}{\rho}$$

in which $e = \frac{M}{F}$ = eccentricity, cm

$\rho = \frac{N}{A}$ = core radius, cm.

The equation for the stress caused by eccentrical compression, the same as in eccentrical tension [see expression (2-26)] contains the

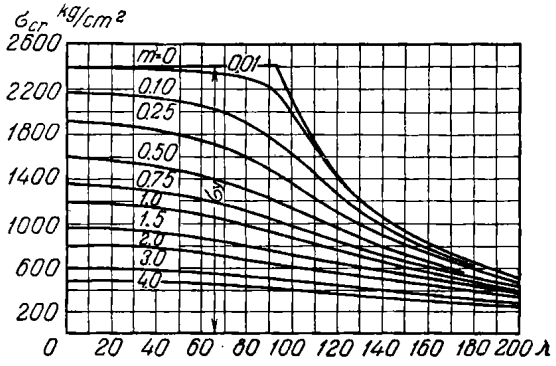


Fig. 2-19. Curves of critical stresses for eccentrically compressed element versus slenderness ratio (for ideal elastoplastic material)

relative eccentricity m (for compression the plus sign has been taken, as usual)

$$\sigma = \frac{F}{A} + \frac{M}{W} = \sigma_0 (1 + m)$$

Figure 2-19 shows theoretical curves of the critical stresses versus the slenderness ratio λ for eccentrically compressed elements having a rectangular shape, which makes possible a general solution in a finite form (Yezhek's solution). The curves are given for various values of the relative eccentricity m. The upper curve in Fig. 2-19 for $m = 0$ corresponds to axial compression and repeats the relevant curve given in Fig. 2-10. With sections other than a rectangular one the influence of the section shape is accounted for in the Building Standards and Regulations by the introduction of an equivalent eccentricity m_1. The latter is equal to the product of a coefficient η allowing for the shape of the section, and the relative eccentricity m, viz., $m_1 = \eta m$. The coefficient η takes into account the development of plastic strains in the element accompanying the

loss of stability. The values of the coefficient η are given in Table 9 of Appendix II.

In the case pictured in Fig. 2-20*a* the coefficient η is greater than unity, since yielding may rapidly propagate along the compressed flange and the element will change to a T section. When the influence of the development of plastic strains in an element is not very great (Fig. 2-20*b*) we have $\eta < 1$.

In such a section the plasticity, penetrating inside the section from its edges, only slightly weakens it.

In the same way as we obtained the buckling factor φ (see Sec. 2-4), let us designate the ratio of the critical stress of an eccentrically compressed element $\sigma_{ec\cdot cr}$ to the yield point σ_y by

$$\varphi_{ec} = \frac{\sigma_{ec\cdot cr}}{\sigma_y} \qquad (2\text{-}35)$$

whence $\sigma_{ec\cdot cr} = \varphi_{ec}\sigma_y$.

Thus the equation for checking the stability of an eccentrically compressed element can be written as follows

$$\sigma = \frac{F}{A} \leqslant \sigma_{ec\cdot cr} = \varphi_{ec}\sigma_y$$

or in the working form (taking $\sigma = R$)

$$\sigma = \frac{F}{\varphi_{ec}A} \leqslant R \qquad (2\text{-}36)$$

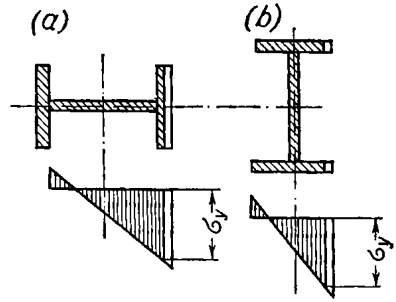


Fig. 2-20. Regions of propagation of plastic strains and weakening of sections in an I element:
a—eccentricity in plane of web; *b*—eccentricity perpendicular to web

where φ_{ec} is a factor that reduces the design strength for the given element to the value of the critical stress for an eccentrically compressed element.

The values of the factor φ_{ec} are given in Tables 7 and 8 of Appendix II against the slenderness ratio of the element λ and the equivalent eccentricity

$$m_1 = \eta m = \eta \frac{e}{\rho} = \eta e \frac{A}{W} \qquad (2\text{-}37)$$

Here $e = \frac{M}{F}$ = eccentricity of force in plane of bending, cm

M = design moment, which is equal to the maximum moment within the limits of the column length for columns of frame systems with a constant section, to the maximum moment over the length of the part with a constant section for stepped columns, to the moment in the constraint for cantilevers, to the values given in Table 2-6 for elements with pinned ends

F = longitudinal force in section under consideration

$\rho = \frac{W_{gr}}{A_{gr}}$ = core radius of section, cm

W_{gr} = section modulus for the most compressed fibre.

Ordinarily when selecting a section the value of ρ is first found from the equation

$$\rho = \frac{W}{A} = \frac{I}{zA} = \frac{r^2}{z} \tag{2-38}$$

where r = radius of gyration of the section (approximate values of r are given in Tables 7-2 and 8-3)

z = distance from neutral axis to farthest compressed fibre.

In Table 2-6 the moment M_1 is the maximum one within the limits of the middle third of the element length; in all cases M is taken not less than $0.5M_{max}$.

TABLE 2-6 **Design Moments M for Elements with Pinned Ends**

m	$0 \leqslant \lambda < 120$	$\lambda \geqslant 120$
$m \leqslant 3$	$M_2 = M_{max} - \frac{\lambda}{120} \times (M_{max} - M_1)$	M_1
$3 < m \leqslant 20$	$M_2 + \frac{m-3}{17} \times (M_{max} - M_2)$	$M_1 + \frac{m-3}{17} \times (M_{max} - M_1)$

With values of m_1 exceeding 4, in addition to the standard formula (2-36), eccentrically compressed elements may be analyzed by means of Yasinsky's binominal expression

$$\sigma = \frac{F}{\varphi A_{gr}} + \frac{M}{W} = \frac{F}{A_{qr}}\left(\frac{1}{\varphi} + \frac{e}{\rho}\right) \leqslant R \tag{2-39}$$

which does not take complete account of the development of plastic strains and entirely omits the influence of the section shape.

The inaccuracy of this expression is insignificant (from five to eight per cent).

With a large equivalent eccentricity ($m_1 > 20$) the influence of the longitudinal force is small and no checking of stability is required. The element is checked for strength in bending by equation (2-17).

Besides checking for stability in the plane of action of the moment, it is always required to check an eccentrically compressed element for stability at right angles to the plane of action of the moment (see Chapter 8).

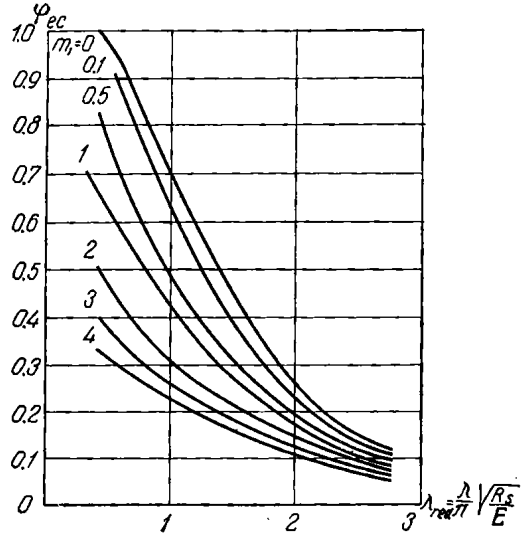


Fig. 2-21. Curves of buckling factors φ_{ec} for aluminium alloy elements

For eccentrically compressed elements made from aluminium alloys the buckling factor φ_{ec} is determined depending upon the equivalent eccentricity $m_1=\eta e\frac{A}{W}$ and the relative slenderness ratio λ_{rel}, the latter being computed from the equation

$$\lambda_{rel}=\frac{\lambda}{\pi}\sqrt{\frac{R_s}{E}}=f\lambda \qquad (2\text{-}40)$$

where $\lambda=\frac{L}{r}$ = slenderness ratio in the plane of action of the moment

$r=\sqrt{\frac{I}{A}}$ = radius of gyration in plane of action of moment

f = factor taken (with mean value of $E=710{,}000$ kg/cm^2) from Table 2-7.

The values of φ_{ec} depending upon m_1 and λ_{red} are given graphically in Fig. 2-21 (for elements with a rectangular section). The coefficient

η is taken according to Table 9 of Appendix II. With relative eccentricities of $m>10$ the factors φ_{ec}, regardless of the type of element section, are taken from Table 2-8.

TABLE 2-7 Values of Factor f

Grade of alloy	f	Grade of alloy	f
АМц-М	0.0093	АВ-Т1 and АМг61	0.0174
АМг-М and АД-31-Т	0.0108	Д1-Т	0.0182
АВ-Т, АД31-Т1 and АМц-П	0.0137	Д16-Т	0.0212
АМг-6 and АМг-П	0.0152	В92 (sheet)	0.017
АД33-Т1	0.0166	В92 (section)	0.0207

TABLE 2-8 Factors φ_{ec} for All Types of Aluminium Alloy Sections with High Eccentricities

m	λ_{red}						
	0.4	0.8	1.2	1.6	2.0	2.4	3.0
10	0.094	0.087	0.081	0.078	0.068	0.061	0.050
15	0.063	0.060	0.058	0.056	0.051	0.042	0.039

2-7. DESIGN STRENGTHS AND ALLOWABLE STRESSES

(1) Design Strengths for Steel Members

Steel members are analyzed in accordance with the Building Standards and Regulations, as indicated in Chapter 1, first of all for the load-carrying capacity of the member elements, i.e., for strength, stability and in some cases for endurance.

The strength and stability are analyzed for the action of the design loads (with the introduction of load factors). Here the design stresses must not exceed the design strengths of the material.

The values of the design strengths of rolled steel, as well as of carbon steel and grey iron castings (for cases when it is not necessary to allow for the factors of service conditions of the elements) established by the standards are set forth in Tables 2-9, 2-10 and 2-11.

TABLE 2-9 **Design Strengths R in kg/cm² for Rolled Steel Shapes**

Stress state	Symbol	Grade of rolled steel					
		Carbon		Low-alloy			
				14Г2, 15ГС		10Г2С, 10Г2СД, 15ХСНД	10ХСНД
		Steel 3, Steel 4	Steel 5	$t \leqslant 20$ mm	$t = 21$-32 mm		
Tension, compression and bending	R	2,100	2,300	2,900	2,800	2,900	3,400
Shear	R_{sh}	1,300	1,400	1,700	1,700	1,700	2,000
Crushing of end face (with matched surfaces)	$R_{cr \cdot e}$	3,200	3,400	4,300	4,200	4,300	5,100
Local crushing with tight contact	$R_{cr \cdot l}$	1,600	1,700	2,200	2,100	2,200	2,500
Diametrical compression of rollers with free contact (in members with restricted movement)	$R_{c \cdot r}$	80	90	110	110	110	130

NOTE. The values of the design strengths indicated in Table 2-9 have been established:
(a) for rolled steel shapes of ordinary quality:
strips, angles, round and square sections with a thickness t up to 100 mm inclusive; I beams and channels with a thickness t up to 20 mm inclusive (in these sections the thickness is measured over the web); steel sheets, plates and wide strips with a thickness t up to 40 mm inclusive;
(b) for rolled low-alloy steel shapes, (strips, angles, round and square sections, I beams, channels, sheets, plates and wide strips) with a thickness from 4 to 32 mm inclusive.

TABLE 2-10 **Design Strengths R in kg/cm² for Carbon Steel Castings**

Stress state	Symbol	Grades of carbon steel castings	
		15Л	35Л
Tension, compression and bending	R	1,500	2,100
Shear	R_{sh}	900	1,300
Crushing of end face	$R_{cr \cdot e}$	2,300	3,200
Local crushing with tight contact	$R_{cr \cdot l}$	1,100	1,600
Diametrical compression of rollers with free contact (in members with restricted movement)	$R_{c \cdot r}$	60	80

TABLE 2-11 **Design Strengths R in kg/cm² for Grey Iron Castings**

Stress state	Symbol	Grades of grey iron castings		
		СЧ 12-28, СЧ 15-32	СЧ 18-36, СЧ 21-40	СЧ 24-44, СЧ 28-48
Axial compression, with bending, if any	R_c	1,600	1,800	2,100
Tension with bending	R_b	450	600	800
Shear	R_{sh}	350	450	600
Crushing of end face	$R_{cr\cdot e}$	2,400	2,700	3,200

(2) Taking Account of Variable Load Influence When Analyzing Endurance of Steel Members

Steel members that are directly subjected to repeated moving loads in buildings and structures with heavy conditions of work [1] (for example, the crane girders of steel foundries) as well as members that are directly subjected to a regular vibrating load, must be analyzed for endurance.

The *endurance* of members subjected to variable loads (Sec. 2-2, para. 4, c, e, f) depends to a considerable degree upon the shape of the member and the related uneven distribution of the force lines causing a concentration of stresses which facilitates brittle failure.

When investigating the endurance of steel members the design strengths are reduced by introducing a factor γ determined from the expression

$$\gamma = \frac{1}{(ak_e \pm b) - (ak_e \mp b)k_{rs}} \qquad (2\text{-}41)$$

Here a and b = coefficients taken from Table 2-12

k_e = effective stress concentration factor, which is the ratio of the fatigue limit for a smooth flat specimen to that of a specimen with the given stress concentration (the values of k_e are given in Table 2-13)

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[1] Buildings and structures with heavy conditions of work include shops and plants with continuous round-the-clock operation of cranes with very heavy and very heavy continuous duties, making it necessary to repair the cranes and the crane tracks without interrupting production. For a more detailed exposition see Chapter 9.

$k_{rs}=\frac{\sigma_{min}}{\sigma_{max}}$ = range ratio of stress cycle, where σ_{min} and σ_{max} are the minimum and maximum absolute values of the stresses in the element being investigated, measured (each with its own sign) from the design load level, without using the dynamic load factor, nor the buckling factors characterizing the stable state of the elements (φ, φ_{ec} and φ_b) and the factors k_{br} contained in Table 6-11.

TABLE 2-12 **Values of Coefficients *a* and *b***

Material of member	Kind of structural member			
	Crane girders and trusses, as well as beams of working platforms and elements of bin and unloading trestle members		Members subjected to action of total continuously applied alternating design load (for example beams supporting an electric motor)	
	a	*b*	*a*	*b*
Carbon steel	0.75	0.3	0.90	0.3
Low-alloy steel	0.80	0.3	0.95	0.3

The upper signs in the denominator of expression (2-41) are taken for cases when the stress with the highest absolute value is tension, and the lower signs—when this stress is compression.

In all instances the value of the factor γ is not taken greater than unity.

The elements of members are checked for stability and endurance separately. The endurance is investigated for the action of the design loads (without regard to the load factors). Thus the working expressions for checking the endurance will have the following form

$$\sigma=\frac{F_s}{A}\leqslant\gamma R \tag{2-42}$$

or

$$\sigma=\frac{M_s}{W}\leqslant\gamma R \tag{2-43}$$

in which F_s and M_s are the force and moment, respectively, determined from the service loads without applying any load factors.

For elements subjected only to alternating compressive loads the factor γ is taken equal to unity.

TABLE 2-13 **Values of Effective Stress Concentration Factors k_e for Base Metal**

Cross sections being analyzed	Values of factor k_e for carbon steel	Values of factor k_e for low-alloy steel
Base metal with rolled, cut off or machined edges	1.0	1.0
Ditto with edges processed by gas cutting:		
(a) machine	1.1	1.2
(b) hand	1.4	1.8
Base metal at unmachined butt weld	1.4	1.8
Ditto, but with machining of weld:		
(a) when joining sheets of identical thickness and width	1.0	1.0
(b) when joining sheets of various width	1.2	1.4
(c) when joining sheets of various thickness	1.3	1.6
Base metal at place of transition to transverse lap fillet weld with a leg ratio of 1:1.5	2.7	3.7
Base metal in lap joint at ends of longitudinal fillet weld	3.4	4.4
Base metal near membranes and webs connected by means of fillet welds to tension flanges of beam and elements of trusses:		
(a) with hand welding	1.6	2.2
(b) with semi-automatic welding	1.3	1.5
Base metal at connections to constructive elements following them:		
(a) rectangular gusset plates welded to elements by means of butt or T joints, without machining	2.5	3.5
(b) gusset plates with smooth curvilinear form and machining of butt or T welded joints to element	1.2	1.4
(c) gusset plates connected to element by means of lap joints with welding along contour, without machining of joints	2.5	3.5
Base metal at cross section:		
(a) of connection with double-shear rivets or bolts	1.6	1.9
(b) ditto, with use of high-strength bolts	1.2	1.4
(c) of connection with single-shear rivets or bolts	2.0	2.4

(3) Allowable Stresses for Steel Members

When employing the method of elastic design (allowable stresses) for analysis, as indicated in Chapter 1, the stresses in a member are determined according to the forces computed for the service loads (without applying the load factors). These stresses must not exceed the allowable ones established for this method by the relevant standards and regulations for the design of steel structural members, and indicated in Table 2-14.

TABLE 2-14 **Allowable Stresses for Rolled Steel**

Stress state	Rolled steel of grades					
	Ст.0 and Ст.2		Ст.3		Low-alloy (НЛ)	
	Allowable stresses in kg/cm² for					
	Main loads	Main and additional loads	Main loads	Main and additional loads	Main loads	Main and additional loads
Tension, compression and bending	1,400	1,600	1,600	1,800	2,250	2,600
Shear (in bending)	900	1,000	1,000	1,100	1,350	1,550
Crushing of end face	2,100	2,400	2,400	2,700	3,400	3,900
Local crushing with tight contact	1,100	1,300	1,300	1,450	1,800	2,050
Diametrical compression of rollers with free contact	50	60	60	70	85	100

(4) Design Strengths for Aluminium Alloy Members

The design strengths for structural members of aluminium alloys established by USSR building standards are given in Tables 2-15 and 2-16.

TABLE 2-15 **Principal Design Strengths *R* (Tensile, Compressive and Bending) for Alloys not Hardened by Heat Treatment, kg/cm²**

Values of *R* for grades and state of alloys					
АМц-М	АМц-П	АМг-М	АМг-П	АМг6-М	АМг61-М
500	1,000	700	1,400	1,400	1,800

NOTE. For the designation of the alloys see page 35.

TABLE 2-16 **Principal Design Strengths R (Tensile, Compressive and Bending) for Alloys Hardened by Heat Treatment, kg/cm²**

Kinds of rolled stock	Values of R for grades and state of alloys								
	АВ-М	АВ-Т	АВ-Т1	Д1-Т	Д16-Т	АД31-Т	АД31-Т1	АД33-Т1	В92-Т
Sheets with thickness:									
$t \leqslant 5$ mm	700	1,100	1,700	1,600	2,500[1]	850	1,200	1,600	1,700
$t = 5.1$ to 10	700	1,000	1,700	1,600	2,500[1]	850	1,200	1,600	1,700
Plates:									
$t = 10.1$ to 25 mm	700	1,100	1,700	1,800	2,400[1]	—	—	—	—
$t = 26$ to 40	700	900	1,600	1,800	2,400[1]	—	—	—	—
Sections:									
$t \leqslant 10$ mm	700	1,000	1,700	1,850[2]	2,500[2]	850	1,200	1,600	2,500
$t = 10.1$ to 20	700	1,000	1,700	1,950[2]	2,500[2]	850	1,200	1,600	2,500
$t = 20.1$ to 40	700	1,000	1,700	2,100[2]	2,700[2]	—	—	—	—
$t > 40$	700	1,000	1,700	2,100[2]	2,900[2]	—	—	—	—
Bars with diameter:									
$d \leqslant 22$	—	—	1,700	1,850	2,200	850	1,200	1,600	—
$d > 22$	—	—	1,700	1,850	2,300	—	—	—	—
Tubes with diameter:									
$d \leqslant 120$ mm	700	1,150	1,750	1,700	2,200	850	1,200	1,600	2,500[3]
$d > 120$	700	1,150	1,750	1,850	2,300	—	—	—	—

[1] The sheets and plates are not plated. For plated sheets and plates the design strengths are reduced by 5%.
[2] Mechanical properties in a longitudinal direction.
[3] Pressed tubes.

The design shear and crushing strengths of aluminium alloys are determined by multiplying the principal design strengths by a correction factor taken from Table 2-17.

TABLE 2-17 **Correction Factors for Transition from Principal Design Strengths R to Shear and Crushing Strengths of Aluminium Alloys**

Stress state	Correction factor
Shear	0.6
Crushing of end face (with matched surfaces). .	1.5
Local crushing with tight contact	0.75

CHAPTER THREE

Standard classification of rolled sections

Steel is employed in construction in the form of rolled stock procured from iron and steel works and having various sections to meet construction requirements. There is distinguished sheet steel and section steel.

Many years of experience in the employment of various shapes have evolved products that give the best results from the viewpoint of erection and economy. These shapes are fit for different types of members in the diverse conditions of erection. They include angles, I sections and channels. These three shapes together with sheets, plates and round bars are the main ones used in construction.

Catalogues are published by iron and steel works showing the standard structural steel products available.

3-1. PRINCIPAL SECTIONS AND THEIR APPLICATIONS

(1) Angles

Two kinds of angles are in use, namely, *equal-leg* or *equal angles* (GOST 8509-57) and *unequal-leg* or *unequal angles* (GOST 8510-57) with a ratio of their legs of about 1 to 1.6. Angles are mainly put to the following uses in members:

(a) connecting elements joining together sheets or plates, or forming a built-up riveted section (Fig. 3-1*a*);

(b) independent sections consisting of one or two angles (Fig. 3-1*b*) designed for resisting axial forces (tension or compression);

(c) construction elements for connecting beams to columns (Fig. 3-1*c*), purlins to the chords of trusses (Fig. 3-1*d*), in the capacity of beam seats, stiffening ribs, etc.

The great variety of applications of angles has resulted in a considerable development of their assortment, from the smallest and lightest sections with leg sizes of 20×20 mm and a thickness of 3 mm, designated 20×3, with a cross-sectional area of 1.13 cm^2, to the largest and heaviest section 250×30 in size with an area of 142 cm^2.

To facilitate designing, connection and splicing of angles their legs have parallel faces.

The section of an angle should be selected with a view to the general design of the element section in which it is used.

In those cases when angles are to be subjected to axial forces as an independent section forming part of a truss or strutted member, it will be a good plan to select the thinnest angles, seeing that the comparative measure of the fitness of a compression element is its radius

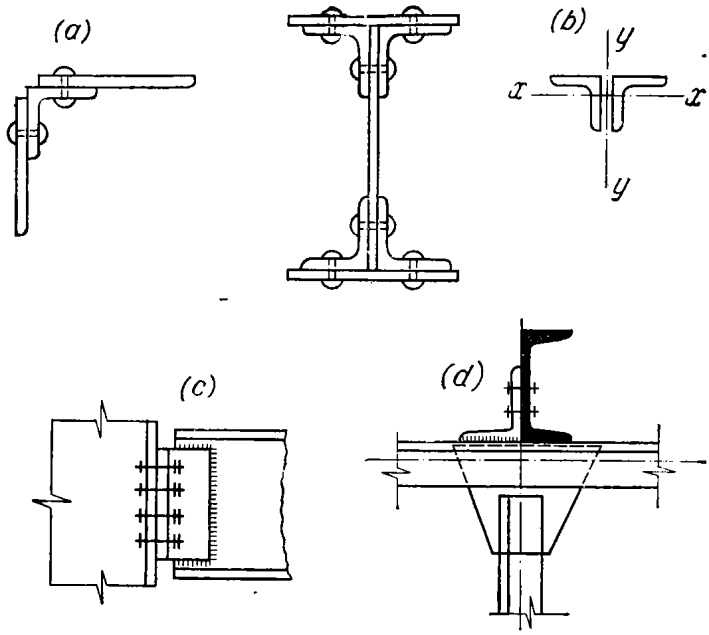


Fig. 3-1. Some applications of angles

of gyration $r = \sqrt{\frac{I}{A}}$. The latter affects the buckling factor φ which, in turn, reduces the design strength. The greater the value of r, the more advantageous will be the section, and it must not be forgotten that the radius of gyration of thin angles is always greater than that of their thicker counterparts. When employing thin angles, local bulging of the legs should not be feared, since the maximum ratio of the width of a leg to its thickness (the slenderness ratio of the leg) used in standard sections, i.e., $\frac{b}{t} \leqslant 15\text{-}17$, will always ensure local stability of the legs. For the tension elements of welded members the thickness of the angles is less important, but in this instance thin

angles are also generally used because of their high stiffness, which simplifies transportation and erection.

In those cases when angles are employed as construction elements (for example as stiffeners in beams or as anchoring parts) that are not subjected to high stresses, use should also be made of thin angles. If the legs of the angle parts, however, are to be subjected to outward bending (beam seats, short members for monorail suspension, etc.), then thick angles should be the rule.

The angles 50×5 and $63\times40\times5$ are taken as the minimum sections for the main elements of load-carrying steel members.

The ordered (marketed) length of angles, depending upon the conditions of rolling and transportation, is taken equal to 6-9 metres for small sections and to 9-12 metres for their large counterparts. This length determines the location of the joints or splices.

(2) I beams

I beams (GOST 8239-56*) are mainly employed to resist bending, and this determines their configuration (Fig. 3-2*a*). I-shaped elements are also used in the built-up sections of columns (Fig. 3-2*b* and *c*).

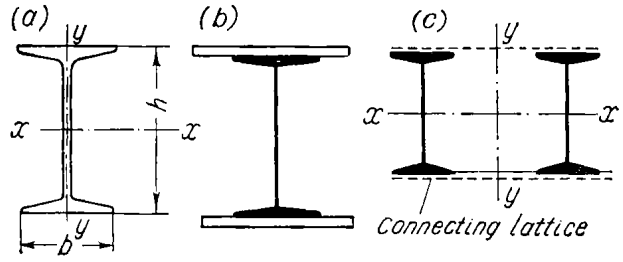


Fig. 3-2. Some applications of I beams

Standard I beams produced in the USSR are designated by numbers ranging from 10 to 70. The number of the I beam corresponds to its depth expressed in centimetres. Beginning with No. 18 and up to No. 30 inclusively, I beams are rolled with different widths and thicknesses of their flanges, the web remaining the same. I beams with thicker and wider flanges are distinguished by adding the letter "a" to their number. It will be more advantageous to use these shapes in beams subjected to bending.

When employing I shapes in a member it should be borne in mind that rolled sections may have deviations from the established dimensions. The allowances in depth are $\pm$ 3 mm for I shapes from No. 20 to No. 36 and $\pm$ 4 mm for No. 40 and above. The I beams have a

12% slope on the inside faces of the flanges, which makes it difficult to connect other elements to them on these faces.

I beams can be ordered in lengths of 6, 9 and 12 metres, but upon agreement with the manufacturers I beams from No. 30 can be delivered with a length of 15 metres and from No. 45 and above with a length of 15-18 metres.

(3) Channels

A channel (GOST 8240-56*) differs from an I beam in the web being shifted to the edge of the flanges, which is convenient for connection to the webs of other elements (Fig. 3-3*a*). Channels can be used as

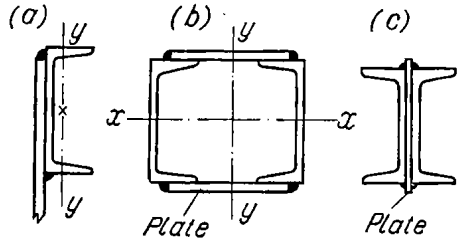


Fig. 3-3. Some applications of channels

elements resisting bending and that is why they are used, for instance, as purlins in the roofings of industrial buildings. Owing to the absence of symmetry of the section with respect to the vertical axis *y-y*, a channel, as a rule, is subjected to torsion and additional stresses appear in it.

Channels are widely employed in members subjected to axial forces in the form of built-up sections of two elements connected by batten plates or lattices, for example, in columns (Fig. 3-3*b*), the chords of trusses (Fig. 3-3*c*), and so on.

The standard channels produced in the USSR are designated by numbers ranging from 5 to 40.

Channels No. 14-24 are rolled with different thicknesses and widths of their flanges, the web remaining the same. The letter "a" is used to designate the channels with the larger flanges. The allowances in depth are ± 3 mm for channels from No. 20 to No. 30 and $\pm$ 3.5 mm for No. 33 and above.

The channels have a slope of 1 in 10 on the inside faces of the flanges. These shapes can be ordered in lengths of 6, 9 and 12 metres, while upon special agreement with the manufacturer this length can be increased to 18 metres.

(4) Steel Sheets and Plates

Sheets and plates are widely employed in construction, often constituting from 40 to 60% of the total weight of a structure, and about 100% in special plate structures—shells. The cause of such widespread employment of sheets and plates even in framework or skeleton systems is the unlimited possibility of producing any sections of the required dimensions, thickness and configuration by welding or riveting together the separate plates.

Sheet steel is classified as follows:

1. *Hot-rolled steel sheets* (GOST 3680-57*) having a thickness ranging from 0.5 to 4.05 mm, a width from 600 to 1,000-1,400 mm (depending upon the thickness), a length from 1.2 to 2, 2.5 and 4 metres. They are used in construction for stamped sections and roofing.

2. *Steel strips* (GOST 103-57*) having a thickness varying from 4 to 60 mm and a width from 12 to 200 mm with 2 mm gradations. It is good policy to use this steel only when a large quantity is to be ordered.

3. *General purpose steel plates* (GOST 82-57*) having smooth and even edges. The plates have a width ranging from 160 to 1,050 mm with 10-50 mm gradations, their thickness is 4 to 60 mm with 2-4 mm gradations beginning from a thickness of 12 mm. The usual lengths ordered vary from 6 to 12 metres. Owing to the general purpose steel plates having even edges, their employment will be very rational.

4. *Steel plates* (GOST 5681-57) having a thickness ranging from 4 to 160 mm with 0.5 mm gradations for thicknesses from 4 to 6 mm, gradations of 1 mm for thicknesses from 7 to 30 mm and 2 mm gradations for thicknesses from 31 to 40 mm. The width of steel plates varies from 600 to 2,500 and even to 3,000 mm with 100 mm gradations. It should be remembered that agreement must be reached with the manufacturer before ordering plates over 1,800 mm wide. The length of the plates, depending on their thickness, is 4 to 8 metres, and up to 12 metres upon agreement with the manufacturer.

3-2. VARIOUS SECTIONS EMPLOYED IN STRUCTURES

(1) Available Sections

Besides the principal sections listed above, a variety of other sections and shapes are employed in structures, namely:

1. *Square steel bars* (GOST 2591-57*) with straight and bevelled edges and with sides ranging from 40 to 250 mm.

2. *Round steel bars* (GOST 2590-57*) with a diameter from 9.5 to 250 mm, employed for the elements of braces, stay bolts, etc., as well as for the reinforcement of concrete members.

3. *High-strength carbon steel wire* for prestressed members (GOST 7348-55*) with a diameter from 2.5 to 10 mm; *rope wire*, bare and galvanized (GOST 7372-55) with a diameter from 0.2 to 5 mm, and so forth.

4. *Seamless steel thick-walled tubes* (GOST 8732-58) with a diameter and thickness from 42×2.5 up to 377×20 and 500×15 mm.

5. *Crane rails* (GOST 4121-62) for crane tracks. The nominal width of the head of the rail in millimetres determines the number of the rail (from KP70 to KP140). The length of these rails ranges from 9 to 12 metres.

6. *Fluted steel sheets* (GOST 8568-57) are used for the landings and treads of staircases. The thickness of the sheets (including the flute) is 6, 8 and 10 mm, the width is 600-1,400 mm and lengths of up to 6 metres can be ordered.

Expanded steel sheets (GOST 8706-58) are also used for flooring where the accumulation of dust is possible. The thickness of the blank is 4-6 mm (overall thickness 13-21 mm, width 500-1,400 mm, length up to 6 metres).

7. *Corrugated sheet steel* (GOST 3685-47) is employed as cold roofing in hot shops. The thickness of this steel ranges from 1 to 1.75 mm, the width from 700 to 1,000 mm, the length is 2,000 mm and the height of the waves or corrugations is 30 and 35 mm.

8. *Special sections* of metal sashes for glazing industrial and certain civil buildings (GOST 7511-58).

In designing structures the trend should be to use the minimum number of various section numbers without permitting however, a noticeable increase in the weight of the members.

(2) Further Development of Rolled Structural Sections

Notwithstanding the relatively large list of standard rolled sections, the iron and steel industry does not always meet the requirements of construction. Hence it becomes necessary to weld or rivet shapes not available on the market from plates and other sections, which increases the cost of members. To meet more completely the requirements of construction, the following sections have been planned for production, some of them already being made:

1. Wide flange (*WF*) beams (Fig. 3-4*a*) with a depth up to 1,000 mm and a ratio between the width of the flanges and the depth of the beam ranging from 1 : 1 to 1 : 2.5 (GOST 6183-52). There has been organized the specialized production of welded wide flange beams.

2. Thin-web I beams (Fig. 3-4*b*) for light girder members resisting bending, with a minimum web thickness (GOST 6184-52). Such sections show an economy over I beams of the normal types of 30-35%.

3. T beams (Fig. 3-4*c*) with a long or short web and wide flanges, which can be employed in the capacity of independent elements of truss chords, in the form of an H shape with one weld (Fig. 3-4*d*) or as part of a welded I shape with a large depth having a plate inserted between two T beams (Fig. 3-4*e*).

4. Sections made of plates or sheets manufactured by bending on special bending machines or by cold rolling (Fig. 3-4*f*). It will be good practice to use these sections in the elements of light members,

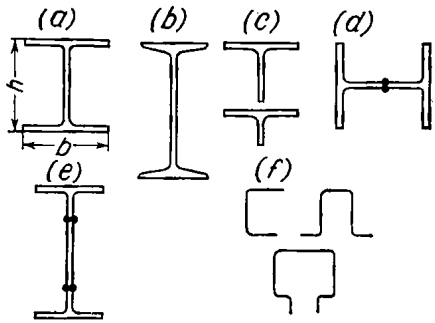


Fig. 3-4. Development of rolled structural shapes

as well as in members that withstand insignificant loads, but are required to have a high stiffness. Such sections can have a great variety of shapes (GOST 8275-57 to 8283-57).

3-3. ALUMINIUM ALLOY SECTIONS

Deformable aluminium alloys in the form of various sections or sheets are the main ones used in construction.

The sections are manufactured either by extrusion, or by bending or stamping rolled sheets.

Extrusion is performed in a hot state (about 400 deg C) in hydraulic presses, in which a round ingot (with maximum dimensions of $d \times L = 345 \times 1{,}450$ mm) is extruded through a die in which a hole of the required section has been made (Fig. 3-5). In view of the size of the ingot indicated above, it should be possible to inscribe the cross section of the shape in a circle with a diameter of 320 mm, and in some instances, when a powerful press is being used, in a circle with a diameter of 530 mm. When a set of dies with different configurations of the holes is available, a great variety of extruded sections can be obtained. The relatively small amount of aluminium members used does not make it possible to establish a catalogue of the most widely

employed sections. Owing to the low modulus of elasticity of aluminium alloys and the small critical ratio between the width of the free overhang of angle legs and their thickness $\frac{b}{t} \leqslant 7\text{-}12$), angle shapes have been proposed with bulbs on the ends of the legs (Fig. 3-6*a*).

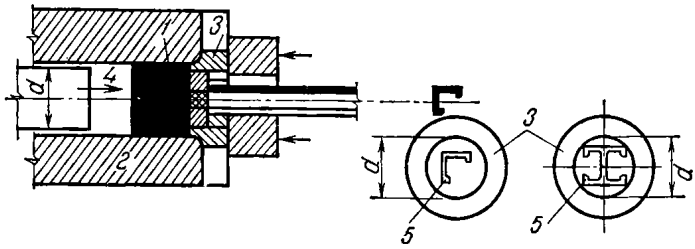


Fig. 3-5. Extrusion of aluminium section:
1—aluminium blank; *2*—cylinder; *3*—die; *4*—punch; *5*—holes in die

This increases the $\frac{b}{t}$ ratio to 10-20 depending upon the shape of the section. The sections can have closed shapes, for example in the form of tubes. An example of an extruded section for the chords of power transmission line masts is shown in Fig. 3-6*b*.

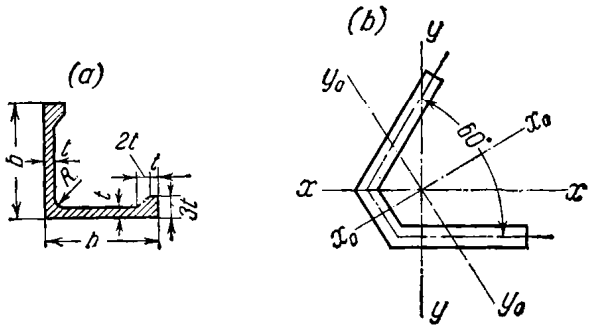


Fig. 3-6. Aluminium sections:
a—angle with bulbs at ends of legs; *b*—section for power transmission line masts

USSR State Standards have been established for a number of small shapes with sectional dimensions up to 120 mm (GOST 8110-56 to 8114-56) and for pipes (GOST 1947-56).

After extrusion the sections are subjected to heat treatment such as hardening, ageing, annealing, etc.

Bent sections are manufactured mainly from thin sheets and strips up to 4 mm thick. The sheets and plates are rolled in a cold (with a

thickness up to 5 mm) or a hot (with a thickness up to 80 mm) state with a width up to two metres and a length up to seven metres (GOST 1946-50). Sheets and plates of duralumin type aluminium alloys to be used in structural members are plated, i.e., coated with a thin layer of pure aluminium to prevent corrosion. Sheets and plates of alloys that are not strengthened by heat treatment, of alloys AB and B92, as well as sections made from alloys of all grades, may be used without plating.

Sheets employed for fencing, guards or for decorative purposes may be given various colours with inorganic pigments by anodizing, which involves a number of electrochemical processes of treating the surface of the sheet.

CHAPTER FOUR

Welded connections

4-1. WELDING PROCESSES

Electric welding is the most widespread method of connecting the elements of steel members. There are distinguished manual, automatic and semi-automatic arc and gas-shielded arc welding.

(1) Manual Arc Welding

Arc welding is based on the phenomenon of an electric arc bridging the gap between a steel rod (electrode) and the steel parts being welded when they are connected to a source of current as pictured in Fig. 4-1*a*. This arc melts and fuses the parent or base metal and the metal of the electrode and forms a weld that connects the original parts into one piece (Fig. 4-1*b*).

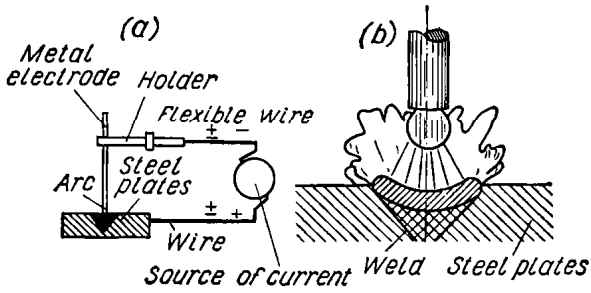


Fig. 4-1. Schematic view of arc welding process and welding arc

Welding with a bare electrode and an unshielded arc leads to the formation of low-quality welds, since oxygen and nitrogen get into the weld metal and form oxides and nitrides. This makes the weld brittle. In order to ensure the formation of a shielding gas medium and to improve ionization of the air gap use is made of coated electrodes. Part of the coating forms a flux on the molten metal that deoxidizes and purifies the latter. A slag crust is also formed that protects the hot surface of the weld against contact with the air and prevents oxidation. Initially the coatings of electrodes consisted of

chalk dissolved in water glass (lime- or wash-coating) which was applied to the electrodes in a thin layer (thin- or lightly-coated electrodes). The quality of the welds, however, remained low. At present special high-quality coatings are in use that are applied to the welding wire (heavy-coated electrodes). These coatings contain various alloying additions that improve the mechanical properties of the welds.

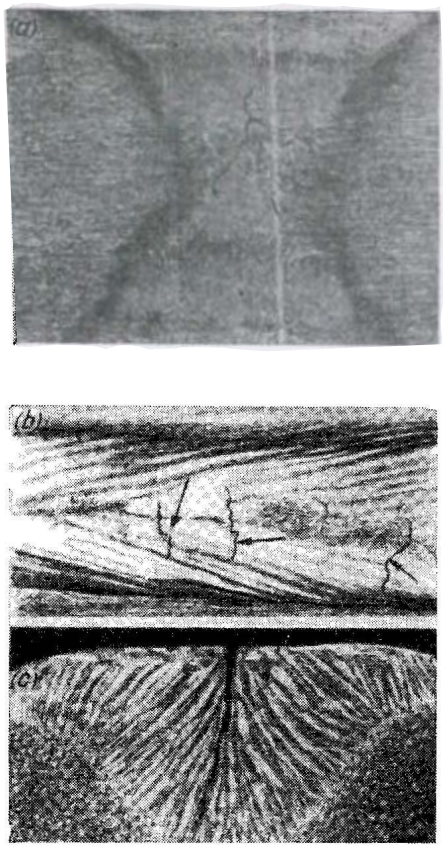


Fig. 4-2. Cracks in welds

The amount of carbon in the metal has a great influence on the weldability of structural steels. For welding steels containing over 0.22% carbon, electrodes with special coatings should be used. Ordinary heavy-coated electrodes can be employed for welding the common grades of structural steel having a carbon content of less than 0.22% (see Table 4-1).

Owing to the great variety of high-quality coatings, the electrodes are distinguished not by the composition of the coatings, but by the results of mechanical tests of the deposited metal and the welded connection. These results must not be less than the values indicated in Table 4-1.

The main types of electrodes employed in construction are as follows: type Э42 for welding members made from steel 3 and type Э42A for heavy-duty members; types Э50A and Э55, which ensure a high-quality weld, for welding members of low-alloy steel. The use of type Э34 electrodes in elements and parts whose strength must be checked in designing is excluded because of the low plasticity and brittleness of the weld.

The strength of welded connections is appreciably affected by the structure of the weld, as well as by nonmetallic inclusions (entrapped slag or fine gas pockets that are formed when the weld cools). The presence of internal micropores leads to a dimensional concentration

TABLE 4-1 **Mechanical Properties of Specimens of Welded Connections and Weld Metal**

Welding method	Kind of electrode	Type of electrode (GOST 9467-60)	Butt weld		Specimens of weld metal			Grades of coatings or flux used	Recommended wire, to GOST 2246-60	Main use of electrode in welding members
			Ultimate tensile strength, kg/mm²	Bend angle, deg	Ultimate tensile strength, kg/mm²	Notch toughness, kg-m/cm²	Unit elongation of short specimens, %			
Manual welding	Thin-coated	Э34	34	30	30	—	6	Lime	—	Steel 0-steel 3 for welds not to be checked for strength
	Heavy-coated	Э42	42	120	42	8	18	ОММ5, ЦМ-7, МЭЗ-04, СМ-5, МР-3	Св-08	Steel 3
		Э42А	42	180	42	14	22	Уони-13/45, УП-1/45, УП-2/45, СМ-11, МР-3	Св-08А	Steel 3 in heavy-duty members
		Э50А	50	150	50	13	20	Уони-13/55, УП-1/55, УП-2/55, ДСК-50	—	Steel 4,5 and НЛ (low-alloy)
		Э55	55	140	55	12	20	ЦЛ-5, К-2, К-5	—	Steel 10ХСНД
Automatic submerged arc welding	—	—	42	120	42	8	18	АН-348А, ОСЦ-45	Св-08ГА	Steel 3-steel 5 and НЛ

of stresses, thus increasing the brittleness of the weld. Figure
illustrates a microsection of a weld where its structure can be
as well as a small crack at the root of the weld connecting two s
slag inclusions. No cracks are tolerated inside a weld.

There are distinguished hot (Fig. 4-2*b* and *c*) and cold cracks.

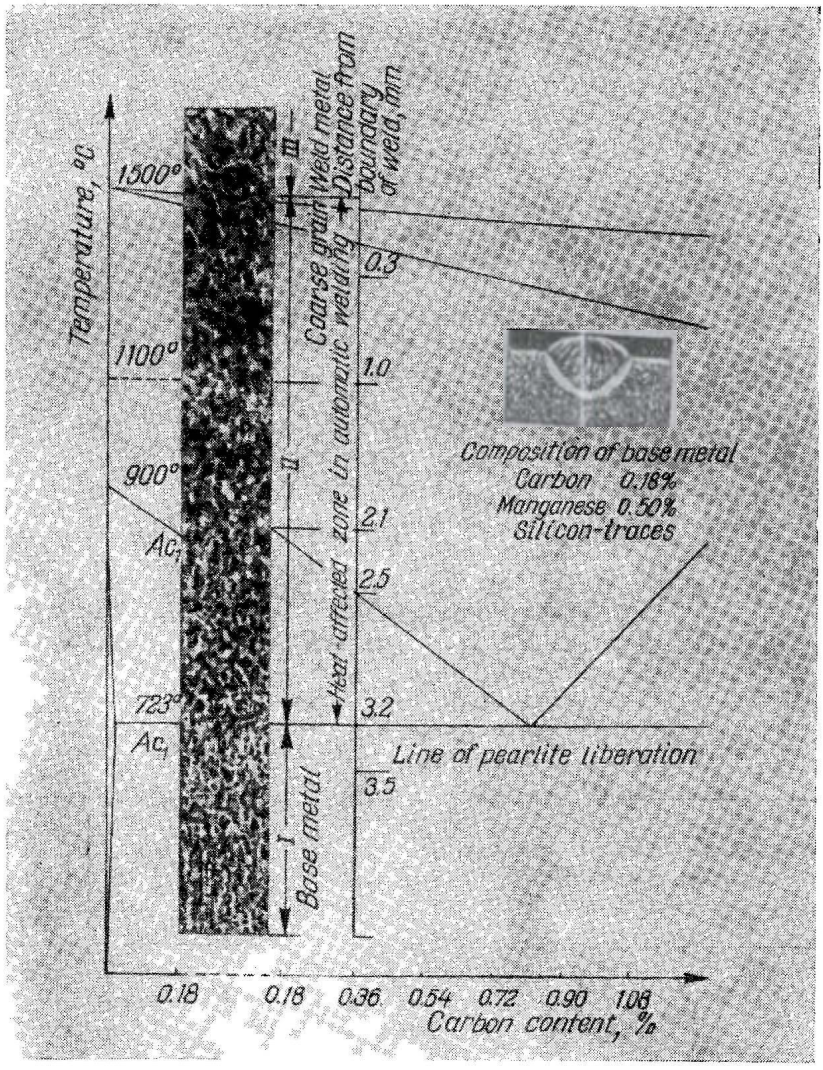


Fig. 4-3. Structure of weld

Hot cracks sometimes appear during cooling of the weld within a temperature range of 1,000 to 1,350 deg C. Cold cracks in the majority of cases are the result of tensile stresses induced in the welds by shrinkage.

Figure 4-3 shows the structure of a weld and the temperatures encountered over its cross section. Here three zones are distinguished, namely, *I*—parent or base metal zone, *II*—transition zone (also known as dilution or fusion zone) and *III*—weld-metal (deposited-metal) zone.

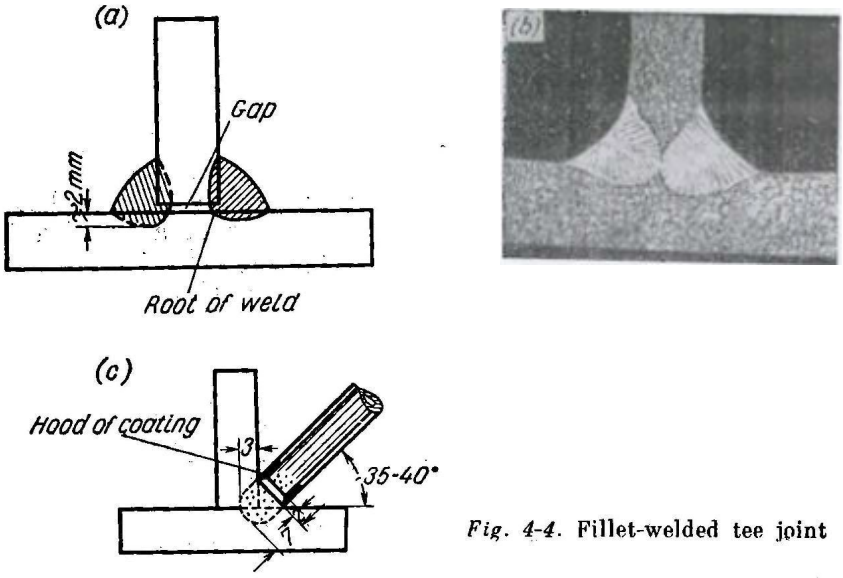


Fig. 4-4. Fillet-welded tee joint

The *base metal zone* is that part of the base metal around the weld, heated at most to the critical temperature (723 deg C), wherein the metal retains its mechanical properties.

The *fusion* or *heat-affected zone* is located between the base and the weld metal. During welding a sharp change of temperature is observed in this zone, from 1,500 deg (melting point) to 723 deg C. The structure of the metal in this zone is not uniform. In the part with a temperature exceeding 1,000-1,100 deg the crystals grow in size, forming a coarse-grain structure with poorer mechanical properties. The fusion zone is the weakest part of the weld.

In welding *penetration* of the weld metal into the base metal takes place, and the deeper this penetration, the higher the quality of the weld. The usual depth of penetration is 1.5-2 mm. Of special importance is deep penetration at the root of fillet welds in members taking alternating loads.

The existence of a gap caused by an unfinished plate edge creates a sharp change in shape that produces stress concentration and the formation of the most minute cracks. They gradually develop under an alternating load, and may be the cause of failure of the weld. Such a connection is illustrated in Fig. 4-4*a*.

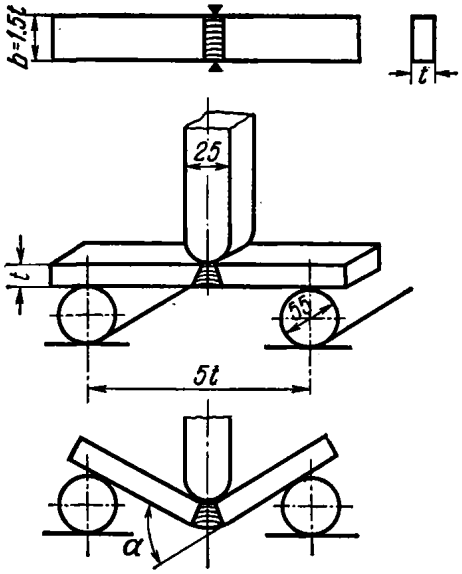


Fig. 4-5. Bending test of weld metal

Deep penetration is achieved either by means of automatic welding (Fig. 4-4*b*), or by employing the so-called method of welding with "deep penetration" ("short arc"), the essence of which will be clear upon glancing at Fig. 4-4*c*.

Besides tensile tests, the weldability and strength of a welded connection are checked by means of a bending test (Fig. 4-5). This test characterizes the static ductility of the connection, which is proportional to the angle of bending, and is a reliable and simple way of simultaneously determining the strength and the plasticity of a weld.

(2) Automatic and Semi-Automatic Welding

Automatic submerged or hidden arc welding is based on the principle of automatically feeding an electrode or welding wire into the welding zone with continuous unrolling of a reel of special welding wire (Fig. 4-6). The welding wire is fed by an automatic head that performs the functions of the welder's hand in manual welding. Instead of the coating, use is made of a loose material having a definite chemical composition (flux), which is spilled over the end of the electrode. The flux completely isolates the welding zone from the air, since it hides the arc. The result is a homogeneous compact weld with deep penetration that possesses high mechanical properties (see Table 4-1).

Owing to the heavy current used in automatic welding (600 to 3,000 amperes) the output is from three to five times (and sometimes even scores of times) greater than that obtained in manual welding. When designing members that are to be connected by automatic welding, such layouts should be employed that will not hinder the movement of the welding head. Automatic welding can be per-

formed on a stationary installation (Fig. 4-7*a*) or with the aid of welding tractors (Fig. 4-7*b*). This method is generally used to make underhand welds (Fig. 4-8), employing revolving jigs (Fig. 4-9) for the purpose.

To weld elements that are over 24 mm thick a method termed *electroslag welding* (Fig. 4-10) is employed, which can be used to weld parts with a great thickness (up to 200-300 mm). The current

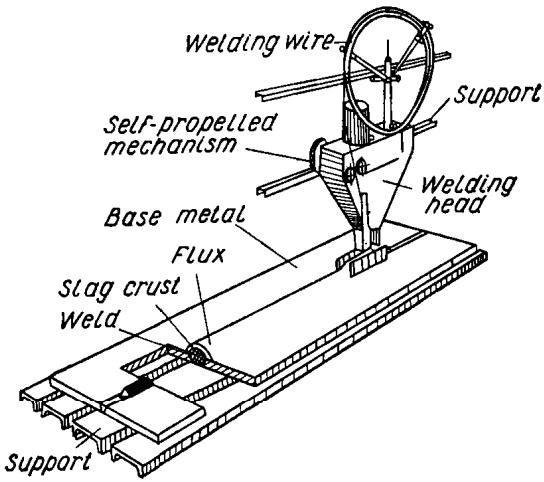
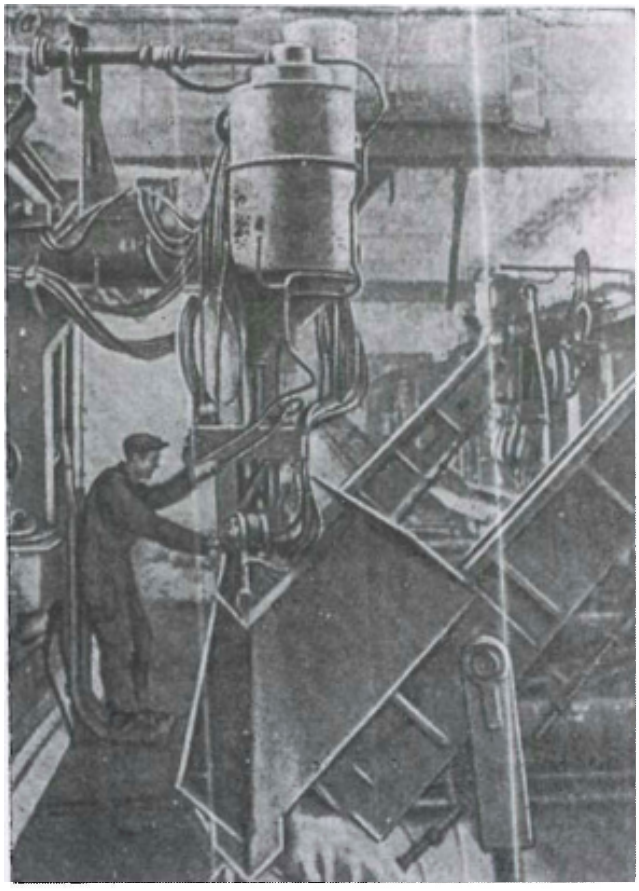


Fig. 4-6. Automatic submerged arc welding

flows through the molten flux (slag) with the liberation of heat sufficient for melting the base metal and the electrode. This method is designed for making vertical welds.

Where it is difficult to employ automatic welding procedures, use can be made of semi-automatic submerged arc welding with a hose-fed wire. In this method, a thin welding wire two millimetres in diameter is fed to the welding zone mechanically along a flexible hose, while movement along the weld is performed manually (Fig. 4-11*a*). The flux is supplied directly from the hopper of the holder, the latter also carrying the control buttons (Fig. 4-11*b*). Figure 4-12 pictures the use of a semi-automatic machine for welding on the stiffening ribs of a welded beam.

The near future should see the introduction of semi-automatic "hose" welding where the filler metal electrode is replaced with a thin tube containing a special powder (powder electrodes). The powder performs the functions of flux. This method of welding is especially convenient for field work.



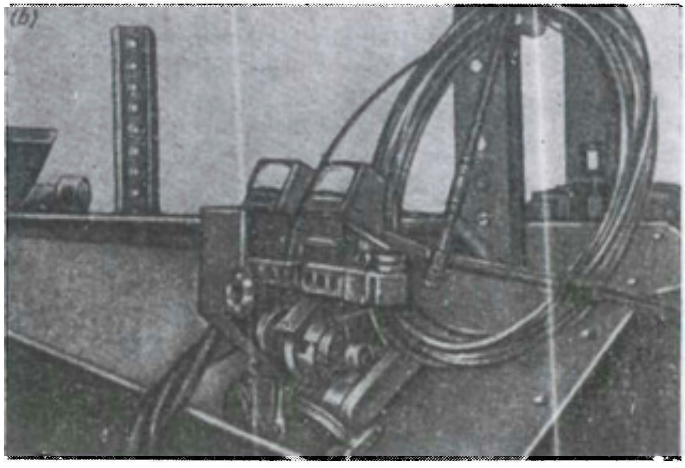


Fig. 4-7. Automatic welding of beam fillet welds:
a—in stationary installation; *b*—using model TC-17 welding tractor

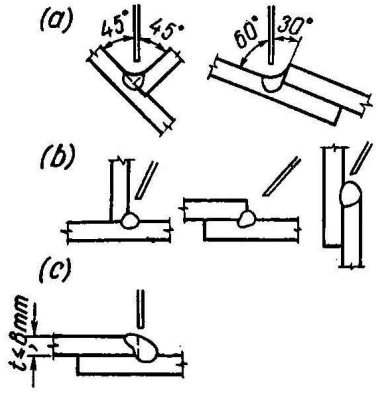


Fig. 4-8. Automatic downhand welding of fillet welds:

a—flat welding; *b*—horizontal welding; *c*—fusion welding

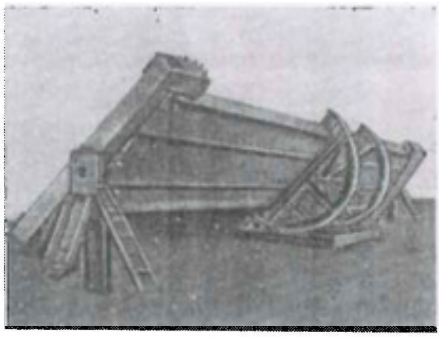


Fig. 4-9. Revolving jig for welding beams

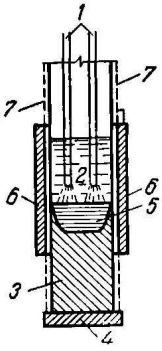


Fig. 4-10. Electroslag welding:

1—electrodes; *2*—flux; *3*—solidified metal; *4*—base-plate; *5*—welding bath; *6*—slides; *7*—direction of slide motion

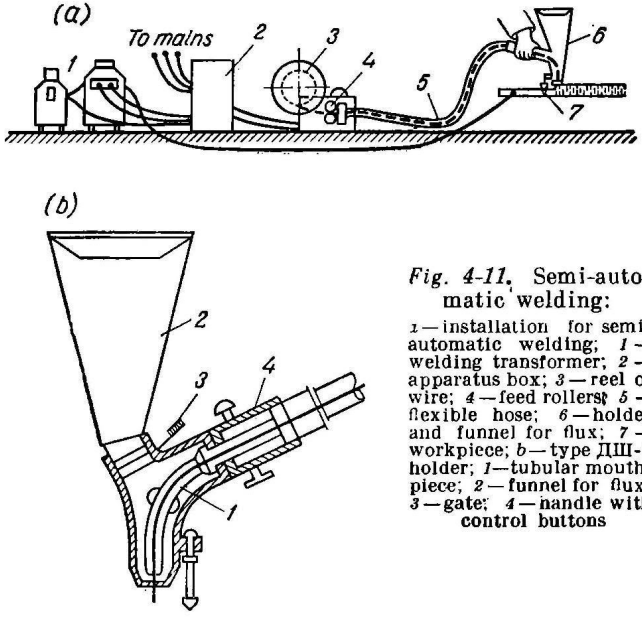


Fig. 4-11. Semi-automatic welding:

1—installation for semi-automatic welding; *1*—welding transformer; *2*—apparatus box; *3*—reel of wire; *4*—feed rollers; *5*—flexible hose; *6*—holder and funnel for flux; *7*—workpiece; *b*—type ДШ-5 holder; *1*—tubular mouthpiece; *2*—funnel for flux; *3*—gate; *4*—handle with control buttons

Fig. 4-12. Semi-automatic welding on of beam stiffening ribs

(3) Gas-Shielded Arc Welding

Among the various methods of arc welding greater and greater preference is being given to gas-shielded arc welding, using carbon dioxide and argon as the shielding gases.

Carbon dioxide is employed in welding low-carbon steels. The gas is fed through a special nozzle and flows around the melting welding wire, which can be fed automatically.

The merits of this method of welding are its high output and low cost, and also the deep penetration. Owing to the high temperature of the stream of shielding gas heated by the arc, the metal cools slowly, and a sound weld is obtained. The possibility of automation of the welding process without the use of fluxes makes this method a very promising one.

Argon is used mainly in welding aluminium members, as well as thin sheets of high-alloy stainless and heat-resisting steels.

Argon-shielded arc welding is performed with the aid of a nozzle containing a nonconsumable tungsten electrode that generates and maintains an electric arc. The shielding gas—argon, surrounds the lower end of the electrode. Filler metal in the form of wire is introduced into the weld.

When welding aluminium members it is also possible to make use of consumable electrodes.

(4) Other Methods of Welding

Spot Welding. At present spot welding is not employed to any noticeable extent for connecting steel structural members, seeing that it is impossible to ensure a stable strength of the weld when connecting thick parts.

Gas Welding. The combustion of acetylene in a stream of oxygen produces a high temperature (3,200 deg C) which causes melting and fusion of the base and filler metal, the latter having the form of wire that is introduced into the gas flame. Gas, or torch, welding yields a low output and is therefore seldom used, mainly for repair jobs.

(5) Welding of Aluminium Members

For a long time aluminium members did not lend themselves to welding, since the molten aluminium readily combines with oxygen and forms an oxide film on the drops of molten metal that hampers fusion. When welding is performed under a blanket of an inert gas, for example argon, such a film does not form. Welding can be carried on using a nonconsumable tungsten electrode with a filler of alu-

TABLE 4-2 **Design Strengths R_w kg/cm² of Welds in Steel Members (According to USSR Building Standards and Regulations)**

Weld	Stress state	Symbol	Automatic and semi-automatic, as well as manual welding with electrodes of type				
			Э42 and Э42А	Э50А		Э55	
			Grade of steel of member being welded				
			Steel 3, steel 4	14Г2, 15ГС		10Г2С, 10Г2СД, 15ХСНД	10ХСНД
				$t \leq 20$ mm	$t=$ =21-32 mm		
Butt welds	Compression	$R_{w \cdot c}$	2,100	2,900	2,800	2,900	3,400
	Tension:						
	(a) in automatic welding	$R_{w \cdot t}$	2,100	2,900	2,800	2,900	3,400
	(b) in semi-automatic and manual welding with the employment for controlling the quality of welds of:						
	highclass methods		2,100	2,900	2,800	2,900	3,400
	ordinary methods		1,800	2,500	2,400	2,500	2,900
	Shear	$R_{w \cdot sh}$	1,300	1,700	1,700	1,700	2,000
Fillet welds	Compression, tension and shear	$R_{w \cdot f}$	1,500	2,000	2,000	2,000	2,400

NOTE. The employment of high-class methods of weld quality control should be stipulated in the project.

minium wire, which is rational for sheets from 1 to 6-8 mm thick. It will be good practice to use a consumable electrode in welding sheets and plates from 6 to 40 mm thick, or still better to weld automatically with an aluminium wire 1.5-3 mm in diameter. Manual arc welding is also employed with a special electrode coating that is able to combine chemically with the oxygen of the oxide film or dissolve it in molten salts.

Good weldability of aluminium members will be ensured by chemically pure elements and wire, as well as proper preparation of the surfaces to be welded (degreasing, pickling and cleaning). The welding of aluminium members requires strict adherence to the established technological conditions, since failure to do this will lead to a reduction in the quality of the welds.

4-2. DESIGN STRENGTHS OF WELDS

The disign strengths of welds (for cases when no allowance for the service conditions factors is necessary) are given in Tables 4-2 and 4-3.

TABLE 4-3 **Design Strengths R_w kg/cm² of Welds Made by Argon-Shielded Arc Welding in Aluminium Members, as well as of Base Metal in Heat-Affected Zone (According to USSR Building Standards)**

Weld	Stress state	Symbol	Grade of alloy of member being welded									
			АМц	АМг	АМг6	АМг61	АД31-Т	АД31-Т1	АД33-Т1	АВ-Т	АВ-Т1	В92-Т
Butt welds	Compression .	$R_{w \cdot c}$										
	Tension	$R_{w \cdot t}$	500	700	1,300	1,600	600	750	1,000	800	1,100	1,500
	Bending	$R_{w \cdot b}$										
Ditto	Shear	$R_{w \cdot sh}$	300	400	800	950	350	450	600	500	650	900
Fillet welds	Compression, tension, bending and shear	$R_{w \cdot f}$	350	450	850	1,050	400	500	650	550	700	1,000

NOTE. The zone in the base metal of the elements being welded, affected by the heat of one weld, is determined as the sum of the values $z = 3.5\ h_w$ laid off to each side from the centre line of the weld.

The compressive and shear design strengths of butt welds made with type Э42 and Э42А electrodes in members of steels 3 and 4, and with

type Э50А and Э55 electrodes in members of low-alloy steels, are taken equal to the design strengths of the base metal. Different values of the tensile design strength of these welds are taken depending upon the method of welding and the requirements which the quality of the electrodes, the weld and its control must meet. If a butt weld is made by automatic welding, or by manual or semi-automatic welding with the use of high-quality electrodes, additional high-class methods being employed to control the quality of the welds (x-ray, gamma-ray, electromagnetic methods, ultrasonic control, etc.), then the design tensile strengths are taken the same as for the base metal. Such connections requiring special attention may be made only in the critical parts of members. With ordinary methods of weld quality control (visual inspection, measurement of the welds, test drilling, hydraulic tests or testing with kerosene, and so on) the design tensile strengths for butt welds made by manual or semi-automatic welding are lower than those for the base metal (Table 4-2).

4-3. TYPES OF WELDED JOINTS. INVESTIGATION OF JOINTS FOR ACTION OF AXIAL FORCES

(1) Classification of Welded Joints and Welds

In the design of welded joints there are distinguished butt joints, lap joints, composite joints, tee joints and corner joints.

The types of welded joints and the types of welds are shown in Table 4-4.

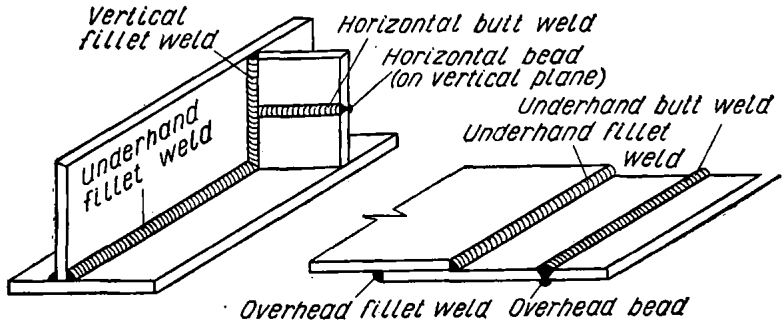
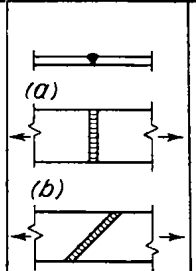
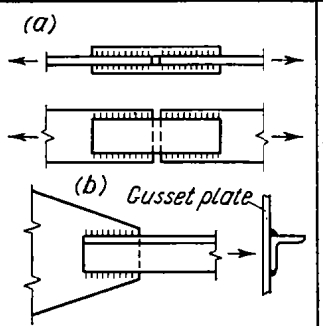
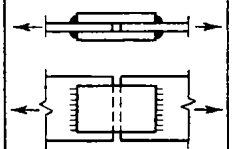
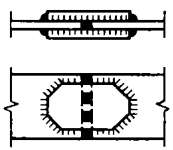
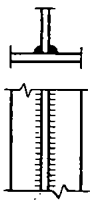
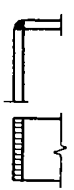
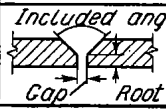
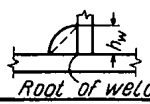
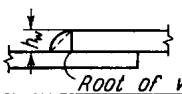


Fig. 4-13. Classification of welds according to position

Welds are classified according to a number of features, namely:

1. With respect to their position on the members being connected—into underhand, vertical, horizontal and overhead welds (Fig. 4-13).

TABLE 4-4 **Types of Welded Joints and Welds**

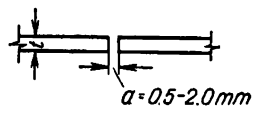
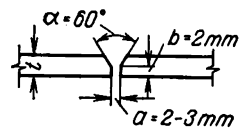
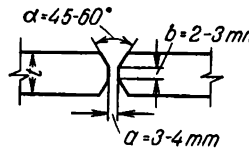
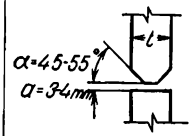
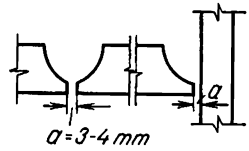
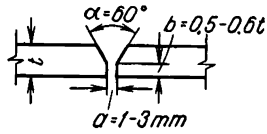
Types of welded joints	Butt joints	Lap joints		Composite	Tee joints	Corner joints
Type of weld	Butt weld	Fillet welds		Side End Butt		
		Side	End			
						
Details of welds				h_w = leg or size of weld		

2. According to the design of the weld—into butt and fillet welds, the latter with respect to their arrangement relative to the acting forces being divided into side and end (edge) welds.

3. According to the method of machining the edges of the plates—into single-V, double-V, double-bevel, J and U groove welds (Table 4-5).

4. With respect to length—into continuous and intermittent welds.

TABLE 4-5 **Classification of Welds According to Method of Machining Plate Edges**

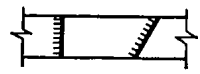
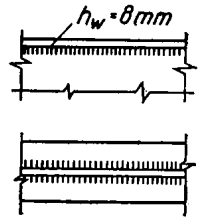
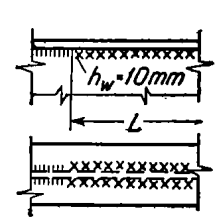
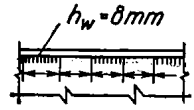
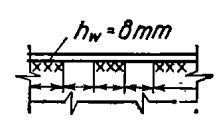
Weld designation according to method of machining edges	Sketch showing how edges are processed	Employed with thickness of plate t, mm	Remarks
Square groove (no machining)		Up to 8-10	a—root opening b—root length α—included angle
Single-V groove		From 10 to 20	
Double-V groove		Over 20	
Double-bevel groove		Ditto	
Single-U and single-J groove		Ditto	
Single-V groove in automatic welding		Over 16	With t up to 16 mm, square joints are used with 1 mm openings

5. As regards the number of layers applied during welding—into single-pass and multipass welds.

6. According to their purpose—into working (force-transmitting) and constructive or connecting welds.

The symbols used in drawings to designate welds and their dimensions are illustrated in Table 4-6.

TABLE 4-6 **Weld Symbols**

Weld	Symbols	
	Welds made at plants (shop welds)	Welds made during erection (field welds)
Butt		
Continuous fillet (h_w — size of weld leg, mm)		
Intermittent fillet		

Generally the size of welds predominant in the given drawing, as well as the type of electrodes, is stated in a note located near the title block (name of drawing) in the lower right-hand corner of the sheet. Only those dimensions that differ from the predominant ones are shown directly in the drawing of the weld.

Instructions relating to the machining of the edges are given in a drawing only if they deviate from the normal ones (Table 4-5).

(2) Butt Joints

It will be the best practice to employ butt joints for connecting plates. These joints can also be used when connecting I beams, channels and angles. When designing butt joints care should be taken to make possible good penetration of the weld and create conditions ensuring free development of the welding deformations (shrinkage) upon cooling of the welds. For this purpose it is necessary in plates

over 10 mm thick to machine one or both edges (see Table 4-5) in order to make possible the inserting of the electrodes to a greater depth, which will ensure the required penetration, and also to leave a gap of a constant size (Table 4-5) between the elements being connected. After welding, the gap completely disappears, which is an indication of the considerable transverse shrinkage of the weld. Failure to provide such a gap leads to warping of the plates and large welding strains. When fastened plates are welded tensile stresses appear in the weld after cooling that lead to the formation of cracks and failure of the weld.

In automatic welding, owing to the high current intensity, the edges are not always machined, which reduces the amount of deposited metal. With a thickness of steel parts up to 16 mm automatic welding is performed from one side and in a single pass, with a square groove, leaving a gap between the parts. With a thickness exceeding 16 mm the edges are bevelled at an angle of about 60 deg. Automatic welding is performed on temporary (copper) or permanent (steel) backing strips, on a flux bed or with a preliminary back bead weld.

For automatic welding from both sides ($t>24$ mm) double-V grooves are used with the length of the root constituting about one-third of the thickness of the plates being welded.

When welding aluminium plates up to 10 mm thick square grooves may be employed, but beginning with a thickness of 12 mm bevelled edges must be the rule.

A gap of one millimetre is used between plates up to 10 mm thick, and one of 1.5-2 millimetres in plates up to 30 mm thick.

The strength of a butt weld subjected to axial forces is analyzed on the assumption that the stresses are uniformly distributed over the cross section of the weld (Fig. 4-14*a*). When the weld is defective (Fig. 4-14*b*) the force field inside the plate is not uniform and induces an additional bending moment and a dangerous concentration of stresses. To eliminate this a bead weld is applied after thorough cutting out of the slag inclusions at the root of the weld.

The section of a butt weld used for investigations, when the weld is located perpendicular to the centre line of the element (a square joint), is taken equal to the area along line *II-II* (Fig. 4-14*a*), but without regard to the strengthening bead weld below and the deposited bead on top, i.e.,

$$A_w = L_w t$$

where t = minimum thickness of elements being welded

l_w = design length of weld, equal to the actual length less 10 mm (allowing for the formation of a crater and poor penetration at the ends of the weld). If the weld is extended to the backing strip these 10 mm may not be subtracted.

The stress in a weld is checked by means of the expression

$$\sigma_w = \frac{F}{A_w} = \frac{F}{L_w t} \leqslant R_w \qquad (4\text{-}1)$$

in which R_w is the design tensile or compressive strength of a butt weld taken from Table 4-2.

The design tensile strengths of square-groove butt joint welds made by manual or semi-automatic welding, and with ordinary

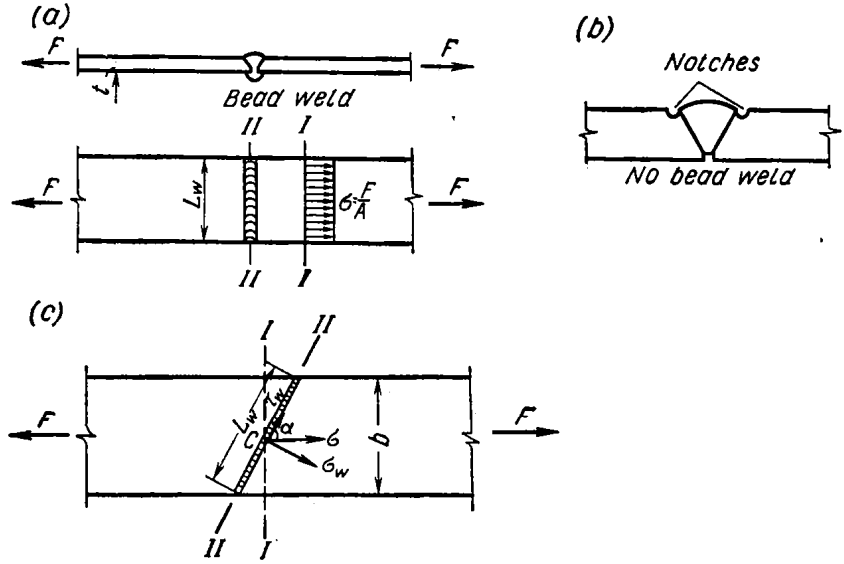


Fig. 4-14. Joint welds:
a—butt joint; *b*—defective butt joint; *c*—skew joint

methods used to control the quality of the welds, are lower than the strength of the base metal at the joint. For this reason the stresses in the base metal in section *I-I* (Fig. 4-14*a*) cannot be utilized completely.

The design of a skew joint having a strength equal to that of the base metal is shown in Fig. 4-14*c*. The angle α between the direction of the axial force and that of the skew weld should not exceed 65 deg (generally an angle of 60 deg is employed). With such an angle the strength of the joint does not have to be checked.

(3) Lap Joints

A lap joint can be made with or without straps, using fillet welds. Depending upon their arrangement relative to the forces acting on the elements, fillet welds can be side (located parallel to the forces) and end or edge ones (located perpendicular to the forces).

Lap Joint with Side Welds (Fig. 4-15). The force is transferred very unevenly from one element to the other along both the length of the weld and the cross section of the joint. Figure 4-15 *a* shows the force field (forcelines) concentrated at the welds in an element, while Fig. 4-15*b* depicts a diagram of stress distribution along the length of the weld. Upon statical loading, however, after stresses equal to

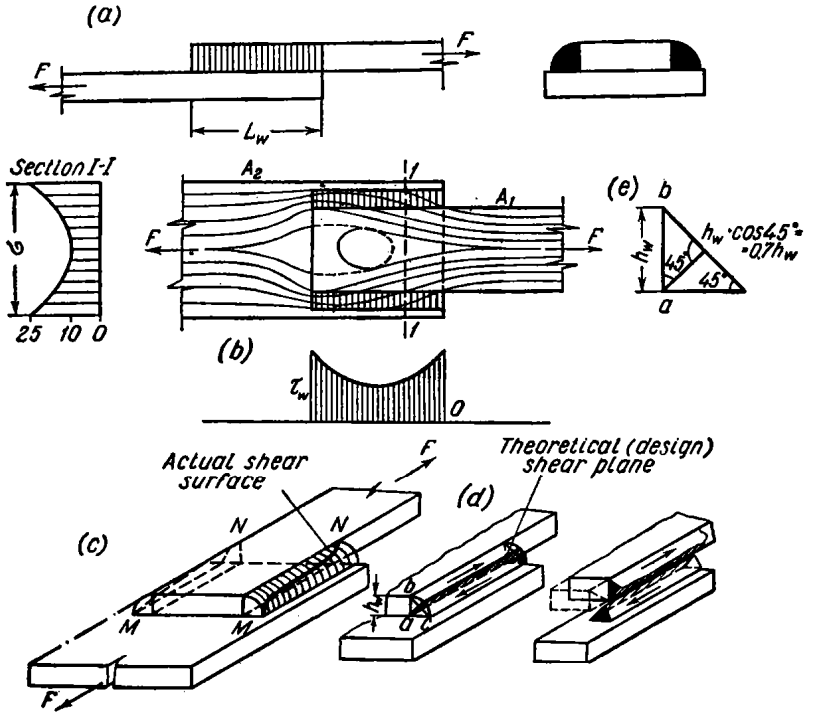


Fig. 4-15. Joints with side welds

the yield point have been reached in the extreme points of the welds, further loading will result in more uniform stress distribution over the length. The stress diagram will level out and failure will occur in the form of shearing along a surface characterized by line *MN* (Fig. 4-15*c*). This line has curved sections at its beginning and end (at the most stressed points) and a straight section at the middle of the weld. Such a form of failure permits analysis to be based on the assumption that the shearing stresses are uniformly distributed over the minimum cross-sectional area of the weld, which passes through its theoretical throat (without taking into account convexity, Fig. 4-15*d*). With equal weld legs this throat is equal to $0.7h_w$, where h_w is the size of the weld (Fig. 4-15*e*).

In view of the fact that in automatic and semi-automatic welding the penetration at the root of the weld is deeper than in manual welding, and that when the weld is resisting shearing stresses part of the base metal is involved in this resistance, the theoretical throat of the weld is in a general form taken equal to $k_w h_w$. Thus the shear area of side welds is found from the expression

$$A_w = k_w h_w \sum L_w \tag{4-2}$$

Here h_w = size of fillet weld (leg of inscribed isosceles triangle)
$\sum L_w$ = sum of design lengths of fillet welds
k_w = factor taken equal to 1 for single-pass automatic welding, to 0.8 for single-pass semi-automatic welding and to 0.7 for manual welding, as well as for multipass automatic and semi-automatic welding.

Because of the poor penetration at the beginning and of the crater at the end of the weld, and also of the influence of chance inclusions, the design length of a fillet weld, according to the standards, must not be less than 40 mm or $4h_w$.

When very long side welds are used, the ultimate strength may be reached at points M and N (Fig. 4-15*c*) before the stresses level out over the whole length of the weld. This is why the standards also limit the maximum design length of side welds to one not exceeding $60h_w$. An exception here are joints where the force is applied over the whole length of the weld, as, for instance, in flange welds of a beam, where the weld length is not limited.

The strength of side welds also depends upon their size, the ultimate strength decreasing somewhat with an increase of the size. The size of a weld is stipulated in the standards and is taken not over $1.2t$ (t is the minimum thickness of the elements being connected). It will be good to avoid the use of welds over 20-25 mm in size. The minimum size of welds is taken equal to 4 mm. The recommended minimum sizes of fillet welds, depending upon the thickness of the elements being welded, are set forth in Table 6-6.

A lap joint with side welds subjected to an axial load, with regard to the assumption of uniform shearing stress distribution, is investigated by means of the expression

$$\tau_w = \frac{F}{A_w} = \frac{F}{k_w h_w \sum L_w} \leqslant R_{w.f} \tag{4-3}$$

where F = design axial load, kg
$R_{w.f}$ = design shearing strength of fillet weld, kg/cm², taken from Table 4-2.

The size of the weld h_w is usually taken equal to the thickness of the elements being connected, or somewhat less. The required total

length of the welds is found from the expression

$$\sum L_w \geqslant \frac{F}{k_w h_w R_{w.f}} \tag{4-4}$$

The actual length of the weld should be greater than the design length by 10-20 mm.

When securing an unsymmetrical section, for example two angles, to a plate (Fig. 4-16) attention is paid to uneven distribution of the load between the welds transmitting the force field from the angles to the plate. The force F extending the angles can be considered as the resultant of a force field of normal stresses applied to the centre of gravity of the angle sections.

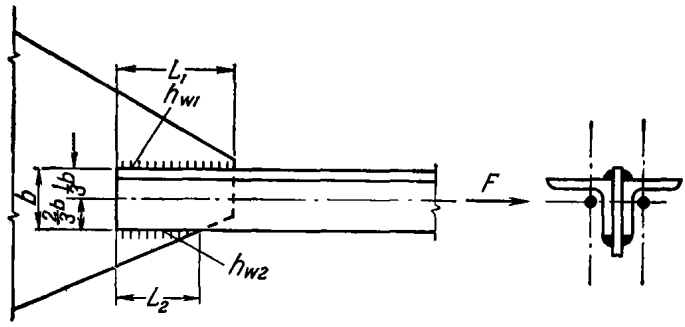


Fig. 4-16. Unsymmetrical lap joint

It is obvious that the force F is distributed in reverse proportion to the distances from the welds to the line of the centre of gravity of the angle. This line is situated at a distance of about one-third of the leg from the back. The force at the back is taken equal to $F_1 = \frac{2}{3}F$ and at the tip equal to $F_2 = \frac{1}{3}F$.

Hence the formula for determining the design length $\sum L_1$ of the weld at the back will be obtained from expression (4-4)

$$\sum L_1 \geqslant \frac{2}{3} \frac{F}{k_w h_{w.1} R_{w.f}} \tag{4-5}$$

in which $h_{w.1}$ is the size of the weld at the back of the angle.

Since the force at the tip of the angle leg is considerably smaller, the weld at this end can have a smaller size. It is not good to cut away the leg of the angle to reduce the length of this weld L_2. Angles (with rare exceptions) should be cut at right angles to their centre line.

Example 4-1. Compute the welds required for connecting two angles 75×8 mm in size to a gusset plate with a thickness of $t=10$ mm (Fig. 4-17). The design tensile load in the angles is $F=32.5$ tons. The material is grade BCт. 3 steel. Manual welding with type Э42 electrodes is to be employed.

Solution. The design length L_1 is determined from expression (4-5), taking the size of the weld $h_w=0.8$ cm. Owing to the presence of two angles and, accordingly, two welds with a length of L_1 located at the backs of the angles, we have

$$L_1=\frac{\sum L_1}{2}=\frac{2F}{2\times 3\times 0.7h_w R_{w.f}}=\frac{2\times 42{,}500}{2\times 3\times 0.7\times 0.8\times 1{,}500}\approx 17 \text{ cm or } 170 \text{ mm}^{1)}$$

We take $L_1=170+10=180$ mm.

If necessary in designing, the dimensions of the gusset plate can be reduced as shown by the dash line in Fig. 4-17, with a view to the allowable reduction in the length of the weld at the tip of the angle.

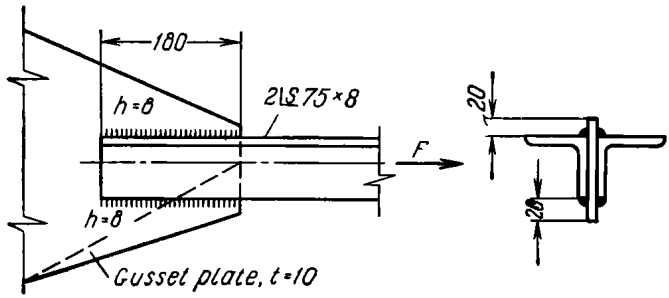


Fig. 4-17. To example 4-1

Notwithstanding certain negative properties of joints with side welds (uneven distribution of the stresses) they are employed very frequently owing to their somewhat greater plasticity in comparison with other kinds of joints, since the shear modulus of elasticity ($G=840{,}000$ kg/cm^2) is less than Young's modulus ($E=2{,}100{,}000$ kg/cm^2).

Lap Joints With End Welds (Fig. 4-18). When the straps are arranged symmetrically (Fig. 4-18*a*), end welds give a sufficiently high strength. Owing to a sharp change in the direction of the forcelines, however (Fig. 4-18*b*), considerable stresses are concentrated at the root of the weld, and this leads to failure with small longitudinal strains ($\varepsilon=4$-6%), i.e., brittle fracture is obtained with failure (in tension) along the plane of contact between the weld and the edge of the plate.

Owing to the combined stressed state and the extremely uneven distribution of the stresses, such joints are of a lower quality. For

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[1]) Here and in the following examples the numerical values were generally obtained with slide-rule accuracy only.

this reason the design strength, regardless of the stresses acting on the weld (compression, tension, shear), is in all instances taken as equal to the design shear strength of a fillet weld.

Thus end welds are provisionally investigated for shear along the minimum section area, which is taken as the design area passing through the theoretical throat equal to $k_w h_w$. The expression used for analysis thus remains the same as for side welds, namely,

$$\tau_w = \frac{F}{A_w} = \frac{F}{k_w h_w \sum L_w} \leqslant R_{w \cdot f}$$

The length a of the strap (Fig. 4-18a) is taken equal to at least $10t$, but not less than 80 mm.

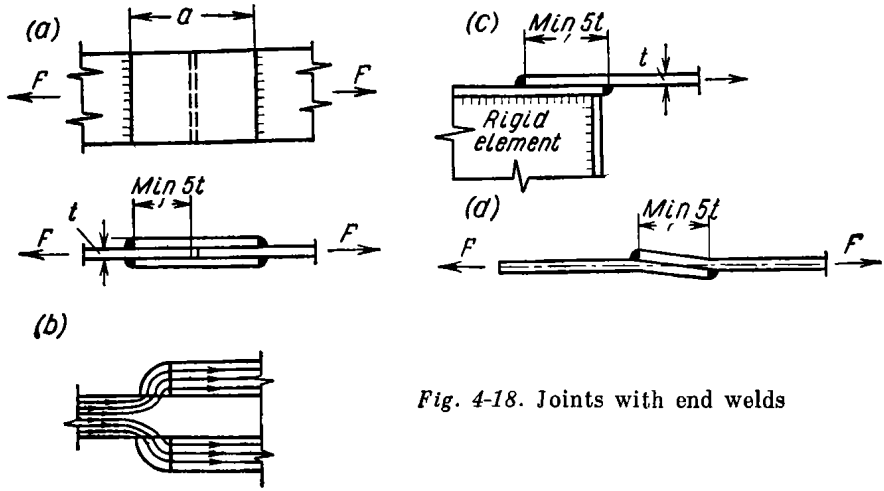


Fig. 4-18. Joints with end welds

It is not recommended to use an unsymmetrical connection with one strap, since the resulting eccentricity will create an additional moment.

The use of a lap joint with end welds is tolerated only when one of the elements is a sufficiently rigid member (Fig. 4-18c) or, on the contrary, in slender (thin) sheet members (Fig. 4-18d). To reduce the influence of the additional bending moment in such joints the length of the lap should be at least $5t$, where t is the thickness of the element being welded on.

(4) Composite Joints

A joint is called *composite* if it contains several different kinds of welds, namely, side, end or butt ones. The simplest example of such joints is a joint with welding done around the whole contour (Fig. 4-19a and b).

The actual behaviour of a composite joint depends on the distribution of the forces between the side and the end welds. It may be described as follows. During the first period after application of the load, the end welds, being more rigid, sustain greater forces, and only after a certain deformation of these welds do the less rigid side welds begin to take the load. Failure occurs in the plastic stage with relative equalizing of the stresses in all the welds.

It is not good practice to use a joint of this type, since it produces overloading of the end welds, especially when alternating loads are applied.

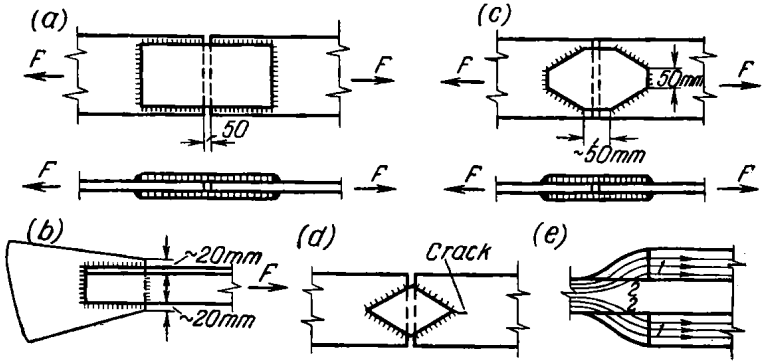


Fig. 4-19. Composite joints

The force field is transmitted more uniformly in joints having straps of a rhombic type with acute angles of 35-45 deg and cut off apexes (Fig. 4-19*c*). In this case, to avoid a great concentration of stresses at the gap of the joint, the weld is not brought up to the middle of the joint, leaving a space of 25 mm at each side. Rhombic straps with uncut apexes are not recommended because of the formation of cracks at the apexes (Fig. 4-19*d*). To ensure smoother transition of the force field, the legs of the welds on the main plate should be extended (Fig. 4-19*e*).

When investigating the welds of a composite joint, uniform distribution of the stresses between all the welds is assumed, and the expression used for calculations remains the same (4-3 or 4-4). Figure 4-20 pictures the distribution of the stresses over the cross section of the base metal and over the straps.

The third type of composite joint used in tension elements of members can be made by butt welding of the elements with the additional use of rhombic straps. Such a joint is more costly, since it requires processing of the edges, and removal of the excessive weld metal after butt welding.

The formula used for analyzing such a joint is

$$\sigma = \frac{F}{A_w + \sum A_s} \leqslant R_{w \cdot t} \tag{4-6}$$

where A_w = cross-sectional area of weld
A_s = area of strap
$R_{w \cdot t}$ = design tensile strength of weld.

Upon comparing various methods of connecting plates the conclusion can be drawn that the best type of joint for plates subjected to

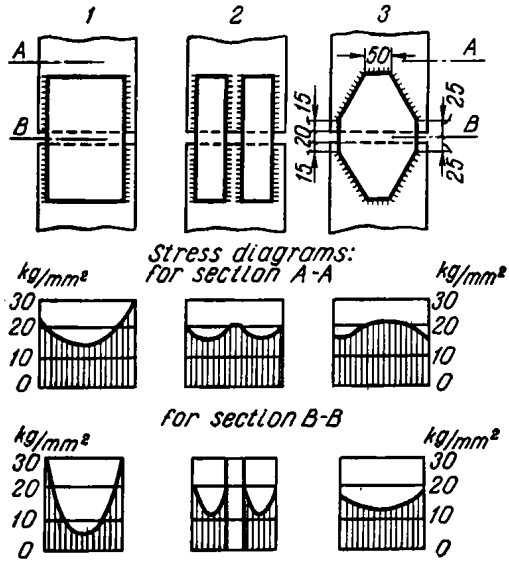


Fig. 4-20. Distribution of stresses in butt joints with straps

axial loads is a square butt joint (or, in tension, a skew joint, which has equal strength). When straps are required in a square butt joint these should be of the rhombic type with cut off apexes. In critical members, however, it will be better to weld the plates without straps, using an automatic welding process (or manual or semi-automatic welding with high-class methods of weld quality control).

It is preferable to connect rigid shapes to plates by means of side welds, which are more plastic.

Example 4-2. Compute the joint of two plates made of grade ВСт.3 steel with a section of 250×12 mm for a design tensile load of $F=62$ tons. Design the joint with two straps of the rhombic type (Fig. 4-21). Manual welding is to be used.

Solution. 1. We find the required section of the straps. Obviously the cross-sectional area of the two straps ($2A_s$) should equal the main section of the plate (A_p) or be somewhat larger than it, i.e., we must take $2A_s \geqslant A_p$. Otherwise it will be impossible to transmit the force F from one plate to the other. From considerations of design the section of the straps is taken equal to 200×8 mm, whence

$$2A_s = 2 \times 20 \times 0.8 = 32 \text{ cm}^2; \qquad A_p = 25 \times 1.2 = 30 \text{ cm}^2$$

Thus $2A_s > A_p$.

2. Type Э42 electrodes will be used. The design shear strength of a fillet weld will therefore, in accordance with Table 4-2, equal $R_{w.f} = 1{,}500$ kg/cm^2. For straps 8 mm thick we take welds with a size of $h_w = 8$ mm or 0.8 cm.

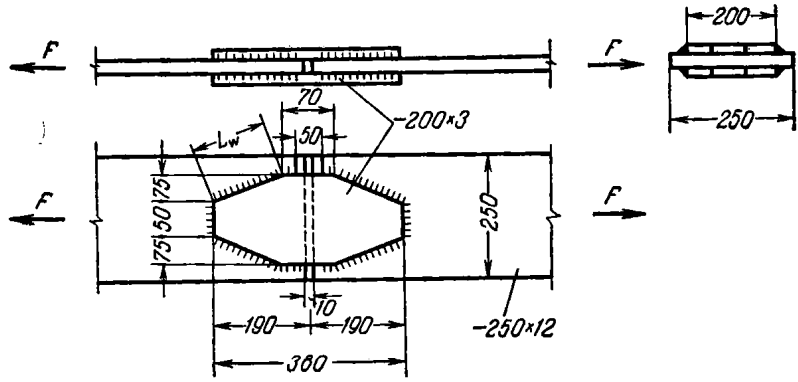


Fig. 4-21. To example 4-2

3. We find the total design length of the welds ΣL_w at one side of the joint per half strap, from expression (4-4)

$$\sum L_w = \frac{F}{2 \times 0.7 h_w R_{w.f}} = \frac{62{,}000}{2 \times 0.7 \times 0.8 \times 1{,}500} \approx 37 \text{ cm}$$

4. We determine the length of a joint half strap. At either side of the joint each half strap is welded on using one continuous weld consisting of two side (slanting) welds and one end weld 5 cm (50 mm) long.

Therefore, the design length of one side weld should be at least

$$L_w = \frac{1}{2}\left(\sum L_w - 5\right) = \frac{1}{2}(37 - 5) = 16 \text{ cm}$$

The constructive length of the weld should be taken somewhat greater ($16+1=17$ cm), taking into account the crater at the end of the weld, and the possible lack of penetration at the beginning of the weld. To reduce the influence of stress concentration at the apexes and at the gap between the plates being connected, part of the joint strap at the gap should be left unwelded over a length of 50 mm, and the design side weld at the apex should be directed parallel to the plate edge over a length of 10 mm (Fig. 4-21). Thus the length of half of a strap will be

$$\frac{1}{2}L_s = \sqrt{17^2 - 7.5^2} + 3.5 \approx 19 \text{ cm} \quad \text{or} \quad 190 \text{ mm}$$

5. We take two straps with a section of 200×8 mm, a length of $L_s = 190 \times 2 = 380$ mm; the size of the weld will be $h_w = 8$ mm.

(5) Tee and Corner Joints

These types of joints are employed, for example, when securing an angle bracket to a column (Fig. 4-22*a*) or connecting plates at right angles to each other (Fig. 4-22*b*), etc. Owing to their simplicity, such joints have found widespread application. They require good penetration over the whole thickness of the plate being connected (Fig. 4-22*c*), since if an opening is left (Fig. 4-22*d*) the direction of

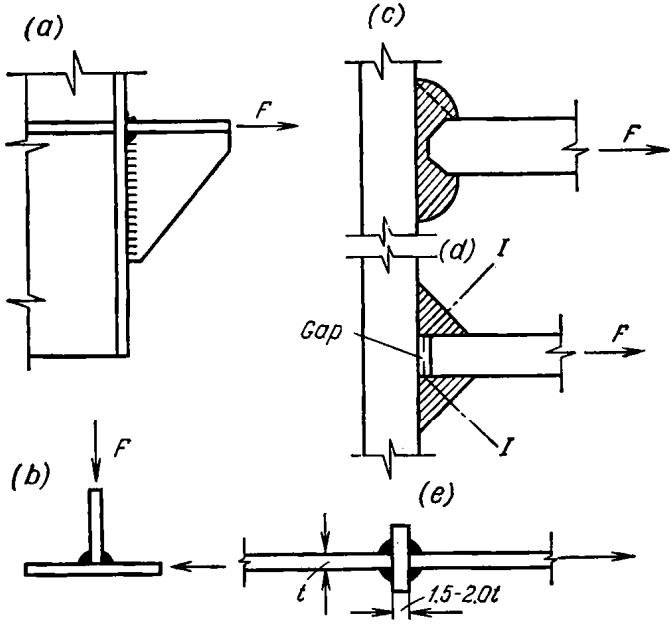


Fig. 4-22. Tee joints

the force lines will sharply change. In the latter instance the design section of the welds is taken with the minimum area (along *I-I*, Fig. 4-22*d*) and the design strength is taken equal to the shear strength for fillet welds $R_{w.f}$. With complete penetration (Fig. 4-22*c*) the size of the weld is taken equal to the thickness of the plate being welded on, while the design strength of the weld is taken with a view to the kind of load (compression, tension).

The employment of a double-tee joint with a transverse plate (Fig. 4-22*e*) cannot be recommended in the general case, since, there is a sharp change in the direction of the force lines passing across the direction of rolling of the inserted plate welded with four welds, and there is a comparatively wide zone with low mechanical properties along the thickness of the inserted plate.

In addition, the mutual displacement of the plates being connected together that often occurs will induce additional bending and shearing stresses at the joint.

Such a design is permissible in compression joints subjected to a statical load, but thick inserted plates must be used with a thickness of 1.5-2t.

4-4. ANALYSIS OF WELDED CONNECTIONS FOR ENDURANCE

In welded steel members directly resisting repeated moving, vibration and other loads that may lead to fatigue failure [see Sec. 2-7 (2)] the endurance of not only the base metal, but also of the welded connections, must be checked.

In analyzing the endurance of welded connections the design strengths are reduced by applying the factor γ determined from equation (2-41)

$$\gamma = \frac{1}{(ak_e \pm b) - (ak_e \mp b)\, k_{rs}}$$

The values of the coefficients a and b, as well as of the range ratio $k_{rs} = \frac{\sigma_{min}}{\sigma_{max}}$, are taken the same as for the base metal (see page 79).

The values of the effective stress concentration factors k_e are taken in accordance with Table 4-7.

TABLE 4-7 **Values of Effective Stress Concentration Factors k_e for Welded Connections**

Design sections	Factors k_e for	
	carbon steel	low-alloy steel
Butt welds with automatic welding	1.0	1.0
Ditto with manual welding	1.2	1.4
End fillet welds along design section with manual welding	2.3	3.2
Ditto with automatic welding	1.7	2.4
Side welds along design section	3.4	4.4

It should be emphasized that constructive measures aimed at reducing any possible stress concentrations are the main method of combatting fatigue phenomena in welded members.

In addition, when checking the endurance of welded connections all the general prescriptions set forth in Sec. 2-7 (2) for the base metal should be complied with.

4-5. INTERNAL STRESSES. SHRINKAGE IN WELDING

As has been mentioned previously, stresses may appear in a material not only as the result of the application of an external load, but also because of heat (thermal) action. It is well known that most

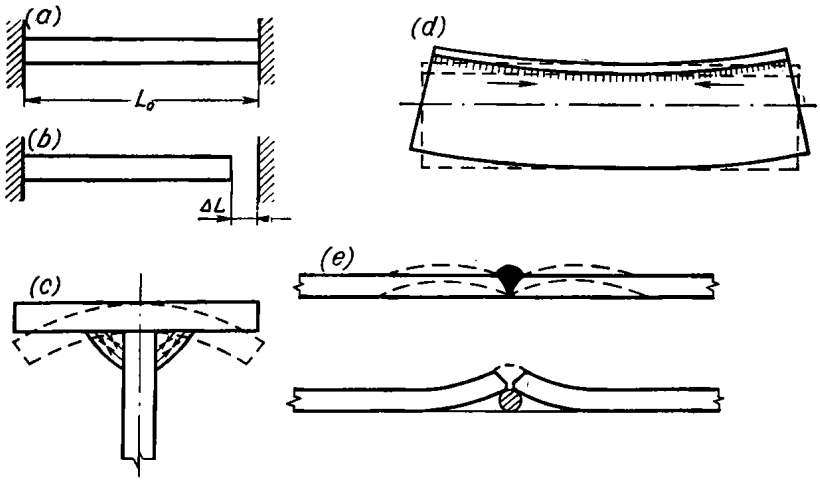


Fig. 4-23. Temperature strains:
a, b—permanent set after heating; *c, d, e*—strain of elements in welding

bodies expand when heated. The elongation in this instance is determined from the equation

$$\Delta L = \alpha L_o t \tag{4-7}$$

in which ΔL = elongation of element
α = coefficient of thermal expansion (for steel $\alpha = 0.000012$ at about 20 deg C and $\alpha = 0.000018$ at about 300 deg C)
L_o = original length of element
$t = t_2 - t_1$ = difference of temperatures.

Let us imagine a bar restrained between absolutely rigid walls (Fig. 4-23*a*). We shall begin to heat it to a temperature below 600 deg C. At such a temperature elastic stresses, equal to [see equation (2-2)]

$$\sigma = E\varepsilon = E\frac{\Delta L}{L_o} = E\alpha t$$

will be induced in the bar since it cannot expand, while reactive forces $F_r = \sigma A$, where A = cross-sectional area of the bar, will appear in the walls.

After cessation of heating, the bar will retain its length (Fig. 4-23*a*), while the stresses in the bar and the reactions in the wall will disappear. If the same bar is heated to a temperature above 600 deg C, then the stresses developing in it will exceed the elastic limit. This is why for low-carbon steels a temperature of about 600 deg is called the *temperature of plasticity*. After heating is stopped, the rod will receive a permanent set—it will shorten (Fig. 4-23*b*). If the bar were restrained in the wall and could not reduce in length, tensile stresses would appear in it after cooling.

Now let us imagine a plate, the middle of which has been heated by applying a small weld.

The sections surrounding the welding zone resist the development of strains and become compressed, as a result of which the stresses in them may reach the yield point. Upon cooling, the heated metal tends to contract. This causes stresses to appear inside the body that are the result of the uneven distribution of the temperature in it. These stresses are balanced within the body itself, forming therein compression and tension zones. These stresses often produce a change in the shape of the body (warping) and lead to such strains that result in a state of equilibrium setting in.

The strains that first appear in members owing to heating in welding and whose development is prevented by the restraints induce stresses at different spots of the member that exceed the yield point. After cooling of the member, the strains are the result of a reduction in the dimensions, since upon cooling of the welds permanent welding sets—shrinkage, appear.

There are distinguished *transverse* shrinkage, across the weld (Fig. 4-23*c* and *e*), and *longitudinal* shrinkage, along the weld (Fig. 4-23*d*).

If the welds in a cross section of an element are located unsymmetrically with respect to the neutral axis, then longitudinal shrinkage leads to bending out of the element that is difficult to remedy in production.

Members are especially unfavourably affected by transverse shrinkage, which is almost ten times greater than longitudinal shrinkage. Figure 4-23*e* shows a transverse weld that not only pulls together the plates, but also bends them out somewhat. Transverse shrinkage of a butt weld is proportional to the area of the weld and can be determined from the following equations: for single-V welds

$$\Delta L = 0.2\left(t \tan\frac{\alpha}{2} + b\right) \qquad (4\text{-}8a)$$

for double-V welds

$$\Delta L = 0.25 \left(\frac{t}{2} \tan \frac{\alpha}{2} + b \right) \qquad (4\text{-}8b)$$

where t = thickness of metal being welded
α = included angle of weld
b = width of gap.

If the plates to be welded are rigidly secured in place along their periphery, then after shrinkage internal tensile stresses will appear in the plates and the weld, which may lead to the formation of cracks. For a plastic material the magnitude of the internal stresses is not so high, since when stresses are reached in the material which are equal to the yield point, the internal stresses partly disappear, and after the very first relieving of the load the member will show quite an elastic behaviour. However, in the conditions of a plane state of stress that promotes the appearance of brittleness (see page 47), internal stresses also play an important part in a plastic material, since they may lead to the formation of cracks.

General prescriptions may be given showing how to reduce welding strains.

1. *Design.* The tendency should be to reduce the total number of welds, avoiding "unnecessary" ones. Seeing that the shrinkage is proportional to the volume of the deposited metal, the minimum thickness of the welds should be determined in strict accordance with the required strength, making them no thicker than required. In a symmetrical member the welds should be arranged symmetrically, or in such a way that the static moments of area of the deposited metal at both sides of the neutral axis be approximately equal.

The intersection of welds or the close location of parallel welds, as well as welds with a closed contour, should be avoided. Finally, the welds should be so arranged as to ensure a rational sequence of welding. For this end a welding technique sheet is drawn up with provision made for the use of special devices in manual or automatic welding.

2. *Technique.* The working out of a welding technique is aimed at establishing a sequence of operations depending on the welding current. It is very important that the pool of molten metal at the weld be filled as quickly as possible, and that there be no interruptions in welding; the smaller the heated zone, the smaller will be the strains.

When necessary, reverse strains (outward bends) should be created, or a definite sequence of welding followed in order to form a preliminary outward bend.

In some instances it will be good practice to employ rigid fastenings (with the aid of clamps or jigs).

When welding at temperatures below freezing point no blows against the metal should be tolerated, since both the base and the deposited metal have a low notch toughness. At a low temperature the heat is rapidly removed from the part being welded; this produces large permanent sets. For this reason it will be good to heat the metal being welded to a temperature perceptible to the hand (70 deg C), and also to take measures ensuring free shrinkage of the welds. The welding zone should be protected from the wind, and favourable conditions of work should be created for the welder.

4-6. LABOUR REQUIREMENTS IN FABRICATION OF WELDED MEMBERS

The fabrication of welded steel members at a plant or shop includes the following main operations:

1. Preparatory operations—selection and matching of the rolled metal, dressing and cutting it, preparation of patterns, marking.
2. Operations in the preparatory shop involving machining of the parts—marking out, cutting, drilling of holes, machining of edges.
3. Operations involving assembly and welding of the members from the prepared parts, quality control, painting, and shipping of the members to the erection site.

The amount of working time required for the manufacture of a member, expressed in man-hours per ton of member weight, is known as the *labour requirements* for the manufacture of the member. The reciprocal of this quantity, namely, the number of tons of members manufactured per unit of time, characterizes *labour productivity*. The lower the labour requirements, the higher will be the labour productivity and the greater the reduction in the cost of the member.

Welded members require a smaller expenditure of time than riveted ones. This is accompanied by a 10-15% reduction in weight.

The average distribution of labour in the manufacture of various kinds of welded members is characterized by Table 4-8. A glance at this table will show that 60-65% of the working time is used for operations connected with assembly and welding of the member parts.

It is clear that the labour requirements will be lower for members consisting of a smaller number of parts (owing to the introduction of larger shapes and reduction of the number required), having a smaller length of the welds and a smaller amount of deposited metal, a reduced number of joints, ribs, etc. In addition, the parts should be simple to assemble. Automatic welding considerably accelerates the process of welding (not to mention the improved quality thereof). The welds should be so arranged that they are easy and convenient to make, mainly using underhand welding.

TABLE 4-8 **Labour Requirements for Separate Operations Involved in Fabrication of Welded Members**

Name of operation	Per cent of total production operation time
1. Preparatory:	
Dressing of steel	3-4
Manufacture of patterns and marking	4-5
Total	7-9
2. Processing of parts:	
Marking out of parts	1-2
Cutting	9-15
Drilling of holes	0-2
Machining of edges	1-3
Dressing and bending	3-4
Forging jobs	1-2
Total	15-28
3. Assembly, welding and shipping of members:	
Assembly	28-32
Welding	32-35
Drilling of assembly holes	2-3
Priming (painting) and shipping	3-4
Total	65-74

When designing the parts of a member the tendency should be to use the greatest possible number of identical ones. This makes it easier to use jigs, which considerably reduce the labour required for assembly and welding, and improve the precision of manufacture.

Thus the member should be technological, i.e., it should be simple and convenient to fabricate, and comply with the requirements of the production process.

Field welding at present does not lead to any difficulties, but it is more costly than shop welding, since it requires special contrivances. For this reason the quantity of such welding should be reduced as much as possible. As a rule field overhead welding should not be tolerated. Before field welding, the element should be hoisted and placed in its final position. It is therefore desirable to so design members that they can be easily installed from above, using special tables or benches if necessary.

CHAPTER FIVE

Riveted and bolted connections

5-1. BEHAVIOUR OF RIVETED CONNECTIONS

(1) General

Riveted joints are a time-honoured way of connecting the parts and elements of steel members. The success of welding during the last decades and its widespread introduction have reduced the sphere of use of riveted connections. The reliability of the latter under reversed and vibration loads, however, necessitates their employment in a number of members and structures with heavy service conditions.

Rivets are manufactured from round calibrated steel in special presses that form the head of the rivet. Depending on their designation, rivets have different shapes of their heads (Fig. 5-1*a-d*).

The diameters of rivets are standardized according to the diameters of the holes. In structural members rivets are used for holes with a diameter of 14, 17, 20, 23, 26, 29.5 and 32.5 mm. The design diameter of rivets is taken in accordance with the diameter of the holes; the diameter of cold rivets before being put in place is less than the diameter of the holes by 1.0-1.5 mm.

Two methods of riveting are in use, namely, hot and cold. In hot riveting the rivets are first heated in special coke or electric furnaces to a temperature of about 1000 deg C. The hot rivet is inserted into the hole, the manufactured rivet head is tightly pressed by means of a knob, jack or dolly bar against the elements being connected, while the end protruding from the hole is shaped into the driven head by blows of a pneumatic hammer (or with riveting clamps). This process is shown in Fig. 5-2. The shank of the rivet is upset and tightly fills the hole. The length of the shank of an inserted rivet corresponding to the thickness of the elements being assembled is known as the *grip* of the rivet.

Upon cooling of a hot rivet, the plates are pulled together by the shank of the rivet. A cooled rivet will be in considerable tension, while the elements joined together will be in compression. The stresses in the rivet will be close to the yield point. This initial contraction is of a very great importance for the behaviour of a riveted joint.

In cold riveting, the rivets are driven with the aid of powerful pneumatic or electrical clamps and fill their holes with sufficient

tightness. This method of riveting is used only at highly mechanized plants.

The principal grades of rivet steels are the comparatively soft grade Ст.2 and Ст.3 rivet steels with an elongation of 26 and 23% and

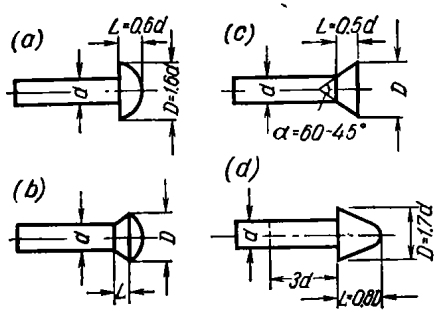


Fig. 5-1. Types of rivets

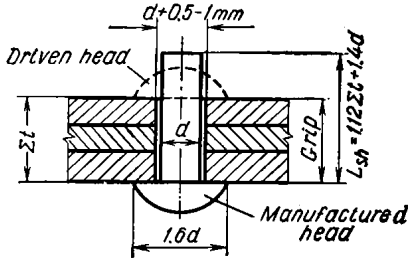


Fig. 5-2. Installation of rivet

a yield point of 22 kg/mm² (for members made from ordinary carbon steels), while rivets of grade 09Г2 steel with an elongation of 18% and a yield point of 30 kg/mm² are used for low-alloy steels.

(2) Behaviour of Rivets in Joint

Riveted joints are mainly subjected to shear. There are distinguished several kinds of behaviour of riveted connections (Fig. 5-3) which must be taken into consideration in designing them.

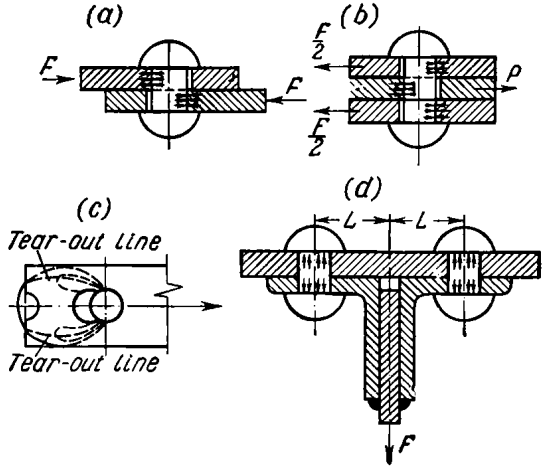


Fig. 5-3. Diagrams showing behaviour of riveted connections

(a) *Behaviour of Rivets in Shear.* Upon relative moving of the riveted elements the rivets may fail due to shearing of their shanks. In this case the rivet shank may have one (Fig. 5-3*a*), two (Fig. 5-3*b*) or more shear planes.

(b) *Behaviour of Rivets in Crushing.* Upon the development of shearing strains in riveted elements, crushing of the rivet hole surfaces (bearing failure) also occurs. In this instance shearing stresses develop in the material between the holes (or between a hole and the edge of the plate) that tend to tear out the plate (Fig. 5-3*c*). Thus, checking of the behaviour of a rivet in crushing is conditional, and actually comprises checking of the plate for tear-out failure.

(c) *Behaviour of Rivets in Tension.* Such behaviour, as a rule, is not characteristic of riveted joints (Fig. 5-3*d*). The initial tension of the rivet due to cooling does not affect its strength, since first the contracting forces must be overcome.

(3) Design Strengths

The behaviour of rivets, besides the design of the joint, the proper arrangement of the rivets and the riveting technique, is also influenced by the technology followed in manufacturing the elements, and first of all by the method of making the holes—punching in presses or drilling.

Punched holes have a ragged and rough surface and do not ensure exact alignment in assembly, which reduces the strength of the joint. With this in view, critical members are designed either with drilling of the holes with the aid of jigs (templates) or with reaming of holes preliminarily punched to a smaller diameter. In less critical members, however, punching of the holes to the full diameter is quite permissible and even desirable owing to the simplicity and high output of the punching process. Holes in plates over 25 mm thick should be drilled.

In respect to the quality of the rivet holes and the conditions of rivet installation, rivets are divided into two groups, B and C.

Group B includes rivets installed in jig-drilled or reamed holes.

Group C covers rivets installed in holes that have been punched or drilled without jigs in each part separately. The design shear and tensile strengths for riveted connections are taken in accordance with the grade of steel of the rivets, while the crushing strength is taken according to the grade of steel of the elements being connected (Table 5-1).

In riveted members whose endurance must be checked, the factor γ is determined from equation (2-41), the corresponding stress con-

TABLE 5-1 **Design Strengths of Riveted Connections, R_r kg/cm²**

Stressed state and group of connection	Symbol	Design strengths R_r							
		For shear and tension of rivets made from		For crushing of elements being connected made from steel of grade					
		rivet steels 2 and 3	grade 09Г2 steel	steel 3, steel 4	steel 5	14Г2 and 15ГС with thickness of rolled stock mm		10Г2С, 10Г2СД, 15ХСНД	10ХСНД
						20 and less	21-32		
Shear B	$R_{r\cdot sh}$	1,800	2,200	—	—	—	—	—	—
Shear C	Ditto	1,600	—	—	—	—	—	—	—
Crushing B	$R_{r\cdot cr}$	—	—	4,200	4,600	5,800	5,600	5,800	6,800
Crushing C	Ditto	—	—	3,800	—	—	—	—	—
Tension (tearing off of head)	$R_{r\cdot t}$	1,200	1,500	—	—	—	—	—	—

centration factors k_e being taken according to the last item of Table 2-13. For riveted connections subjected only to compressive alternating loads, the factor γ is taken equal to unity.

5-2. ANALYSIS AND DESIGN OF RIVETED CONNECTIONS

The analysis of a riveted connection should always be performed together with its designing.

(1) Selection of Rivet Diameter

Both the compactness of a joint and the labour requirements for manufacturing the members depend upon the rational selection of the diameter of the rivets.

The most widely used rivet diameters are as follows:

in members of medium thickness	20-23 mm
in heavy members	26-29.5 mm

The general trend is to use rivets of one diameter throughout the whole structure. Only in heavy structures with a great difference in the stresses induced in the separate members and connections may two or three diameters of rivets be tolerated.

Riveted connections are made using rivets with a normal button head (Fig. 5-1*a*), with a total thickness of the elements being connected together of up to 5*d*. In connections from 5*d* to 7*d* thick, rivets with a high button head (Fig. 5-1*d*) should be used. With a further increase in the thickness of the connection, turned bolts should be used.

(2) Analysis of Rivets for Shear and Crushing

In the investigation of riveted connections it is assumed that the load is distributed uniformly between all the rivets.

When *checking rivets for shear* it should be seen that the shearing stress in the rivets does not exceed the design shear strength of the rivet material

$$\tau = \frac{F}{\sum A_{r \cdot sh}} = \frac{F}{N n_{sh} \frac{\pi d^2}{4}} \leqslant R_{r \cdot sh} \qquad (5\text{-}1)$$

where F = design load applied to connection, kg
N = number of rivets in connection
n_{sh} = number of shear planes in one rivet
$\frac{\pi d^2}{4}$ = cross-sectional area of installed rivet, cm².

Hence the number of rivets N required to withstand the design load will be

$$N \geqslant \frac{F}{n_{sh} F'_{r \cdot sh}} = \frac{F}{n_{sh} \frac{\pi d^2}{4} R_{r \cdot sh}} \tag{5-2}$$

in which $F'_{r \cdot sh}$ is the maximum design load which may be applied to one shear plane of a rivet.

The number N is rounded off to the nearest whole number.

In *checking rivets for crushing* it should be seen that the crushing stress induced by the rivets in the elements being connected does not exceed the design crushing strength of the material

$$\sigma_{cr} = \frac{F}{NA_{cr}} = \frac{F}{Nd \sum t} \leqslant R_{r \cdot cr} \tag{5-3}$$

where $A_{cr} = d \sum t$ is the design crushing area of one rivet, conditionally taken equal to the product of the rivet diameter d and the least total thickness $\sum t$ of the plates being crushed at one side of the rivet shank.

The required number of rivets obtained when checking for crushing of the plates will be

$$N \geqslant \frac{F}{F'_{r \cdot cr}} = \frac{F}{d \sum t R_{r \cdot cr}} \tag{5-4}$$

in which $F'_{r \cdot cr}$ is the maximum design load that may be tolerated on one rivet from the standpoint of crushing.

The method of analysis set forth above is known as the method of analysis in accordance with the load applied to the connection.

It is also possible to investigate rivets not in accordance with the actual force or load applied to the connection, but according to the maximum possible one that can be withstood by the section of the element being connected. In this case the design load in the connection will be

for elements in tension

$$F = A_n R \tag{5-5}$$

for elements in compression

$$F = A_{gr} \varphi R \tag{5-6}$$

where A_n and A_{gr} = net and gross area of the element being connected

R = design strength of base metal

φ = buckling factor.

This method is known as the method of analysis according to the section area and provides a margin of safety. It is used in investigat-

ing butt joints and connections when the stresses in an element are close to the limit value or when the actual (design) load on the element is not known.

(3) Analysis of Rivets for Tension (Tearing off of Heads)

Rivets are checked for tension by means of the expression

$$\sigma = \frac{F}{N \frac{\pi d^2}{4}} \leqslant R_{r.t} \tag{5-7}$$

whence the required number of rivets will be

$$N \geqslant \frac{F}{\frac{\pi d^2}{4} R_{r.t}} \tag{5-8}$$

When rivets are simultaneously subjected to shear (crushing) and tensile stresses, they are checked for their resistance to both of them separately.

(4) Arrangement of Rivets and Conventional Symbols

When designing a connection, the rivets should be so arranged that they will ensure adequate strength and the best possible transmission of the load from one element to the other. Close spacing of the rivets (less than $3d$) causes excessive stresses in the base metal.

With a view to the conditions of fabrication, the rivets should have a simple arrangement, for which end the rivet centres are located on straight lines (gauge lines) running parallel and perpendicular to the centre line of the element.

The rivets can be arranged on the connection in rows or chains (Fig. 5-4*a*) or in a staggered order (Fig. 5-4*b*). The distance between rivet centres in the direction of load action is called the pitch, and across this direction is named the transverse pitch, or gauge.

The spacing of rivets should be taken as prescribed in Table 5-2.

The minimum pitches of rivets indicated in Table 5-2 are determined by the condition of adequate strength of the base material. The maximum pitches are established from the conditions of stability of the compression parts of the elements in the space between the rivets, and of a tight contact of parts in tension to avoid dust and moisture getting into the joint, which will facilitate increased corrosion.

TABLE 5-2 **Arrangement of Rivets and Bolts in Steel Members**

Characteristics of distance	Distance
Distance between rivet and bolt centres in any direction:	
(a) minimum	For rivets $3d$ For bolts $3.5d$
(b) maximum in extreme rows in the absence of edging angles—for tension and compression	$8d$ or $12t$
(c) maximum in middle rows, as well as in extreme rows in the presence of edging angles:	
for tension	$16d$ or $24t$
for compression	$12d$ or $18t$
Distance from centre of rivet or bolt to edge of element:	
(a) minimum, along direction of force	$2d$
(b) minimum, across direction of force:	
with cut off edges	$1.5d$
with rolled edges	$1.2d$
(c) maximum	$4d$ or $8t$

NOTATION: d=diameter of hole for bolt or rivet
t=thickness of thinnest external element.

At joints and connections of elements, the minimum pitches of the rivets should be so selected as to avoid the use of excessive metal. When arranging constructive rivets used to connect separate shapes into a single section, the maximum rivet pitches should be selected to reduce their number to the greatest possible extent.

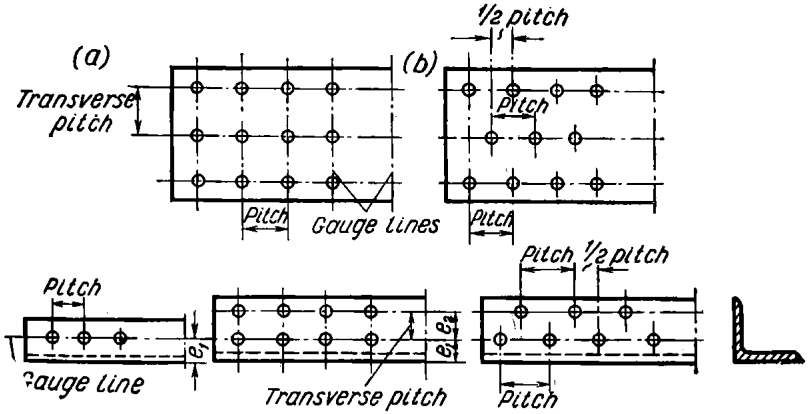


Fig. 5-4. Arrangement of rivets

The minimum number of rivets used to secure a working element of a member is taken equal to two irrespective of the force acting in the connection (an exception may be a connecting lattice of built up elements).

The distance from the rivet gauge lines to the back end of angles is taken in accordance with Table 5 of Appendix III.

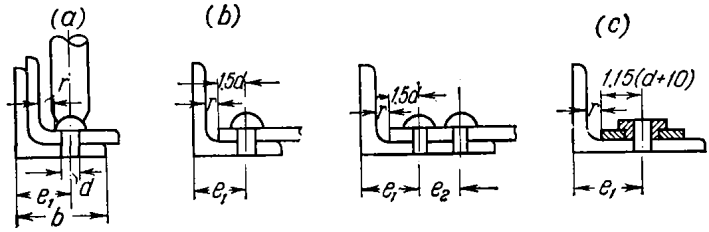


Fig. 5-5. Arrangement of rivet gauge lines on angles

The rivet gauge lines are so arranged as to make possible riveting and positioning of the dolly when joining angles (Fig. 5-5*a*). When using straps inside an angle (Fig. 5-5*b*) or employing special jigs for drilling (Fig. 5-5*c*) it should be seen that the appointed rivet gauge lines permit the work to be performed.

TABLE 5-3 **Conventional Symbols of Rivets and Holes**

Kind of rivet and hole	Symbol	Remarks
Normal button head rivets		—
Rivets with countersunk and chipped or not chipped head at one or both sides		Accompanied by explanatory note, for instance "at far side", etc.
Round hole		—
Hole counterbored at one or both sides		Accompanied by explanatory note, for instance, "counterbore at nearest side", etc.

Rivet gauge lines are arranged on I sections and channels in accordance with Tables 6 and 7 of Appendix III.

The conventional symbols used to indicate rivets and their holes on drawings are shown in Table 5-3.

The diameters of rivets prevailing on the given working drawing, as well as various deviations or instructions on the methods of performing the work are indicated in special notes.

(5) Types of Riveted Connections

Single- or double-strap butt and lap joints can be used for connecting metal sheets and plates (Fig. 5-6). The double-strap butt joint, which ensures symmetrical transfer of the load from one element to

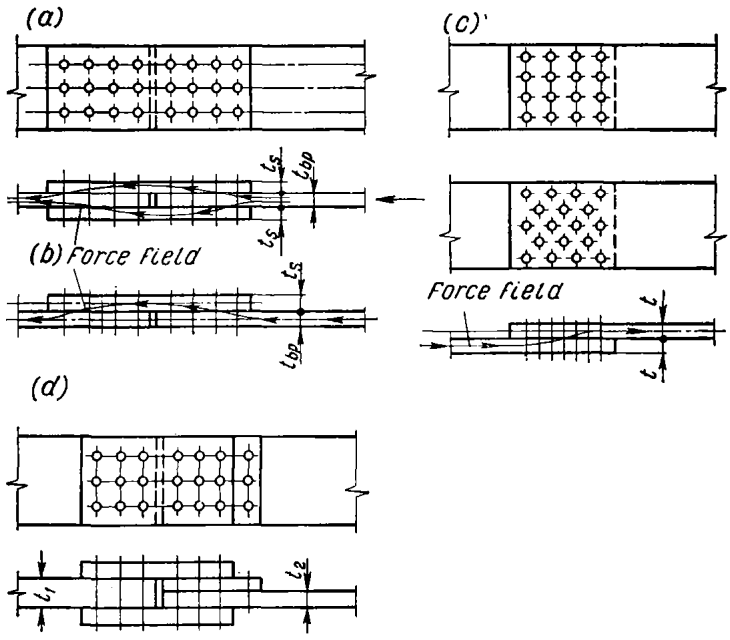


Fig. 5-6. Types of riveted plate connections

the other (Fig. 5-6*a*) should be considered as the main kind of joint. The employment of single strap butt joints (Fig. 5-6*b*), as well as lap joints (Fig. 5-6*c*) results in an eccentric connection, in which the forcelines deviate to a side of the centre line; this may cause bending of the plates under load.

For this reason when single-strap joints are used, as well as when the connections are made with filler plates (Fig. 5-6*d*), the standards

require a 10% increase in the number of rivets over the normal number found from analysis.

Angle sections, as a rule, are joined together with angle straps of the same sections. In this case the sharp back tip of the strap angle is bevelled to ensure tight fitting of the strap angle legs (Fig. 5-7*a*). Unequal leg and also large equal leg angles can be connected by means

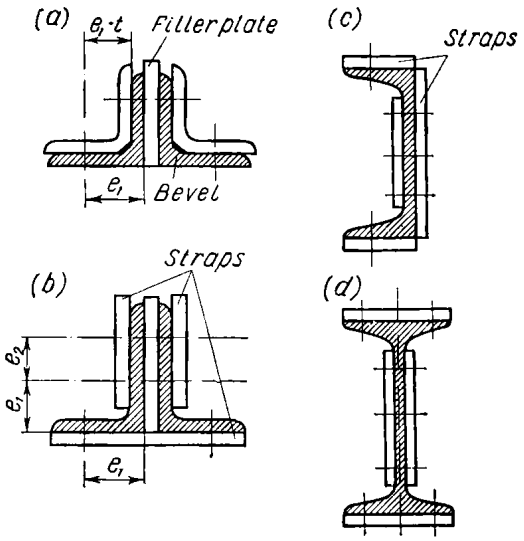


Fig. 5-7. Riveted connections of shapes

of plate straps if two rows of rivets can be arranged on each strap (Fig. 5-7*b*). Channels and I sections can be strapped as shown in Fig. 5-7*c* and *d*.

The total section area of the straps should not be less than the area of the basic section. The cross sections of the straps should be designed in proportion to the areas of the covered parts of the section.

(6) Exercises

Example 5-1. Design the connection of a compression diagonal of a roof truss to an angle gusset (Fig. 5-8). The section of the diagonal is composed of two angles 140×90×10. The design load is 63 tons, the material is grade ВСт.3 steel, the thickness of the gusset is $t=12$ mm or 1.2 cm. The holes are drilled (group B).

Solution. We shall use rivets 20 mm in diameter ($d=20$ mm or 2 cm). We find the number of rivets required to resist shear and crushing under the applied load. The number of rivets (double-shear) necessary to resist shear is found from

equation (5-2)

$$N=\frac{F}{n_{sh}\frac{\pi d^2}{4}R_{r\cdot sh}}=\frac{63{,}000}{2\times\frac{3.14\times 2^2}{4}\times 1{,}800}=5.6\ (6\ \text{rivets})$$

The number of rivets required to withstand crushing is determined from equation (5-4)

$$N=\frac{F}{dtR_{r\cdot cr}}=\frac{63{,}000}{2\times 1.2\times 4{,}200}=6.3\ (7\ \text{rivets})$$

The greater number of rivets obtained from the above two calculations is taken (in this instance to resist crushing) and the seven rivets are arranged as shown in Fig. 5-8.

To reduce the total length of the connection the rivets are arranged at minimum distances from each other, taking the distance from the edge of the angle to the nearest rivet (the edge distance) equal to 40 mm ($2d$) and half of the rivet pitch equal to 50 mm (the diagonal pitch will be $3.5d=70$ mm).

Example 5-2. Design the connection of a tension vertical to a heavy double-plane truss composed of two No. 24 chan-

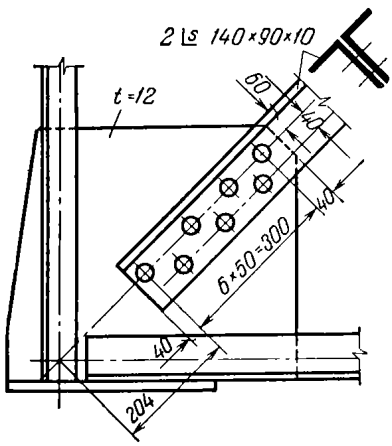


Fig. 5-8. To example 5-1

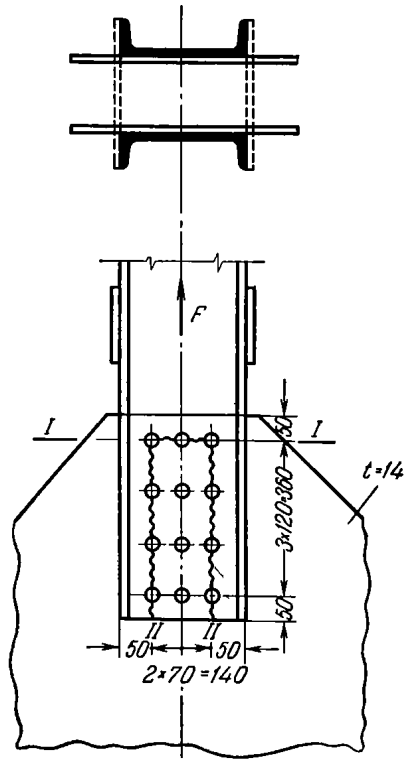


Fig. 5-9. To example 5-2

nels connected with batten plates (Fig. 5-9). The design load in the vertical is 107 tons, the material is grade ВСт.3 steel, the thickness of the gussets to which the branches of the vertical are secured is $t=14$ mm. The holes for the rivets are jig-drilled.

Solution. We shall use rivets 23 mm in diameter ($d=23$ mm or 2.3 cm). Equation (5-2) is used to find the number of rivets required to secure each branch

with a view to resisting the shearing stresses (single-shear rivets)

$$N=\frac{F}{2n_{sh}\frac{\pi d^2}{4}R_{r.sh}}=\frac{107{,}000}{2\times1\times\frac{3.14\times2.3^2}{4}\times1{,}800}=7.2\,(8\text{ rivets})$$

To withstand crushing of the channel web it is necessary to have the following number of rivets for connecting each branch (with a thickness of the channel web of 0.56 cm)

$$N=\frac{F}{2dtR_{r.cr}}=\frac{107{,}000}{2\times2.3\times0.56\times4{,}200}=9.9\,(10\text{ rivets})$$

We design the connection as illustrated in Fig. 5-9. The rivets are arranged in three rows with a transverse pitch of 70 mm, and 12 rivets with a diameter of $d=23$ mm (2.3 cm) will be used. The distance from the edges to the longitudinal gauge lines of the rivet rows along the web of channel No. 24 is 50 mm, the distances from the extreme rivets to the edges of the elements are 50 mm. The pitch of the rivets, 120 mm, was determined after checking the resistance of the channel web to tearing out of the metal (see below).

Let us check the strength of the vertical branches in the weakened or critical sections. The section of each channel is weakened by three holes; the area of the critical section (*I-I*) of the channel will be

$$A_n=30.6-0.56\times2.3\times3=26.74\text{ cm}^2$$

The stress over the critical section *I-I* will be

$$\sigma=\frac{107{,}000}{2\times26.74}=2{,}000\text{ kg/cm}^2<2{,}100\text{ kg/cm}^2$$

We take the second critical section along the inverted U-shaped line (Fig. 5-9, section *II-II*), along which tearing out of the web may take place. Owing to the webs of standard channels being relatively thin, this check is very essential. In the critical inverted U-shaped section *II-II* (Fig. 5-9) the horizontal part of the area of the element section (A_n') located perpendicular to the line of action of the load is subjected to normal stresses, while the parts of the area that are parallel to the line of action of the load (A_n'') are subjected to shearing stresses. Therefore, the condition must be complied with that

$$F\leqslant A_n'R+A_n''R_{sh}=A_n'R+A_n''\times0.6R=A_{red}R$$

where the reduced area $A_{red}=A_{red}'+0.6A_n''$ and $R_{sh}=0.6R$.

We plan four rows of rivets with a pitch of 120 mm (Fig. 5-9). Hence

$$A_{red}=A_n'+0.6A_n''=(14-2\times2.3)\,0.56+0.6\times0.56\times2\,[(3\times12+5)-3.5\times2.3]=$$
$$=27.4\text{ cm}^2$$

The stress over the critical section *II-II* will be

$$\sigma=\frac{F}{2A_{red}}=\frac{107{,}000}{2\times27.4}=1{,}955\text{ kg/cm}^2<2{,}100\text{ kg/cm}^2$$

With a rivet pitch less than 120 mm the stress over section *II-II* will be greater than the design strength.

(7) Labour Requirements for Fabrication of Riveted Members

The process of fabricating riveted steel members consists of the following principal operations, which are mainly the same as in welded member fabrication:

1. Preparatory operations—selection and matching of the rolled metal, dressing it in plate and angle rolls or gag presses, and cutting of the metal.
2. Preparation of patterns and templates in the marking department, and the transfer of the dimensions required for processing from the patterns or templates to the metal parts.
3. Machining of parts in the preparatory shop: gas or power shear cutting of the metal, the making of holes either by punching or drill-

TABLE 5-4 **Labour Requirements for Separate Operations in Fabrication of Riveted Members**

Name of operation	Per cent of total production operation time
1. Preparatory:	
Dressing of steel	3-4
Manufacture of patterns and marking	5-6
Total	8-10
2. Processing of parts:	
Marking out of parts	2-3
Cutting	6-8
Formation of holes	12-15
Milling and other machining of edges	2-3
Dressing and bending	2-3
Total	24-32
3. Assembly, riveting and shipping of members:	
Assembly	27-28
Reaming of holes	4-5
Riveting	20-22
Drilling of assembly holes	3-4
Priming (painting) and shipping	4-5
Total	58-64

ing in the respective machines, machining of the edges in edge-planing machines.

4. Assembly and riveting in the assembly and riveting shop, control of the quality of fabrication, painting and shipping of the members to the erection site.

The average distribution of labour in the fabrication of various kinds of riveted members is shown in Table 5-4. About 60% of the working time is used for operations connected with assembly and riveting. It is obvious that the labour requirements for the fabrication of riveted members will be reduced if the designer uses the smallest possible number of parts and the minimum number of rivets that will ensure normal service of the members.

(8) Riveted Connections of Aluminium Members

Riveted connections of aluminium members are mainly designed and investigated in the same way as their steel member counterparts, but using the design strengths indicated in Table 5-5.

TABLE 5-5 **Design Strengths of Cold-Riveted Aluminium Member Connections, R_r kg/cm²**

Stressed state and group of connection	Symbol	Design strengths R_r for cold rivets made from alloy						
		АВТ-1	Д18п			В65		
		in members made from alloy						
		АВТ-1	Д16-Т	Д1-Т	АД33-Т1	Д16-Т	Д1-Т	АД33-Т1
Shear B	$R_{r \cdot sh}$	1,000	1,100	1,100	1,100	1,450	1,450	1,450
Shear C	Ditto	800	900	900	900	1,150	1,150	1,150
Crushing B . . .	$R_{r \cdot cr}$	2,400	3,900	2,900	2,400	3,900	2,900	2,400
Crushing C . . .	Ditto	2,150	3,100	2,300	1,900	3,100	2,300	1,900
Tension (tearing off of head) . .	$R_{r \cdot t}$	—	500	500	500	700	700	700

The rivets should be arranged in accordance with Table 5-6.

The diameter of the rivet hole is taken as the design diameter of the rivet.

The diameter of a rivet must not be less than the thickness of the thickest element of a connection, and must not exceed the tripled thickness of the thinnest element.

The overall thickness of the connected elements should not exceed three rivet diameters.

TABLE 5-6 **Arrangement of Rivets and Bolts in Aluminium Members**

Characteristics of distance	Distance
Distance between rivet and bolt centres in any direction:	
(a) minimum	For rivets $3d$ For bolts $3.5d$
(b) maximum in extreme rows in the absence of edging angles—for tension and compression	$5d$ or $10t$
(c) maximum in middle rows, as well as in extreme rows in the presence of edging angles:	
for tension	$12d$ or $20t$
for compression	$10d$ or $14t$
Distance from centre of rivet or bolt to edge of element:	
(a) minimum, along direction of force and along diagonal	$2.5d$
(b) minimum, across direction of force:	
with cut off edges	$2.5d$
with rolled or pressed edges	$2d$
(c) maximum	$6t$

NOTATION: d = diameter of hole for bolt or rivet
t = thickness of thinnest external element

5-3. BOLTED CONNECTIONS

(1) General

Bolted connections are a very widespread method of securing members in erection and assembly, especially in industrial construction. In field conditions it is important to have a simple and sufficiently reliable way of fastening elements and members that does not require any special equipment consuming power.

Bolts, the same as rivets, are subjected to shear, crushing and tension. A property that noticeably distinguishes a bolted connection from its riveted counterpart is its greater give or compliance (deformability). This is the result of a lower preliminary tension of the bolts (caused by tightening of the nuts), and also of the presence of clearances between the bolt and its hole.

When bolts are to work in tension, their preliminary tensioning is of great significance, and it should be greater than the external

tension load. Since it is difficult to ensure equal tightening of several bolts in a connection, and that they will be subjected to uneven stresses, lower design strengths are established for bolts.

Excessive tightening of bolts (which is possible when spanners or wrenches are used for assembly that have been extended by fitting lengths of tubes onto them) leads to plastic strains and improper behaviour of the connection

The following types of bolts are employed in steel members:

1. Black or unfinished bolts (GOST 7798-57) with nuts (GOST 5909-51).

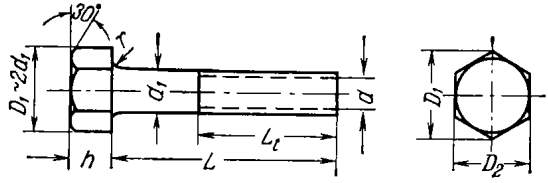


Fig. 5-10. Unfinished bolt

2. Semifinished bolts (GOST 7798-57) with nuts (GOST 5915-51).

3. Bright or finished (turned) bolts (GOST 7805-57) with nuts (GOST 5926-51).

4. High-strength or heavy bolts.

The washers for all the bolts are taken in accordance with GOST 6957-54.

Unfinished bolts (Fig. 5-10) are stamped from round steel bars, deviations from the nominal diameter of ± 0.75 to 1 mm being tolerated. Unfinished bolts may be fitted into holes with a clearance of 2-3 mm.

Semifinished bolts with a hexagonal head are manufactured from calibrated round steel bars. For this reason the tolerance in the diameter is only a negative one ranging from -0.5 to -1 mm.

Bright or *turned* bolts with a hexagonal head have a turned shank with a negative tolerance of up to -0.34 mm. They can be fitted into holes with a clearance of 0.3 to 0.5 mm, which means tight fitting. A bolt can be placed into such a hole only by means of light blows of a hammer.

Bolts are manufactured with a diameter of 10, 12, 14, 16, 18, 20, 22, 24, 27, 30, 36, 42 and 48 mm and a total length ranging from 40 to 200 mm. The length of the thread is L_t=20-25 mm for bolts with d=10-14 mm, L_t=28-30 mm for d=16-20 mm and L_t=35-50 mm for d = 22-30 mm.

Bolts are manufactured of grade ВСт.3, ВСт.5 carbon steels, grade 14Г2, 15ГС low-alloy steels, etc.

High-strength or *heavy* bolts are employed in field joints and transfer the load by means of friction. They are made from carbon or alloy steels (for example grade 40X chromium steel to GOST 4543-57) with an ultimate strength after heat treatment of at least 100 kg/mm² for carbon and 130 kg/mm² for alloy steels.

(2) Behaviour of Bolted Connection

When unfinished bolts are subjected to shearing stresses, the connection has a great deformability owing to the presence of clearances. Investigations carried out by G. Shapiro have shown that the shear of rivets at stresses near the yield point constitutes 0.24 mm on the average, while the shear of unfinished bolts with clearances from 1 to 4 mm varies correspondingly from 2.65 to 3 mm (i.e., about ten times greater). At the same time the statical strength of a bolted connection is only about 10% less than that of a riveted one, which is explained by equalizing of the loads on the separate bolts when they are stressed above the yield point. The vibration strength of a bolted connection is considerably (about 50%) less than that of its riveted counterpart. For this reason it will be good practice to use unfinished bolts when they are subjected to shear only under statical loads, where the principal requirement is strength, while the deformability of the connection is not of great significance. An effective but costly means of reducing the deformability of a bolted connection is the transition to bright bolts that will tightly fill their holes.

It is more rational to use bolts when they are subjected to tension. For reliable behaviour of a bolt, good tightening thereof is essential, since the external tensile force must first reduce the compression of the elements due to the initial tensioning. To reduce the deformability of a connection, either the stress in the bolts must not be allowed to reach the design strength, or the tightened nuts must be reliably secured with locknuts or by welding them to the bolts.

(3) Analysis and Design of Bolted Connections

The following expressions are used for analyzing bolted connections: for shear

$$N \geqslant \frac{F}{n_{sh} \frac{\pi d^2}{4} R_{b \cdot sh}} \qquad (5\text{-}2')$$

for crushing

$$N \geqslant \frac{F}{d \sum t R_{b \cdot cr}} \qquad (5\text{-}4')$$

for tension (taking into account the net section area of the bolt)

$$N \geqslant \frac{F}{\frac{\pi d_m^2}{4} R_{b \cdot t}} \tag{5-8'}$$

Here F = design force applied to the connection
d = diameter of unthreaded part of bolt
d_m = root diameter of shank thread
R_b = design shear, crushing and tension strengths of bolts taken from Table 5-7.

Heavy bolts are employed for obtaining a less deformable field connection. The bolts should be so tightened that the forces acting in the joint will be transmitted through the mating bearing surfaces. This is attained by providing the required friction between these surfaces by means of mechanical wrenches that are calibrated to control the tensioning of the bolts.

It is obvious that when heavy bolts are employed, a greater clearance can be tolerated between the hole and the bolt than when finished bolts are used, and this is also one of their merits.

When field connections are made with heavy bolts instead of rivets, the necessity of hot pneumatic riveting disappears.

Of great importance for the behaviour of connections made with heavy bolts is the surface cleanliness of the elements being connected. Steel wire brushes are sometimes used for cleaning the surfaces, but it is better to make use of pneumatic cleaning (with sand, metal powder, etc.) or roasting.

Heavy bolts are investigated as follows. The axial tensioning force P_b is determined for a completely tightened bolt. This force depends upon the mechanical properties of the bolt material after its heat treatment. Assuming a homogeneity factor of 0.8 for heavy bolts and a service condition factor of 0.75 for the material in the member, the maximum force of tensioning the bolt will be

$$P_b = 0.6\sigma_{b \cdot u} A_{b \cdot n} \tag{5-9}$$

in which $A_{b \cdot n}$ is the net area of the bolt section (in the threaded part).

The design load or force, which can be applied to each friction surface of the elements being connected by means of one heavy bolt, will be

$$F'_{b \cdot h} = P_b f k_2 \tag{5-10}$$

where f = coefficient of friction, taken according to Table 5-8
k_2 = factor of service conditions for connection with heavy bolts, taken equal to 0.9.

TABLE 5-7 **Design Strengths of Bolted Connections, R_b kg/cm²**

Kind of bolted connection	Stressed state and group of connection	Symbol	Design strengths R_b									
			For tension and shearing of bolts made from steel of grade				For crushing of elements being connected made from steel of grade					
			ВСт.3, ВСт. 3кп	ВСт.5	09Г2	14Г2, 15ГС, 15ХСНД	steel 3 and steel 4	steel 5	14Г2 and 15ГС with thickness of rolled stock, mm: 20 and less	14Г2 and 15ГС with thickness of rolled stock, mm: 21-32	10Г2С, 10Г2СД, 15 ХСНД	10ХСНД
Bright and semi-finished (high precision) bolts	Tension	$R_{b\cdot t}$	1,700	1,900	2,000	2,300	—	—	—	—	—	—
	Shear B	$R_{b\cdot sh}$	1,700	1,800	2,000	2,200	—	—	—	—	—	—
	Crushing B	$R_{b\cdot cr}$	—	—	—	—	3,800	4,100	5,200	5,000	5,200	6,100
Unfinished bolts (normal precision): in single-bolt connections	Tension	$R_{b\cdot t}$	1,700	1,900	2,000	2,300	—	—	—	—	—	—
	Shear	$R_{b\cdot sh}$	1,500	1,600	—	—	—	—	—	—	—	—
	Crushing	$R_{b\cdot cr}$	—	—	—	—	3,800	—	—	—	—	—
in multibolt connections	Tension	$R_{b\cdot t}$	1,700	1,900	2,000	2,300	—	—	—	—	—	—
	Shear	$R_{b\cdot sh}$	1,300	1,400	—	—	—	—	—	—	—	—
	Crushing	$R_{b\cdot cr}$	—	—	—	—	3,400	—	—	—	—	—
Anchor bolts	Tension	$R_{b\cdot t}$	1,400	1,500	1,700	1,900	—	—	—	—	—	—

Thus the number of heavy bolts required in a connection (assuming uniform distribution of the load between them) will be

$$N \geqslant \frac{F}{F'_{b.h}} = \frac{F}{0.54\sigma_{b.u} A_{b.n} f} \tag{5-11}$$

in which F is the design load on the connection.

Bolts are positioned in accordance with the rules for the arrangement of rivets set forth in Table 5-2 (except that the minimum distance between bolts must always equal $3.5d$). Besides, it is necessary to take into account the possibility of screwing on the nuts with standard spanners or wrenches.

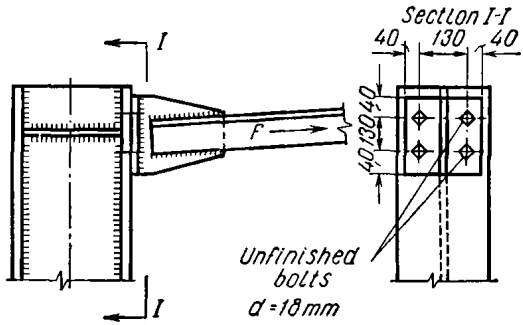


Fig. 5-11. To example 5-3

Table 5-9 contains the recommended dimensions of holes for unfinished bolts both of the standard metric and inch classifications.

TABLE 5-8 **Values of Coefficient of Friction *f***

Method of preliminary cleaning of surfaces being connected	Material of member elements being connected	
	Carbon steel	Low-alloy steel
Pneumatic cleaning (with sand, metal powder, etc.) or roasting	0.45	0.55
Cleaning with steel wire brushes	0.35	—
No cleaning (with partial replacement of rivets or bolts)	0.25	—

The conventional symbols used to designate bolts in drawings are given in Table 5-10.

TABLE 5-9 **Recommended Dimensions of Holes for Unfinished Bolts**

Metric standard	Diameter of bolt, mm . .	12	14	16	18	20	22	24	27	30	36
	Area of section A_{gr}, cm²	1.13	1.54	2.01	2.54	3.14	3.80	4.52	5.73	7.07	10.18
	Area of section over thread A_n, cm² . .	0.76	1.04	1.44	1.75	2.25	2.81	3.24	4.27	5.18	7.58
Inch standard	Diameter of bolt:										
	in inches . . .	$\frac{1}{2}$		$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$		1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$
	in mm	12.7		15.7	19.1	22.2		25.4	26.8	31.8	34.9
	Area of section A_{gr}, cm² . .	1.27		1.98	2.85	3.88		5.07	6.41	7.92	9.55
	Area of section A_n, cm² . .	0.78		1.31	1.96	2.72		3.57	4.93	5.77	7.07
Recommended diameter of hole, mm		14	16	18	20	22	24	26	29.5	32.5	29.5

TABLE 5-10 **Conventional Symbols for Bolts and Holes**

Kind of bolts and holes	Symbol	Remarks
Permanent bolts in shop connections		—
Temporary bolts connecting elements into one assembly for shipping		In the assembly diagrams and drawings of multiple joints denotes permanent bolts in field connections
Heavy bolts		—
Oval hole		—

Example 5-3. Investigate the connection of the top chord of a roof truss to a column using unfinished bolts (Fig. 5-11). The design tensile load in the chord $F=10$ tons, the members and the bolts are made from grade BCr. 3 steel.

Solution. We shall use bolts with a diameter of $d=18$ mm and a net section area of $A_n=1.75$ cm^2.

We find the number of bolts required for securing the chord from expression (5-8′)

$$N=\frac{F}{A_nR_{b\cdot t}}=\frac{10{,}000}{1.75\times1{,}700}=3.4\,(4\text{ bolts})$$

Four bolts with a diameter of $d=18$ mm will be used, with bolt holes having a diameter of 20 mm.

TABLE 5-11 **Design Strengths of Aluminium Member Connections Using Aluminium Bolts, R_b kg/cm^2**

Stressed state	Symbol	Values of R_b for bright bolts of alloy							Values of R_b for bolts with grip rings of alloy			
		АМг5п	АВ-Т1		В94		Д16п		Д16п		В65	
		in members of alloy							in members of alloy			
		АМг6	АВ-Т1	АД33-Т1	Д1-Т	Д16-Т	Д1-Т	Д16-Т	Д1-Т	Д16-Т	Д1-Т	Д16-Т
Shear *B*	$R_{b\cdot sh}$	900	900	900	1,600	1,600	1,300	1,300	1,700	1,700	1,700	1,700
Crushing B	$R_{b\cdot cr}$	1,900	2,500	2,150	2,600	3,500	2,600	3,500	2,600	3,500	2,600	3,500
Tension (tearing off of head) . .	$R_{b\cdot t}$	1,250	1,700	1,700	2,950	2,950	2,000	2,000	1,600	1,600	1,600	1,600

NOTE. The data for bolts with grip rings relate to bolts with a diameter of 6-12 mm.

(4) Bolted Connections of Aluminium Members

The elements of aluminium members can be connected by means of aluminium or steel bolts, taking a number of measures to prevent corrosion.

There have been recently introduced aluminium bolts with grip rings (lock bolts), made from a high-strength alloy (Fig. 5-12*a*). A grip ring made from a light plastic alloy is fitted onto the bolt, and the ring is set with a special tensioning gun, thus pulling together the plates being connected. Tensioning is performed with such a

force that the end of the bolt is torn off along the critical section (Fig. 5-12*b*).

The arrangement, designing rules and the analysis of aluminium bolts remain the same as for aluminium rivets. The material and the design strengths of aluminium bolts are indicated in Table 5-11.

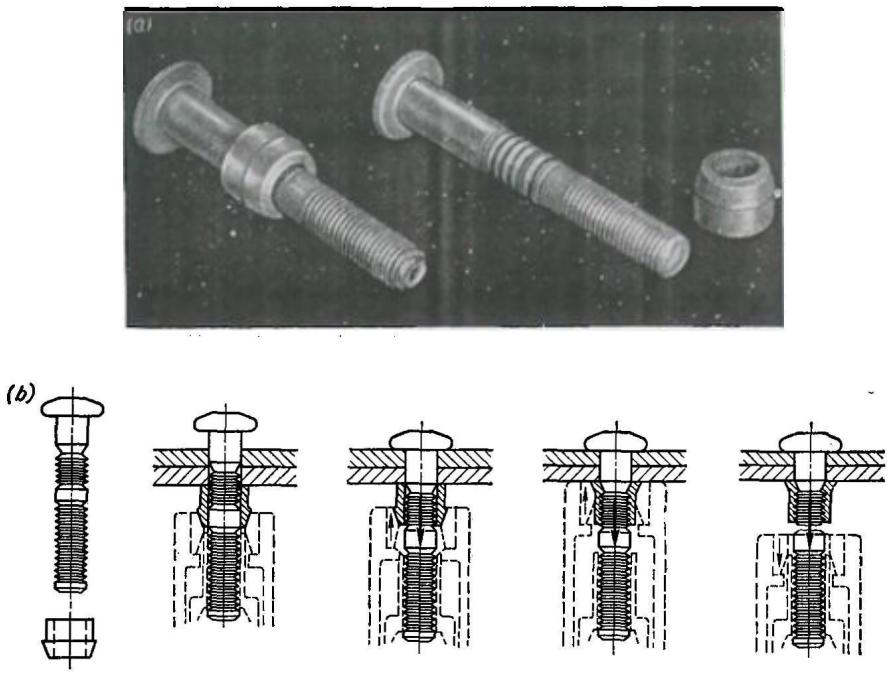


Fig. 5-12. Aluminium bolts with grip rings:
a—view of bolt; *b*—sequence of operations in installation

Should steel bolts be employed for connecting aluminium members, the bolts must be zinc- or cadmium-plated in order to prevent corrosion at the place of contact between the aluminium alloys and the steel. To prevent local damage to the aluminium alloys when the bolts are being tightened, use must be made of washers.

CHAPTER SIX

Beams and girders

6-1. GENERAL CHARACTERISTICS. BEAM GRILLAGES

(1) Types

Metal beams and girders, which are subjected to bending, are either of the *rolled* or *built up* type. Preference should be given to the use of rolled beams, which will reduce labour requirements (Fig. 6-1*a*). Due to the limited number of standard sections, however, it becomes necessary to design heavy beams (and girders) of a riveted or welded built up type. Welded beams are made up of three plates, namely, one vertical one, called the *web*, and two horizontal ones, known as the *flanges*, which are welded to the web (Fig. 6-1*b*). Riveted I beams are composed of a vertical plate (web) and flange angles that are riveted to the web (Fig. 6-1*c*). If it is necessary to use heavy riveted

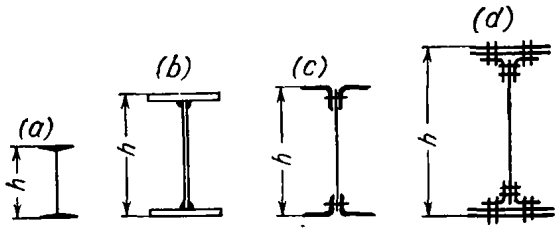


Fig. 6-1. Beam sections

beams, then horizontal plates are riveted to the legs of the angles to increase the section modulus (Fig. 6-1*d*). Welded beams are more economical than their riveted counterparts, and this is why the latter are used only to a limited extent, mainly in heavy members, as well as in members that are subjected to large dynamic or vibration loads.

(2) General Dimensions

The general dimensions of a beam or a girder are its effective span length and the depth of its section. The effective span length L of a beam is the distance between the centres of its supporting parts (Fig. 6-2).

The actual length of a beam L_a is always somewhat greater than its effective span length. The length L_u is called the unsupported or clear length. It is usually determined by the service conditions of the structure and is selected with a view to economical considerations.

The depth of the section h is selected with regard to ensuring the optimal relations between the section dimensions which will result in a minimum consumption of steel (if there are no special design requirements limiting the overall dimensions of the beam), as well as to providing the required stiffness of the beam, which is determined

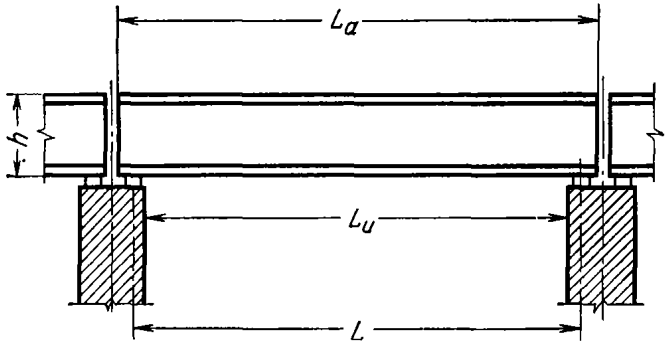


Fig. 6-2. General dimensions of beam

by the ratio between its deflection and the span, i.e., by the unit deflection $\frac{\delta}{L} \leqslant \frac{1}{r_0}$. The maximum values of the quantity $\frac{1}{r_0}$ are established by standards (see Appendix I, Table 3). The ratio between the deflection of a beam and its span directly depends on the ratio between the depth of the section and the span. If the uniformly distributed dead load is p and the movable or live load is q, then the maximum bending moment for a simple beam will be

$$M = \frac{(pn_p + qn_q) L^2}{8} = \sigma W = \frac{\sigma 2I}{h} \tag{6-1}$$

where n_p and n_q are the respective load factors.

As has been mentioned in Chapter 1, when deflections are being determined, only the design load is taken into consideration, without using any load factors. Hence, with a view to equation (6-1), let us find the relation between the unit deflection on one hand, and the design load and the depth of the beam on the other, expressed in terms of the design stresses

$$\frac{\delta}{L} = \frac{5}{384} \times \frac{(p+q) L^3}{EI} = \frac{5}{48} \times \frac{ML}{EI} \times \frac{p+q}{pn_p + qn_q} =$$

$$= \frac{5}{24} \times \frac{\sigma L}{Eh} \times \frac{p+q}{pn_p + qn_q} \leqslant \frac{1}{r_0} \tag{6-2}$$

From the above expression the minimum depth of a beam can be determined for a given unit deflection

$$h_{min} = \frac{5}{24} \times \frac{\sigma L r_0}{E} \times \frac{p+q}{pn_p + qn_q} \tag{6-3}$$

In view of the necessity of completely utilizing the design strength of the material ($\sigma = R$) in the beam and taking into account that for steel 3 we have $\frac{R}{E} = \frac{2{,}100}{2.1 \times 10^6} = \frac{1}{1{,}000}$, and also assuming that $n_p = n_q$, expression (6-3), after the substitutions, will become

$$h_{min} = \frac{L r_0}{4{,}800} \times \frac{1}{n_q} \tag{6-4}$$

Table 6-1 contains values of the ratio between the minimum depth of a beam section and the span length depending on the values of the unit deflection $\frac{\delta}{L} = \frac{1}{r_0}$, which can be used to determine the minimum depth of a beam section.

TABLE 6-1 **Ratio Between Minimum Depth of Beam Section and Span Length Depending Upon Unit Deflection (for Beams of Steel 3)**

$\frac{\delta}{L} = \frac{1}{r_0}$	$\frac{1}{1{,}000}$	$\frac{1}{750}$	$\frac{1}{600}$	$\frac{1}{500}$	$\frac{1}{400}$	$\frac{1}{250}$	$\frac{1}{200}$
$\frac{h_{min}}{L}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{25}$	$\frac{1}{30}$

With a depth of the beam less than that indicated in Table 6-1, it is also possible to obtain the stipulated unit deflection of the beam, but only with reduced stresses, i.e., with an increased consumption of metal.

The minimum required depth of a beam is not always the most advantageous one from the viewpoint of steel consumption. The most advantageous depth of a beam will be considered separately (Sec. 6-3).

(3) Beam Grillage

When designing beam (and girder) members it is essential, depending upon the designation of the beams, to draw up a diagram showing their arrangement, plan the general dimensions and determine the

load to be carried by the beams. If it is necessary to cover a certain area, the beams supporting the member are generally arranged in two directions. Such a member, sometimes consisting of a whole system of intersecting beams, is referred to as a *beam grid* or *grillage.* The grillage is covered with a flooring in the form of a metal sheet (the working areas of shops, hydraulic engineering structures), reinforced concrete slabs, etc. A grillage consists of *main* beams (girders) bridging a large span or bay, and *secondary* beams or *joists* (Fig. 6-3*a*,

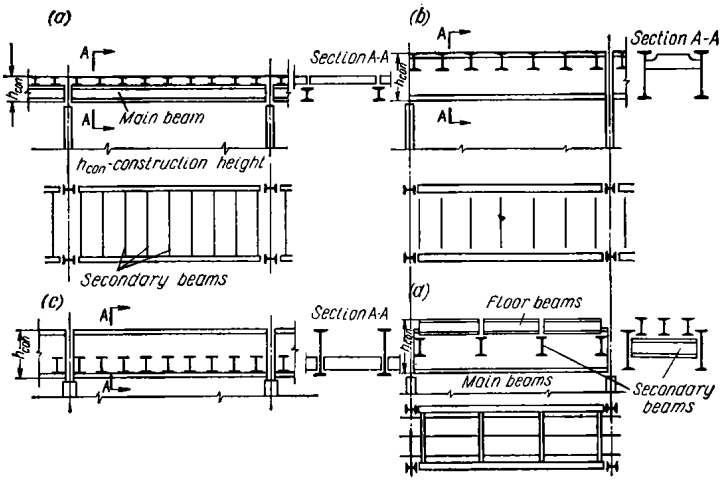


Fig. 6-3. Types of beam grillages

b and *c*). The main beams rest on supports, while the secondary ones are supported by the main beams. The following types of grillages are distinguished:

1. With a tier arrangement of the secondary beams (Fig. 6-3*a*).
2. With the secondary beams arranged on one level with the main ones (Fig. 6-3*b*).
3. With the secondary beams located at a lower level (Fig. 6-3*c*).
4. A beam grillage of a complicated type consisting of three systems of beams, namely, main, secondary and floor beams (Fig. 6-3*d*).

The type of grillage to be used is selected with a view to economical considerations, as well as to the established overall dimensions and clearances, which depend on the service conditions. The maximum height limiting the dimensions of a member is referred to as the *construction height.*

Determining Load on Beams. To determine the load on a beam (when the grillage is subjected to a uniformly distributed load) the

tributary area of this beam must be found. In Fig. 6-4 tributary area *1* of a secondary beam (cross-hatched) has a width of b (beam spacing) and a length of L_s (span of beam). A main beam is subjected to a load from tributary area *2*, while tributary area *3* shows the load to which

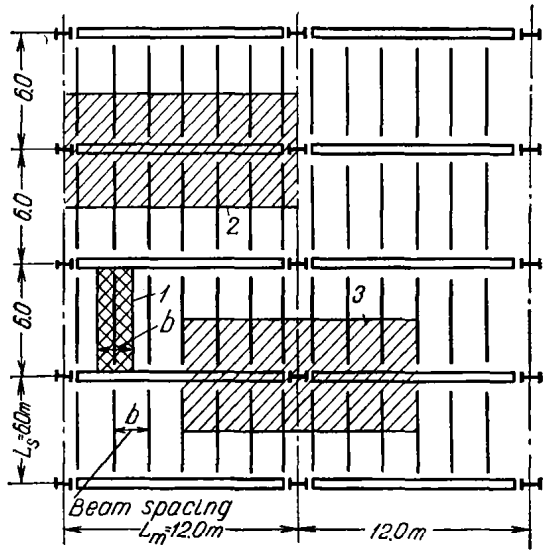


Fig. 6-4. Distribution of load on beams of a grillage

a column is subjected. When investigating rolled beams the influence of their own weight may be neglected owing to its insignificant value.

(4) Analysis of Sheet Flooring

A flat metal sheet flooring is placed on the flanges of the beams and welded to them (Fig. 6-5*a*). The thickness of the flooring is most frequently established depending upon the allowable deflection, and for this reason sheet flooring is analyzed in accordance with the loads stipulated in building standards.

As regards the nature of its behaviour, a sheet flooring occupies an intermediate position between a plate and a membrane. While a plate under a load is subjected only to bending (Fig. 6-5*b*), a membrane is subjected only to axial tension, for which condition fixed supports are required. A flooring may be subjected to both bending and axial tension, behaving as an elastic suspended member (Fig. 6-5*c*).

We shall consider only cases of a sufficiently long flooring supported at two sides (with a ratio of the length to the span of the sheet exceeding 2).

Axial tension may appear in such a flooring only if the plates are fixed at their edges and the constraints are able to take up thrust loads F_h. This phenomenon is known as bending with membrane action. If there is no thrust load (membrane action) or if it is very small,

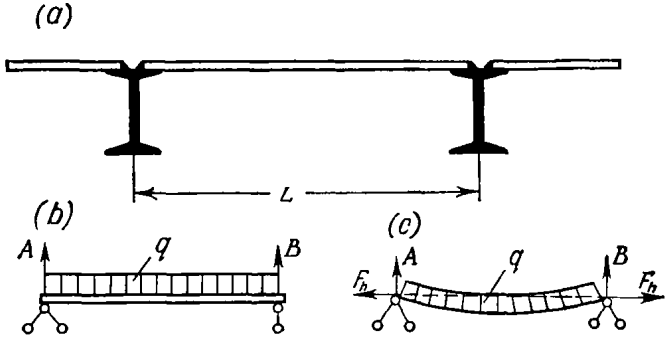


Fig. 6-5. Flat sheet flooring

the sheet may be considered to be subjected only to bending, as a plate. Let us consider both of these cases of bending.

First Case—Bending with Membrane Action. Let us cut out a strip b centimetres wide from a sheet and consider its behaviour under the action of a uniformly distributed load q kg/cm (i.e., considering the load on 1 linear centimetre of the span with a width of the plate of b centimetres). The design span L can be taken equal to the unsupported length between the flanges of the beams. Hence with a given unit deflection of $\frac{1}{r_0}=\frac{\delta}{L}$ the thickness of the sheet will be determined with an accuracy sufficient for practical purposes from the equation

$$t=\frac{qLr_0^3}{16\,bE}\ \text{cm} \tag{6-5}$$

The stress in the sheet will be equal to

$$\sigma=\left(\frac{qLr_0^2}{11.3bt\sqrt{E}}\right)^2+\frac{qr_0^2}{3b}\ \text{kg/cm}^2 \tag{6-6}$$

The intensity of the thrust or membrane action F_h with a thickness of the sheet t will be

$$F_h=\sigma t\ \text{kg/cm} \tag{6-7}$$

With small deflections of the order of $\frac{1}{r_0} < \frac{1}{250}$, equilibrium of the bent sheet can be ensured by its bending as a pure plate, with no membrane effect.

Second Case—Bending with No Membrane Action. In this case the thickness of the sheet is determined from the condition of bending of a simple beam, using an expression similar to equation (6-2)

$$\frac{\sigma}{L} = \frac{5}{384}\frac{qL^3}{EI_{cyl}} = \frac{5}{36}\frac{qL^3}{Ebt^3} = \frac{1}{r_0}$$

where $EI_{cyl} = \frac{Ebt^3}{12(1-\mu^2)}$ = cylindrical stiffness of the plate, in which the quantity $(1-\mu^2)$ is a correction allowing for the absence in the plate of a transverse linear deformation (μ is Poisson's ratio, taken equal to 0.3 for steel).

Next we find

$$t = \frac{L}{1.93}\sqrt[3]{\frac{qr_0}{bE}} \text{ cm} \tag{6-8}$$

$$\sigma = \frac{M}{W} = \frac{qL^2}{8}\frac{6}{bt^2} = \frac{3}{4}\frac{q}{b}\left(\frac{L}{t}\right)^2 \text{ kg/cm}^2 \tag{6-9}$$

The approximate boundary limiting the application of the equations deduced for the first and the second cases will be such a ratio $\frac{L}{t}$ in which the same thickness and the stresses in the sheet will be obtained from equations (6-5) and (6-8), and respectively from (6-6) and (6-9). This corresponds to the value of the load of

$$q = \frac{23.8bE}{r_0^4} \text{ kg/cm} \tag{6-10}$$

For values of q less than those obtained from equation (6-10), the analysis should be performed for the first case, and for greater values—for the second case.

The value of $\frac{1}{r_0}$ used in designing various structures generally ranges from $\frac{1}{100}$ to $\frac{1}{200}$. Table 6-2 contains expressions for calculating flooring for two values of the curvature $\frac{1}{r_0}$, viz., $\frac{1}{150}$ and $\frac{1}{200}$.

It is not recommended to design a thickness of sheet flooring less than 6 mm.

Example 6-1. Analyze a flooring for a design load $q_d = 400$ kg/m^2 and a design length of the flooring span of $L = 80$ cm.

TABLE 6-2 **Expressions for Analysis of Flooring**

$\frac{\delta}{L}=\frac{1}{r_0}$	$\frac{1}{150}$		$\frac{1}{200}$	
Linear load q (kg/cm) on strip 100 cm wide	$q \leqslant 10$	$q > 10$	$q \leqslant 3$	$q > 3$
Kind of deformation in equilibrium	Bending with membrane action	Bending	Bending with membrane action	Bending
Thickness t, cm	$\frac{qL}{1{,}000}$	$\frac{L\sqrt[3]{q}}{216}$	$\frac{qL}{420}$	$\frac{L\sqrt[3]{q}}{197}$
Stress σ, kg/cm^2	$\left(\frac{qL}{73t}\right)^2+75q$	$\frac{3qL^2}{400t^2}$	$\left(\frac{qL}{41t}\right)^2+135q$	$\frac{3qL^2}{400t^2}$
Thrust (membrane action) F_h, kg/cm	σt	0	σt	0

NOTE. The span of the flooring L is taken in centimetres.

Solution. 1. We find the load per linear centimetre of a strip with a width of $b=100$ cm

$$q=q_d b=\frac{q_d\times 100}{10{,}000}=\frac{q_d}{100}=\frac{400}{100}=4 \text{ kg/cm}$$

2. We select the unit deflection

$$\frac{\delta}{L}=\frac{1}{r_0}=\frac{1}{150}$$

3. The thickness of the sheet according to the expression in Table 6-2 for $\frac{1}{r_0}=\frac{1}{150}$ and $q=4$ kg/cm <10 kg/cm will be

$$t=\frac{qL}{1{,}000}=\frac{4\times 80}{1{,}000}=0.32 \text{ cm, or about 4 mm}$$

4. We find the stress in the sheet and the magnitude of the membrane action

$$\sigma=\left(\frac{qL}{73t}\right)^2+75q=\left(\frac{4\times 80}{73\times 0.4}\right)^2+75\times 4=420 \text{ kg/cm}^2$$

$$F_h=\sigma t=420\times 0.4=168 \text{ kg/cm}$$

If we wanted to avoid the membrane effect and make the flooring behave as a plate, then a different sheet thickness would be obtained for the same allowable deflection, namely

$$t=\frac{L\sqrt[3]{q}}{216}=\frac{80\sqrt[3]{4}}{216}=0.59 \text{ cm, or about 6 mm}$$

In this instance the stress in the sheet would be equal to

$$\sigma=\frac{3qL^2}{400t^2}=\frac{3\times4\times80^2}{400\times0.6^2}=534 \text{ kg/cm}^2$$

Example 6-2. Analyse a flooring for a design load $q_d=1{,}800$ kg/m^2 and a design length of the flooring span $L=80$ cm.

Solution. 1. We find the load per linear centimetre of a strip with a width of $b=100$ cm

$$q=q_d b=\frac{q_d\times100}{10{,}000}=\frac{1{,}800}{100}=18 \text{ kg/cm}$$

2. We determine the thickness of the sheet for $\frac{1}{r_0}=\frac{1}{150}$; since $q=18>10$ kg/cm we have pure bending, without any membrane action.

$$t=\frac{L\sqrt[3]{q}}{216}=\frac{80\sqrt[3]{18}}{216}=0.97 \text{ cm, or about 10 mm}$$

3. The stress in the sheet will be

$$\sigma=\frac{3\times18\times80^2}{400\times1^2}=864 \text{ kg/cm}^2$$

6-2. ROLLED STEEL BEAMS

The rolled beams generally used are of an I or a channel shape. It is better policy to use an I shape owing to its symmetry. At the same time a channel shape behaves better in oblique bending.

The investigation of rolled beams consists in determining the required number (size) of the rolled shape, after which the strength, stiffness and stability (resistance to buckling) of the beam are checked.

(1) Analysis and Selection of Rolled Beam Section

After having selected the type of beam, determined the design span and the design load acting on the beam, the maximum design bending moment M is found. The latter is used to compute the minimum section modulus required, i.e.,

$$W_{req}=\frac{M}{R} \tag{6-11}$$

while when it is possible to take the development of plastic deformations into account (see Sec. 2-5)

$$W_{req} = \frac{M}{1.12R} \tag{6-11'}$$

Having determined the required section modulus W_{req}, the nearest shape number with an actual section modulus W greater than or equal to the required one is selected from the catalogue of standard shapes.

Checking for Adequate Strength. When the section has been selected, the actual stress in the beam is determined, which must comply with the condition that

$$\sigma = \frac{M}{W} \leqslant R \text{ or } \quad \sigma = \frac{M}{1.12W} \leqslant R \tag{6-12}$$

When a concentrated load is applied through the flange of a beam at a spot not strengthened with a rib (Fig. 6-6) the strength of the

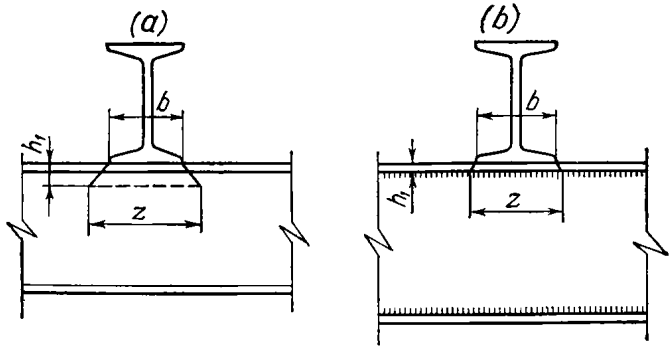


Fig. 6-6. Distribution of local load on beam

beam web should be checked for the local load according to the expression

$$\sigma_{loc} = \frac{P}{tz} \leqslant R \tag{6-13}$$

Here σ_{loc} = stress in web under load perpendicular to centre line of beam

P = design concentrated load

t = thickness of web

$z = b + 2h_1$, where b is the length of the loaded part of the beam, while h is the distance from the outer surface of the beam flange to the beginning of the web fillet in rolled beams (see Fig. 6-6*a*, as well as Table 3 of Appendix III).

Checking for Stiffness of a beam consists in determining the unit deflection, which should not exceed the design value.

Checking for General Stability. If the upper flange of a beam is not secured to prevent lateral buckling, then the beam may lose its general stability after a critical load has been reached.

The following expression is used for checking the general stability of a beam

$$\sigma = \frac{M}{\varphi_b w_{gr}} \leqslant R \tag{6-14}$$

The values of the factor φ_b for rolled I beams are determined as prescribed in Sec. 6-4 (1).

With low values of φ_b it is necessary to provide for horizontal ties that will strengthen the upper flange. When a statical load is transmitted through a solid rigid flooring continuously supported on a compression flange of a beam (reinforced concrete slabs, corrugated steel, etc.), no checking of the general stability is required.

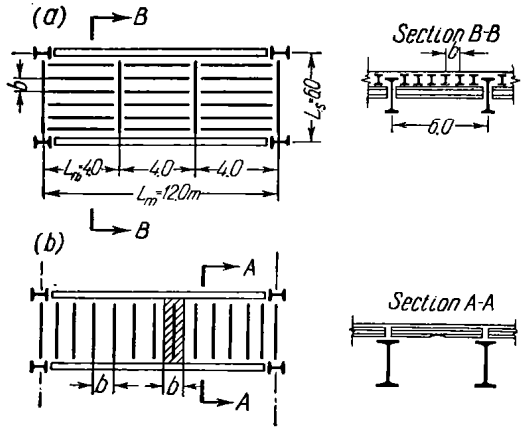


Fig. 6-7. To example 6-3

Example 6-3. Design and analyze a beam grillage (except for the main beams) covered with a metal flooring, with dimensions of the openings of 12×6 metres. The service load uniformly distributed over the whole area is q_s=2,000 kg/m², the load factor n=1.2. The thickness of the flooring is at least t=8 mm, the material is grade ВСт.3кп steel. The allowable unit deflections are $\frac{\delta}{L}=\frac{1}{r_0}=\frac{1}{150}$ for the flooring and $\frac{1}{r_0}=\frac{1}{250}$ for the secondary beams.

Solution. Let us consider two alternatives of the grillage, the first one of a complicated type (Fig. 6-7*a*) and the second one with a tier arrangement of the secondary beams (Fig. 6-7*b*).

First Alternative. 1. *Analysis of the Flooring and Flooring Beams.* Since the minimum thickness of the flooring is given ($t=8$ mm), then this will determine its maximum span L_f equal to the spacing of the beams b. From the expressions given in Table 6-2 we find

$$L_f = b = \frac{t \times 216}{\sqrt[3]{q}} = \frac{0.8 \times 216}{\sqrt[3]{20}} = 63.5 \text{ cm}$$

In the above equation q is the linear load in kg/cm on a strip 100 cm wide, i.e.,

$$q = \frac{q_s \times 100}{10{,}000} = \frac{2{,}000 \times 100}{10{,}000} = 20 \text{ kg/cm}$$

We take $L_f = b = 60$ cm.

The span of the flooring beams $L_{fb} = 4$ metres. The design load on a beam

$$q_1 = q_r n_q L_f = 2{,}000 \times 1.2 \times 0.6 = 1{,}440 \text{ kg/m}$$

The maximum bending moment

$$M = \frac{q_1 L_{fb}^2}{8} = \frac{1{,}440 \times 4^2}{8} = 2{,}880 \text{ kg-m}$$

The required section modulus

$$W_{req} = \frac{M}{1.12R} = \frac{288{,}000}{1.12 \times 2{,}100} = 123 \text{ cm}^3$$

We shall take an I section No. 18 with a section modulus of $W = 143$ cm^3 and a weight $g_1 = 18.4$ kg/m.

The design stress in a beam

$$\sigma = \frac{M}{1.12W} = \frac{288{,}000}{1.12 \times 143} = 1{,}800 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

Next we check the stiffness of a beam, which is ensured by the minimum depth determined from expression (6-4)

$$h_{min} = \frac{L_{fb} r_0}{4{,}800} \times \frac{1}{n_q} = \frac{400 \times 250}{4{,}800 \times 1.2} = 17.4 \text{ cm} < 18 \text{ cm}$$

Thus the section selected complies with the conditions of adequate strength and stiffness.

2. *Analysis of Secondary Beams.* Owing to the frequent location of the flooring beams, the load on a secondary beam can be considered to be a uniformly distributed one. The span of a secondary beam is $L_s = 6$ metres. The design linear load on a beam

$$q = q_s n_q L_{fb} = 2{,}000 \times 1.2 \times 4 = 9{,}600 \text{ kg/m}$$

The maximum bending moment

$$M = \frac{9{,}600 \times 6^2}{8} = 43{,}200 \text{ kg-cm}$$

The section modulus required for these conditions

$$W_{req} = \frac{M}{1.12R} = \frac{4{,}320{,}000}{1.12 \times 2{,}100} = 1{,}835 \text{ cm}^3$$

We shall take an I section No. 55 with a section modulus of $W=2{,}000$ cm³ and a weight of $g_2=89.8$ kg/m. The minimum depth of the beam to ensure the required stiffness will be

$$h_{min}=\frac{L_s r_0}{4{,}800}\times\frac{1}{n_q}=\frac{600\times250}{4{,}800\times1.2}=26 \text{ cm} < 55 \text{ cm}$$

The design stress in a beam

$$\sigma=\frac{M}{1.12W}=\frac{4{,}320{,}000}{1.12\times2{,}000}=1{,}930 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

No checking of the general stability of the beams is necessary, since they are adequately secured in a horizontal plane.

3. *Finding Consumption of Metal.* The amount of metal used in the members will be characterized by their weight per square metre, namely, for the flooring $78.5\times0.8=62.8$ kg/m², for the flooring beams $\frac{g_1}{L_f}=\frac{18.4}{0.6}=30.7$ kg/m², for the secondary beams $\frac{g_2}{L_{fb}}=\frac{89.8}{4}=22.5$ kg/m².

The total amount of steel required for the first alternative

$$g=62.8+30.7+22.5=116 \text{ kg/m}^2$$

Second Alternative. According to the conditions of adequate stiffness $\left(\frac{1}{r_0}\leqslant\right.$ $\left.\leqslant\frac{1}{250}\right)$ and with the same span of $b=L_f=60$ cm, use can be made of I sections having a weight of 31.5 kg/m for the flooring beams.

The total amount of steel required for the second alternative will be

$$g=62.8+\frac{31.5}{0.6}=115.3 \text{ kg/m}^2$$

The grillage will be designed in accordance wtth the second alternative, since it is simpler to manufacture and requires a smaller amount of labour, while as regards the quantity of metal necessary, both alternatives are approximately equivalent.

(2) Joints of Rolled Beams

Rolled beams can be connected together by welding or riveting. The simplest design of a welded joint (splice) is a square butt weld (Fig. 6-8*a*). Such a weld, however, can be located along the length of a beam only at places where the stresses in the beam do not exceed the allowable values for a weld, i.e., where the moment does not exceed the value of

$$M=M_{max}\frac{R_{w\cdot t}}{R}=0.85\,M_{max}$$

If a welded joint is required at the middle of a beam, i.e., at the spot where the maximum moment is observed, it is strengthened with horizontal straps (Fig. 6-8*b*). The dimensions of these straps are determined from the condition that the stress in butt welds should not exceed the design strength $R_{w\cdot t}$.

Taking into account that part of the moment is taken by the butt weld and part by the straps, we get $WR_{w\cdot t}+F_s h=M_{max}$ whence we find the force in a strap $F_s=\frac{M_{max}}{h}-\frac{WR_{w\cdot t}}{h}$ and the required minimum area of the section of the straps

$$A_s \geqslant \frac{F_s}{R_{w\cdot t}}=\frac{M_{max}}{hR_{w\cdot t}}-\frac{W}{h} \tag{6-15}$$

in which $h=$ depth of beam

$W=$ section modulus of the entire rolled I-beam section.

The design force of the welds when straps are welded on will be

$$F=A_s R_{w\cdot t} \tag{6-16}$$

Attention must be drawn to the required sequence of welding a joint, which will noticeably affect its strength. Thus, for example, when joining I sections, first their webs and then their flanges should be welded, since otherwise great internal tensile stresses will appear in the web that often lead to the formation of cracks.

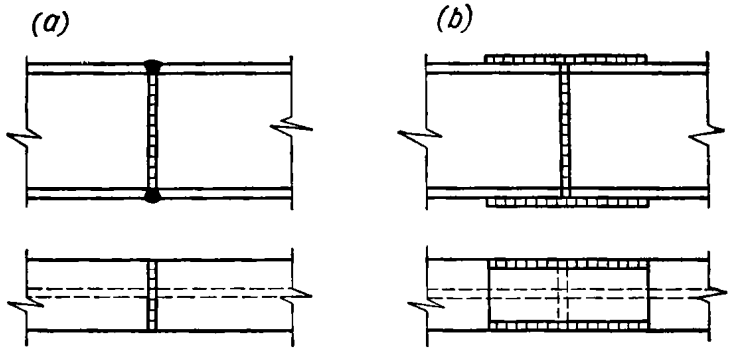


Fig. 6-8. Welded joints of rolled beams

It is also possible to use a connection that is more simple to make, wherein straps are used without butt welding of the I section. In this case the straps must transmit the moment M and the shear force Q acting at the joint.

Almost the whole bending moment is taken by the flanges of the I beam (see diagram of the resultant internal forces F_i in Fig. 6-9*a*). The force in the flanges (with a certain margin)

$$F=\frac{M}{h}$$

where h is the depth of the I section.

The area of the straps is found from the expression

$$A_s \geqslant \frac{F}{R} = \frac{M}{hR}$$

It is also necessary to transmit the shear force Q at the joint, as otherwise the deformation illustrated in Fig. 6-9*b* will appear.

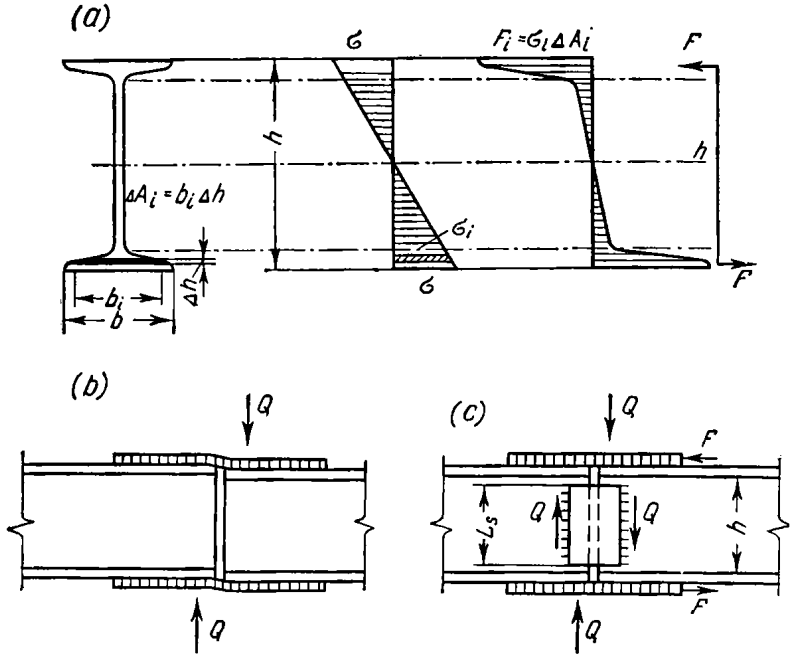


Fig. 6-9. Splicing rolled beams with the aid of straps

To take this shear force, straps are welded to both sides of the web (Fig. 6-9*c*). Here the welds are investigated for shear induced by the shear force

$$\tau = \frac{Q}{2 \times 0.7 h_w L_w}$$

Owing to the presence of a gap that creates a concentration of stresses, this type of joint should be used only when subjected to a statical load.

(3) Support Connections of Beams

When beams rest directly on masonry walls or concrete members, a sufficient bearing surface must be provided for transmitting the load to the support. Generally a bearing plate (Fig. 6-10*a*) is welded

to the beam. The dimensions of this plate should be sufficient to ensure that the pressure under the plate does not exceed the design compressive strength of the wall material.

The thickness of the bearing plate is found by analyzing its bending under the reaction acting on the plate from below (Fig. 6-10*b*). To avoid deformations, a long plate may be reinforced with stiffening ribs (Fig. 6-10*c*).

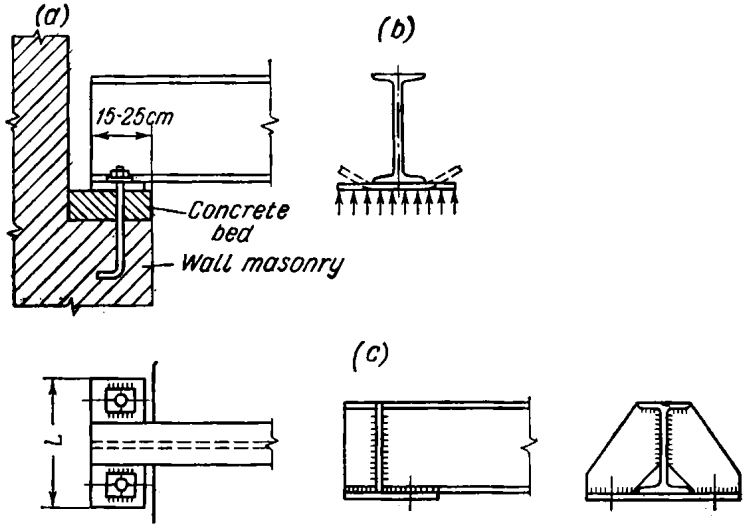


Fig. 6-10. Support connections of beams

When simple beams are supported on steel members, the following kinds of connections are possible:

1. Tier connection (Fig. 6-11*a*).
2. Bolted or riveted connection (Fig. 6-11*c* and *e*).
3. Seated connection (Fig. 6-11*f* and *g*).

Tier connection is the simplest kind and can be made both with bolts (Fig. 6-11*b*) and by means of field welding.

The design of a connection using end angles joined by means of unfinished bolts or rivets (Fig. 6-11*c*) is encountered quite frequently. The load is transmitted to the supports in this kind of connection through the angles. The latter are usually welded to the web of the I beam with the aid of jigs, seeing that deviation of the angles from a plumb or vertical line will necessitate costly field alteration jobs.

In the connection pictured in Fig. 6-11*c*, the end angles are welded on using a weld that is subjected not only to shear caused by the support reaction F_r, but also to bending by the moment $F_r e$. The weld at the edge of the beam web is not considered in analysis.

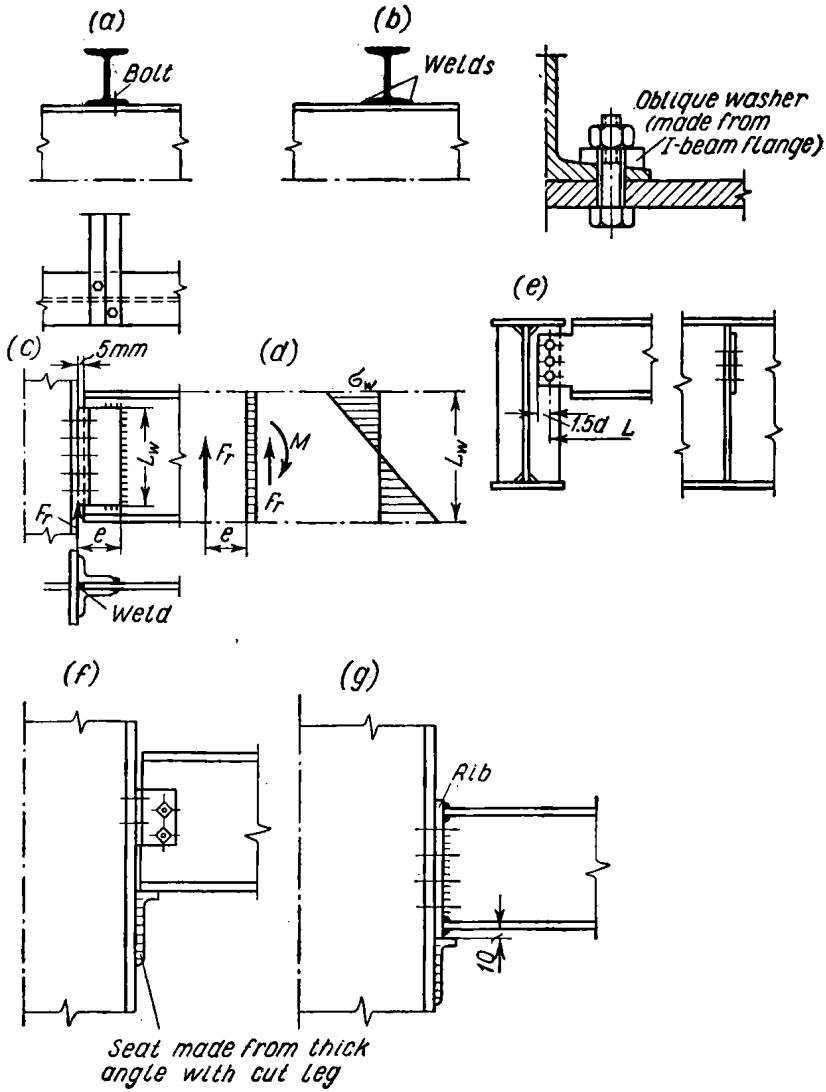


Fig. 6-11. Pinned connections of rolled beams

The behaviour of the weld should be checked for the action of the stress resulting from the common action of the forces in accordance with the conditional expression (Fig. 6-11*d*)

$$\sigma_{res} = \sqrt{\sigma_w^2 + \tau_w^2} \leqslant R_{w \cdot f} \tag{6-17}$$

in which

$$\sigma_w = \frac{M}{W_w} = \frac{F_r e}{2 \times \frac{0.7 h_w L_w^2}{6}} = \frac{3 F_r e}{0.7 h_w L_w^2}$$

$$\tau_w = \frac{F_r}{2 \times 0.7 h_w L_w}$$

Beams can also be secured in place by directly connecting the web of the beam to ribs (Fig. 6-11*e*) or other protruding parts of the member. Here it is necessary to cut away the flanges of the beam, doing this cutting when possible along the web, and not the flanges.

In the connections illustrated in Fig. 6-11*c* and *e* the fastening bolts are ordinarily investigated for shear induced by the reaction of the beam support increased by 20% (allowing for slight fixing of the ends).

A connection that is convenient for field erection is shown in Fig. 6-11*f* and *g*. Here all the load is transmitted to the support through a previously welded on beam seat.

To improve the transmission of the bearing load to the seat, the design illustrated in Fig. 6-11*g* is generally used. To ensure better contact, the lower end of the bearing rib should be milled. The possibility of direct welding of the beam to the member by making a weld around its contour is excluded, inasmuch as this would require the simultaneous performance of three operations in erection, namely, hoisting, trueing and welding, which is almost impossible, not mentioning the necessity of an increased accuracy of cutting when manufacturing such beams.

6-3. BUILT-UP STEEL BEAMS

(1) General Dimensions

As has been previously indicated, built-up beams are either of a welded (mainly) or riveted design.

The general dimensions—the span and the depth—are established with a view to the most advantageous (optimal) relations between the dimensions of the structure.

In industrial structures the length of the span is established depending upon the production process for which the structure is designed.

The minimum depth of a beam determined from the condition of adequate rigidity in accordance with Table 6-1 or expression (6-4) is not, as a rule, the optimal one from the standpoint of the amount of material required: The determination of the most advantageous cross section of a beam consists in finding the minimum area A of the section for the given section modulus $W_{req}=\frac{M}{R}$, and in the most efficient distribution of this sectional area between the web and the flanges depending on the depth h and the thickness of the web.

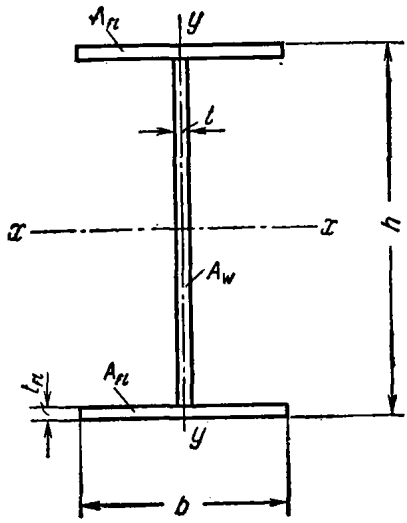


Fig. 6-12. Section of built-up welded I beam

Let us introduce the concepts of the web slenderness ratio (the ratio between the depth of the web and its thickness)

$$\lambda_w=\frac{h}{t} \qquad (6\text{-}18)$$

and a factor showing the distribution of the material over the section of the beam

$$k_d=\frac{A_w}{A}=\frac{ht}{A} \qquad (6\text{-}19)$$

Let A denote the section area of a welded I beam (Fig. 6-12). The section area of one flange will be

$$A_{fl}=0.5\,(A-ht)$$

By neglecting, owing to their insignificant magnitudes, the moments of inertia of the flanges with respect to their own axes, and also assuming that the depth of the web equals that of the beam, the section modulus of a beam can be expressed with sufficient accuracy as follows

$$W=2\left[A_{fl}\left(\frac{h}{2}\right)^2\right]\frac{2}{h}+\frac{th^3}{12}\times\frac{2}{h}=A_{fl}h+\frac{th^2}{6}=\frac{Ah}{2}-\frac{th^2}{3} \qquad (6\text{-}20)$$

Substitution of $\frac{h}{\lambda_w}$ for t in equation (6-20) yields

$$A=\frac{2W}{h}+\frac{2h^2}{3\lambda_w} \qquad (6\text{-}21)$$

By taking the first derivative in respect to the depth and equating it to zero (with the selected constant slenderness ratio of the web λ_w), i. e.,

$$\frac{dA}{dh}=-\frac{2W}{h^2}+\frac{4}{3}\frac{h}{\lambda_w}=0$$

we shall find the optimal depth of a symmetrical beam section

$$h_{opt} = \sqrt[3]{\frac{3}{2}\lambda_w W_{req}} \tag{6-22}$$

Having selected the slenderness ratio of the web λ_w and determined the optimal depth of the beam for this ratio, we have thus also established the best distribution of the material over the cross section. In a symmetrical I beam having the optimal depth, the material is distributed equally between the web and the flanges ($k_d=0.5$). Indeed, on the basis of equation (6-20) and assuming $A_{fl}=k_dA$, we obtain

$$W = \frac{Ah}{2} - \frac{th^2}{3} = Ah\left(\frac{1}{2} - \frac{k_d}{3}\right)$$

or, seeing that

$$h = \lambda_w t \quad \text{or} \quad h^2 = \lambda_w th = \lambda_w k_d A$$

we obtain

$$W = A\sqrt{\lambda_w k_d A}\left(\frac{1}{2} - \frac{k_d}{3}\right) =$$

$$= \sqrt{A^3}\sqrt{\lambda_w k_d}\left(\frac{1}{2} - \frac{k_d}{3}\right) = W_u\sqrt{A^3} \tag{6-23}$$

where

$$W_u = \frac{W}{\sqrt{A^3}} = \sqrt{\lambda_w k_d}\left(\frac{1}{2} - \frac{k_d}{3}\right) \tag{6-24}$$

The quantity W_u, known as the unit section modulus, depends on λ_w and k_d and, being a dimensionless quantity, serves to indicate whether a section resisting bending is advantageous or not. Figure 6-13 pictures a surface illustrating the growth of the unit section modulus depending upon the web slenderness ratio λ_w and the factor k_d. For each value of λ_w there is a relative maximum value of k_d. By taking the first derivative in respect to the factor k_d and equating it to zero (with a constant value of λ_w) we shall obtain the relative maximum value, i. e., the most advantageous distribution of the material over the cross section of a beam

$$\frac{dW_u}{dk_d} = \sqrt{\lambda_w}\left(\frac{1}{4\sqrt{k_d}} - \frac{\sqrt{k_d}}{2}\right) = 0$$

whence

$$k_d = \frac{1}{2}$$

Any deviation from the value of $k_d=0.5$ (with a constant value of λ_w) increases the area of the section, while a growth of λ_w (with a constant value of the factor $k_d=0.5$) results in a reduction of the

section area. With a view to a smooth function having only slight deviations near its minimum, it will be advantageous to take the depth of a beam somewhat smaller than its optimal value (if this is allowable with a view to the maximum deflection). In case of deviation from the optimal depth by a quantity amounting up to 10%, but with retaining of the established value of $\lambda_w = \frac{h}{t}$, the area of the section will be greater by about 1.5%, whereas when this deviation reaches 20% the area will grow by 5-6%.

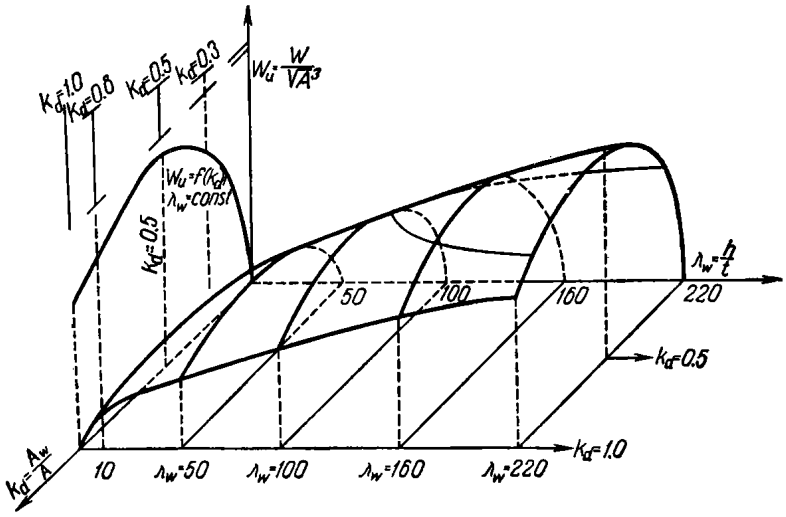


Fig. 6-13. Curves of unit section modulus

In order to standardize members, it will be good practice to take the depth of a built-up beam in round numbers that are a multiple of 100 mm.

In individual projects, where the depth of the beams is not directly connected with insignificant deviations in the height of the structure, it will be better to round off the depth of the beam webs, which will simplify the work involved in the fabrication of the members.

Thus it can be said that the greater the web slenderness ratio $\lambda_w = \frac{h}{t}$, the more economical is the beam. In practice, however, the value of this ratio is limited by the necessity of ensuring stability of the web and its adequate resistance to shear. The selection of the web slenderness ratio is also noticeably affected by the minimum thickness of the web permitted by conditions of corrosion and the local pressure caused by a live load (see page 219).

Generally the minimum thickness of the web is taken equal to t=8 mm, or sometimes 6 mm. The thickness of the web is increased

in steps of 2 millimetres. When a large order is being placed for steel, however, the thickness may be stipulated in odd millimetres (but not above 15 mm).

Designing practice has resulted in the values of the web slenderness ratio $\lambda_w = \frac{h}{t}$ being established that are set forth in Table 6-3.

TABLE 6-3 **Practical Values of λ_w**

h in m	0.8	1	1.25	1.5	1.75	2	2.5	3	4	5
t in mm	8-6	10-8	10-9	12-10	14-12	14	16-14	18-16	20	24-22
λ_w	100-133	100-125	125-140	125-150	125-146	143	156-178	166-187	200	208-227

The thickness of the web can also be determined from an empirical equation that well reflects the increase in λ_w with a growth of the depth of a beam, namely,

$$t = 7 + 3h \tag{6-25}$$

where t is in millimetres and h in metres.

With large concentrated loads the minimum web thickness has to be checked for adequate strength on the basis of the maximum shear force Q (reaction of support)

$$t_{min} \geqslant \frac{3}{2} \frac{Q}{hR_{sh}} \tag{6-26}$$

in which R_{sh} is the design shear strength of steel.

This expression has been obtained from the condition that only the web of the beam takes the shearing stresses.

It must be stressed that from an economical viewpoint very great attention should be paid to selecting the thickness of the web, seeing that after the thickness t has been established there will be only one solution (with $k_d = 0.5$) for the optimal depth and, therefore, the minimum sectional area of a beam.

Indeed, substitution of $\frac{h}{t}$ for λ_w in equation (6-22) yields an expression for the optimal depth which depends on the web thickness as follows

$$h_{opt} = 1.23 \sqrt{\frac{W_{req}}{t}} \tag{6-27}$$

or, assuming the depth of beams somewhat less than the optimal one, we obtain

$$h_{opt} = 1.15 \sqrt{\frac{W_{req}}{t}} \tag{6-28}$$

It is obvious that the minimum possible web thickness complying with the conditions of sufficient strength and stability should be selected (Sec. 6-4). For beams in which a smaller optimal depth is obtained than that essential to ensure the required stiffness (i. e., to keep within the maximum allowable deflection) a different efficient distribution of the material will be obtained than with $k_d=0.5$. In this instance determination of the most efficient section consists in finding the minimum section area A for the given moment of inertia I_{req}. The latter determines the section required for keeping within the limits of the established unit deflection $\frac{1}{r_0}=\frac{\delta}{L}$. On the basis of expression (6-2) we find approximately that

$$I_{req}=\frac{MLr_0}{10E} \tag{6-29}$$

By expressing the moment of inertia similarly to equation (6-20) and the section area A in terms of I, taking the first derivative and equating it to zero, i. e.,

$$\frac{dA}{dh}=-\frac{8I}{h^3}+\frac{4}{3}\frac{h}{\lambda_w}=0$$

we obtain

$$h_{opt}=\sqrt[4]{6I\lambda_w}=\sqrt[3]{3W\lambda_w}=1.73\sqrt{\frac{W}{t}} \tag{6-30}$$

Similarly to the section modulus, the moment of inertia can be expressed in terms of the unit (dimensionless) moment of inertia

$$I=I_uA^2, \text{ where } I_u=\lambda_w k_d\left(\frac{1}{4}-\frac{k_d}{6}\right) \tag{6-31}$$

When $\frac{dI}{dk_d}=0$, we obtain $k_d=3/4$ and $A_w=3/4A$.

For beams with a given (not optimal) depth the factor k_d showing the distribution of the material over the cross section can be found from the equation

$$k_d=\frac{h^2}{\lambda_w A}=\frac{3}{2\left(1+\frac{3W_{req}\lambda_w}{h^3}\right)} \tag{6-32}$$

Example 6-4. Find the optimal depth of a beam with a span of $L=12$ metres for a uniformly distributed live load of $q_r=16$ t/m; the live load factor $n=1.3$. The beam is made from grade BCт.3 steel. The established unit deflection is $\frac{1}{r_0}=\frac{1}{600}$.

Solution. From Table 6-1 we find the minimum depth of the section to provide the required stiffness

$$h_{min}\geqslant\frac{1}{10}L=\frac{1}{10}\times 12=1.2 \text{ m}$$

We take the weight of the beam equal to 0.3 t/m (from similar designs). The total design load on the beam will be

$$q=16\times1.3+0.3\times1.1=21.13 \text{ t/m}$$

Here 1.1 is a dead load factor.

The maximum design bending moment at the middle of the span

$$M=\frac{qL^2}{8}=\frac{21.13\times12^2}{8}=380 \text{ t-m}$$

The required section modulus

$$W_{req}=\frac{38{,}000{,}000}{2{,}100}=18{,}000 \text{ cm}^3$$

From Table 6-3 we select $\lambda_w=125$ or 150.

For $\lambda_w=125$ we have

$$h_{opt}=\sqrt[3]{\frac{3}{2}\lambda_w W_{req}}=\sqrt[3]{\frac{3\times125\times18{,}100}{2}}=151 \text{ cm}$$

and the theoretical thickness of the web $t=\frac{h_{opt}}{\lambda_w}=\frac{151}{125}\cong1.2$ cm.

For $\lambda_w=150$ we obtain

$$h_{opt}=\sqrt[3]{\frac{3\times150\times18{,}100}{2}}=160 \text{ cm}$$

and the thickness of the web $t=\frac{h_{opt}}{\lambda_w}=\frac{160}{150}=1.065$ cm.

If we select the thickness of the web, for example, $t=10$ mm (1 cm), then from equation (6-28)

$$h_{opt}=1.15\sqrt{\frac{W_{req}}{t}}=1.15\sqrt{\frac{18{,}100}{1}}=155 \text{ cm}$$

If a thickness of $t=12$ mm (1.2 cm) is taken, then

$$h_{opt}=1.15\sqrt{\frac{18{,}100}{1.2}}=142 \text{ cm}$$

From expression (6-26) we find the minimum web thickness required to resist the maximum shearing stresses possible on the support of the beam for $h=150$ cm ($Q=127$ tons, see example 6-5):

$$t_{min}=\frac{3}{2}\frac{Q}{hR_{sh}}=\frac{3\times127{,}000}{2\times150\times1{,}300}=0.975 \text{ cm}$$

We take a beam depth of $h=1{,}500$ mm and a web thickness of $t_w=10$ mm.

(2) Selection of Section

Welded Beams. When the required section modulus W_{req} is known and the optimal depth of the section and the thickness of the web have been determined, we find the most economical section of a welded I beam. For a beam that is symmetrical with respect to its neutral axis (with identical upper and lower flanges) the problem

consists in finding such dimensions of the flanges that will give a total section modulus of the beam equal to the required one W_{req}. The total section modulus of a beam is expressed approximately in terms of the section moduli of the web and the flanges (see equation 6-20).

Hence the approximate minimum cross-sectional area of one flange required (with $k_d=0.5$) will be

$$A_{fl}=\frac{W_{req}}{h}-\frac{t_w h}{6}=\frac{3}{4}\frac{W_{req}}{h} \tag{6-33}$$

or for beams with an insignificant asymmetry of the sectional area of the two flanges

$$2A_{fl}=\frac{3}{2}\frac{W_{req}}{h} \tag{6-34}$$

If the width of a flange is b and its thickness t_{fl}, then $A_{fl}=bt_{fl}$.

The ratio between the width and the thickness of a flange ensuring the required area must comply with certain design and production requirements. The wider a flange with a given area, the greater the

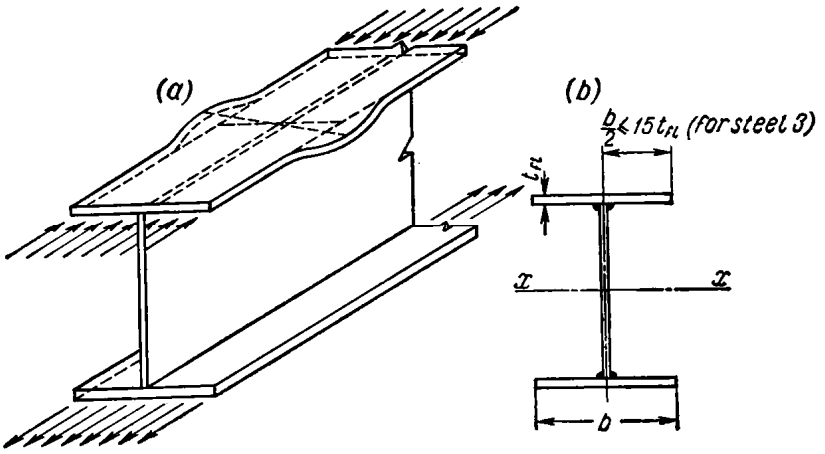


Fig. 6-14. Relation between width of upper beam flange and flange thickness

stability of the beam and the more convenient is the beam for transportation and erection. However, with a too great width and, therefore, a small thickness, the compression flange of a beam may lose stability under the action of normal stresses (Fig. 6-14*a*). Local stability of the overhanging part of the flange will be secured if this part does not exceed $15t_{fl}$ for steel 3 (Fig. 6-14*b*) and $15t_{fl}\sqrt{\frac{2{,}100}{R}}$

for low-alloy steel. Consequently, the total width of a flange must comply with the following relations

$$\text{for steel 3} \quad b_{fl} \leqslant 30 t_{fl}$$

$$\text{for low-alloy steel} \quad b \leqslant 30 t_{fl} \sqrt{\frac{2{,}100}{R}}$$

in which R is the design strength of steel, kg/cm^2.

The minimum thickness of the flange should ensure such general stability of the beam against buckling in a horizontal plane, when it will be unnecessary to introduce the buckling factor φ_b into the expression being used for analysis (see page 191). This minimum width is determined by the ratio of the design length of the beam L to the width of the flange b, which should not exceed the values contained in Table 6-4.

TABLE 6-4 Maximum Values of L/b Ratio for Which No Checking of the General Stability of Steel Beams is Required

Type of beam	$\frac{h}{b}$	$h/t_{fl} = 100$			$h/t_{fl} = 50$		
		with load applied to		with intermediate constraints of top flange regardless of place of load application	with load applied to		with intermediate constraints of top flange regardless of place of load application
		top flange	bottom flange		top flange	bottom flange	
Welded	2	18	28	22	19	30	23
	4	16	26	19	18	27	21
	6	15	24	18	16	25	19
Riveted	2	21	30	24	30	42	33
	4	18	28	21	25	35	27
	6	16	25	19	21	32	24

The design length of a beam L is taken equal to the distance between the points where the compression flange is secured against lateral displacements.

The values of the L/b ratio given in the table are valid for beams of steel 3 and steel 4. For beams of other grades of steel, the tabulated values of L/b must be multiplied by a factor of $\sqrt{\frac{2{,}100}{R}}$.

In those instances when the L/b ratio is greater than the values given in Table 6-4, the general stability of the beams must be checked [Sec. 6-4(1)].

From considerations of design a flange width less than 180 mm should not be used.

The thickness of a flange is established within the limits of 8 to 40 mm, but not less than the thickness of the web and not over 2.5-3.0 times this thickness, since when thick plates are welded on, considerable residual tensile stresses due to shrinkage develop. It will be good practice to select the width and the thickness of the flange in accordance with the USSR Standards for general-purpose steel sections, taking the following dimensions: for the thickness, in steps of 2 mm up to 22, and then thicknesses of 25, 28, 30, 33, 36 and 40 mm; for the width in steps of 20 mm up to 420, and then widths of 450, 480, 500, 530, 560, 600 mm, etc.

After the section has been selected its actual section modulus W is calculated and the strength of the beam and the web is checked by means of the expressions

$$\sigma = \frac{M}{W} \leqslant R$$

and

$$\tau = \frac{QS}{I_b t_w} \leqslant R_{sh} \tag{6-36}$$

Here M = maximum design moment
Q = maximum shear force
$W = \frac{I}{\frac{h}{2}}$ = section modulus of beam (Fig. 6-15)
h = depth of beam
I_b = moment of inertia (second moment of area) of entire beam with respect to neutral axis x-x.

$$I_b = I_w + I_{fl} = \frac{t_w h_w^3}{12} = 2\,(I_0 + c^2 A_{fl}) \tag{6-37}$$

where I_w = moment of inertia of web with respect to neutral axis
I_{fl} = moment of inertia of flanges with respect to neutral axis
I_0 = moment of inertia of one flange with respect to own axis x_0-x_0; the quantity I_0 is usually neglected
h_w = depth of web
c = distance from centre of gravity of flange to neutral axis
$S = A_{fl}c + \frac{A_w}{2}\frac{h_w}{4}$ = statical moment (first moment of area) of half-section with respect to neutral axis.

Simple welded beams with a constant cross section that are stiffened to prevent the loss of general stability, and which carry a statical load, are investigated with regard to the development in them of plastic strains as prescribed in Sec. 2-5 (1).

Example 6-5. Find the section of a welded beam according to the data of example 6-4, i.e., for a span of $L=12$ m, a load of $q=21.13$ tons per metre, a design moment of $M=380$ t-m, a maximum design shear force of $Q=\frac{qL}{2}=\frac{21.13\times12}{2}=$ $=127$ tons and a required section modulus of $W_{req}=18{,}100$ cm^3. In example 6-4 the dimensions of the beam were taken equal to $h=1{,}500$ mm and $t_w=10$ mm.

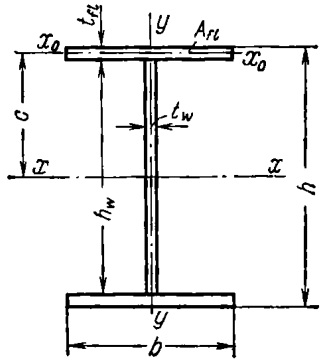


Fig. 6-15. To selection of a welded beam section

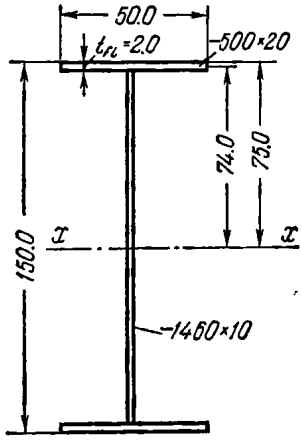


Fig. 6-16. To example 6-5

Solution. We find the required flange cross-sectional area from equation (6-33), assuming that $h=h_w=146$ cm and that the thickness of each flange is 2 cm

$$A_{fl}=\frac{3W_{req}}{4h_w}=\frac{3\times18{,}100}{4\times146}\cong93\ \text{cm}^2$$

We select a flange section of 500×20 mm with $A_{fl}=100$ cm^2 (Fig. 6-16), introducing a certain margin of safety to allow for the simplifications made when deducing the equation. We check the strength of the section. To do this it is first necessary to find the moment of inertia of the beam from equation (6-37)

$$I_b=\frac{t_w h_w^3}{12}+2c^2A_{fl}=\frac{1\times146^3}{12}+2\times74^2\times100=1{,}355{,}000\ \text{cm}^4$$

The section modulus

$$W=\frac{I_b}{\frac{h}{2}}=\frac{1{,}355{,}000}{75}=18{,}000\ \text{cm}^3$$

The stress

$$\sigma = \frac{M}{W} = \frac{38,000,000}{18,100} = 2,100 \text{ kg/cm}^2$$

We check the strength of the web for shear at the support, for which purpose it is first essential to determine the statical moment of the half-section

$$S = A_{fl}c + \frac{A_w}{2}\frac{h_w}{4} = 100 \times 74 + \frac{146 \times 1 \times 146}{2 \times 4} = 10,060 \text{ cm}^3$$

The shearing stress

$$\tau = \frac{QS}{I_b t_w} = \frac{127,000 \times 10,060}{1,355,000 \times 1} = 945 \text{ kg/cm}^2 < 1,300 \text{ kg/cm}^2$$

The total area of the beam section $A = 200 + 146 = 346$ cm^2.

If a vertical plate were taken with a thickness of 12 mm (see example 6-4) the area of the beam section would have been equal to $A = 192 + 175 = 367$ cm^2, i.e., greater by 6%.

With an unsymmetrical beam section, the centre of gravity of the contemplated section through which the neutral axis passes is

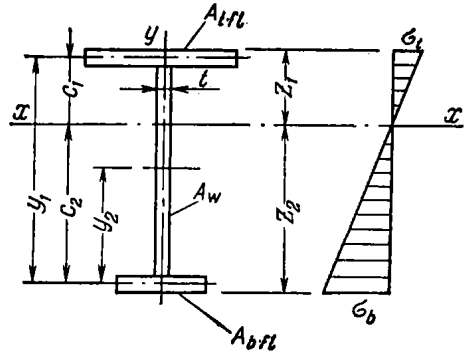


Fig. 6-17. To selection of an unsymmetrical welded beam section

first determined. The distance to the centre of gravity from the centre line of the bottom flange plate c_2 (Fig. 6-17) is found from the equation

$$c_2 = \frac{\sum Ay}{\sum A} = \frac{A_{t.fl}y_1 + A_w y_2}{A_{t.fl} + A_w + A_{b.fl}} \qquad (6\text{-}38)$$

All the notation will be clear from Fig. 6-17.

The moment of inertia of such a beam is equal to

$$I_b = I_w + I_{t.fl} + I_{b.fl} = \frac{t}{3}(z_1^3 + z_2^3) + c_1^2 A_{t.fl} + c_2^2 A_{b.fl}$$

while the section moduli for the top and bottom fibres are

$$W_{top}=\frac{I_b}{z_1} \quad \text{and} \quad W_{bot}=\frac{I_b}{z_2}$$

The stresses in the extreme fibres of the section are checked with the aid of the expressions

$$\sigma_{bot}=\frac{M}{W_{bot}}\leqslant R \quad \text{and} \quad \sigma_{top}=\frac{M}{W_{top}}\leqslant R$$

Besides checking the strength of the beam and the web, the stability of the web and the general stability of the beam are also investigated (see Sec. 6-4).

When a concentrated load is applied through the flange of a beam, at a spot not strengthened with a stiffening rib, the web of the beam must be checked for resistance to local pressure by means of expression

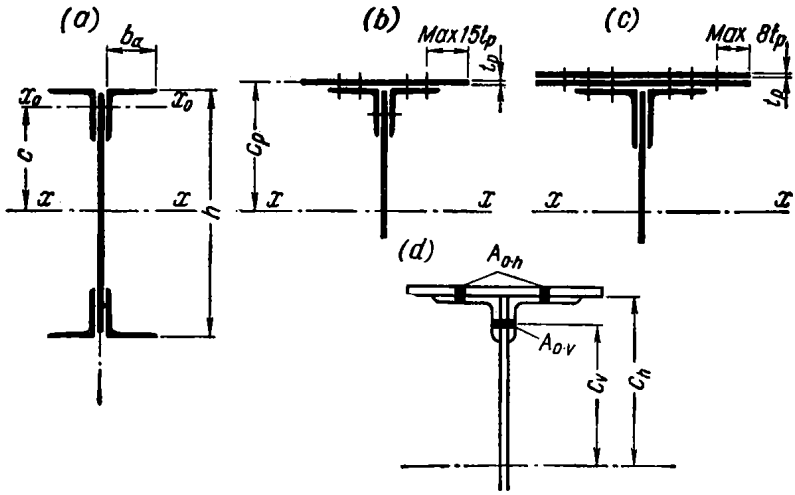


Fig. 6-18. Riveted beam sections

(6-13). In this case h_1 is taken equal to the distance from the outer surface of the beam flange to the upper edge of the web (see Fig. 6-6*b*).

Riveted Beams. The simplest riveted beam section consists of a vertical plate and four flange angles (Fig. 6-18*a*). In most high-strength beams the section can be developed by installing horizontal plates (Fig. 6-18*b* and *c*).

The general dimensions of riveted beams are selected in the same way as those of welded ones. An additional feature in their analysis

is the necessity of taking account of the weakening of the cross section by the rivet holes, which may be preliminary taken equal to 15%, i. e.,

$$W_n = 0.85 W_{gr} \tag{6-39}$$

The section of a symmetrical riveted beam is selected as follows.

1. From the established bending moment we find the required net and gross section moduli

$$W_n = \frac{M}{R} \quad \text{and} \quad W_{gr} = \frac{W_n}{0.85} \tag{6-40}$$

We use the value of W_{gr} and the selected slenderness ratio of the web λ_w to determine the general dimensions of the section—the optimal depth and the thickness of the web. Having established in round numbers the final depth h of the beam (in riveted beams we shall take the depth h equal to the distance between the back legs of the flange angles, i. e., $h = h_w$), we find the required moment of inertia of the section

$$I_{req \cdot gr} = W_{gr} \frac{h}{2} \tag{6-41}$$

2. We select the size of the standard flange angles to be used, generally taking equal leg angles with a leg width of

$$b_{ang} = \left(\frac{1}{10} \text{ to } \frac{1}{11}\right) h$$

and a leg thickness of

$$t_{ang} = \left(\frac{1}{10} \text{ to } \frac{1}{11}\right) b_{ang}$$

The moment of inertia of a beam consisting of a vertical plate and four angles will be

$$I_b = I_w + I_{ang} = \frac{t_w h^3}{12} + 4 (I_0 + c^2 A_{ang}) \tag{6-42}$$

where (Fig. 6-18*a*) I_{ang} = moment of inertia of four flange angles with respect to neutral axis x-x

I_0 = moment of inertia of one angle with respect to its own axis x_0-x_0

c = distance from centre of gravity of the flange angles of one flange to the neutral axis

A_{ang} = sectional area of one angle.

When there is a small difference between the computed values of I_b and $I_{req \cdot gr}$, the required relation $I_b \geqslant I_{req \cdot gr}$ can be satisfied by changing the size of the flange angles.

3. With a great difference between I_b and $I_{req\cdot gr}$, it is necessary to develop the section of the beam flanges by riveting horizontal plates to the horizontal legs of the angles.

The total moment of inertia of such a beam will be

$$I_b = I_w + I_{ang} + I_p \tag{6-43}$$

and it should not be less than the required moment of inertia of the beam $I_{req\cdot gr}$ (I_p is the moment of inertia of the horizontal plates with respect to the neutral axis x-x).

Hence the required moment of inertia of the horizontal plates

$$I_{req\cdot p} = I_{req\cdot gr} - (I_w + I_{ang}) \tag{6-44}$$

and the minimum required area of the horizontal plates of one flange

$$A_{req\cdot p} = \frac{I_{req\cdot p}}{2\left(\frac{h}{2}\right)^2} = \frac{2I_{req\cdot p}}{h^2} \tag{6-45}$$

The horizontal plates should have such a width as to ensure their projecting beyond the edges of the angles. The overhang of a single plate (from the extreme rivet gauge line), owing to the possibility of a loss of stability of the plate, must not exceed 15 plate thicknesses, i.e., $15t_p$ (Fig. 6-18*b*). With a thickness t_p over 20-25 mm it will be good to use two plates (Fig. 6-18*c*). When horizontal plates are employed it is essential that the flange angles ensure reliable transmission of the forces to the web. For this purpose it will be good practice to design the cross-sectional area of the two flange angles equal to at least 30% of the total flange area.

4. After establishing the final dimensions of the beam section, we check its strength. For this purpose we compute the normal stresses at the place of action of the maximum bending moment and the maximum shearing stresses at the support by means of the expressions

$$\delta = \frac{M}{W_n} \leqslant R \tag{6-46}$$

$$\tau = \frac{QS_{gr}}{I_{gr}t_w}\frac{a}{a-d} \leqslant R_{sh} \tag{6-46a}$$

in which a = pitch of rivets

d = diameter of rivet hole

$W_n = \frac{I_n}{\frac{h}{2}}$ = net section modulus (h=total depth of beam).

The moment of inertia of a section weakened by rivet holes is

$$I_n = I_{gr} - I_r \tag{6-47}$$

where I_r = the moment of inertia of the rivet hole areas with respect to the neutral axis of the beam, which can be approximately determined from the equation (Fig. 6-18*d*)

$$I_r = 2\,(A_{o\cdot h}c_h^2 + A_{o\cdot v}c_v^2) + 0.15I_w \qquad (6\text{-}48)$$

Here $A_{o\cdot h}$ = total area of the rivet holes in the horizontal legs of the flange angles and the horizontal plates of one half the beam

$A_{o\cdot v}$ = total area of the rivet holes in the vertical legs of the flange angles of one half the beam

c_h and c_v = distance from the centres of gravity of the respective holes to the neutral axis of the beam

$0.15I_w$ = weakening of the moment of inertia of the beam web, generally taken equal to 15%, since it is possible that a joint will be made at the middle of the beam.

With an unsymmetrical section, it is necessary to determine separately the stresses for the top and the bottom fibres of the beam after having found the respective section moduli $W_{n\cdot t}$ and $W_{n\cdot b}$ (in the same way as described on page 179 for welded beams).

Besides checking the strength of a beam, its stability is also investigated (see Sec. 6-4).

When designing a riveted beam, provision is generally made for the outer faces of the flange angles to overhang 5 millimetres beyond the edge of the vertical plate, except for cases when special flush milling of the web is required (see page 222).

Example 6-6. Select the section of a riveted beam using the data given in examples 6-4 and 6-5, namely, the span is $L=12$ m, the design moment $M=380$ t-m and the required section modulus $M=18{,}100$ cm^3.

Solution. 1. We find the required gross section modulus from equation (6-40)

$$W_{gr} = \frac{W_n}{0.85} = \frac{18{,}100}{0.85} = 21{,}300\ \text{cm}^3$$

We assume the same dimensions of the beam as in the preceding examples i.e., $h=1{,}500$ mm, $h_w=1{,}490$ mm and $t_w=10$ mm.

The required gross moment of inertia is found from equation (6-41)

$$I_{req\cdot gr} = W_{gr}\frac{h}{2} = 21{,}300\times\frac{150}{2} = 1{,}600{,}000\ \text{cm}^4$$

2. We select the size of the angles

$$b_{ang} = \frac{h}{11} = \frac{1{,}500}{11} = 137\ \text{mm}$$

$$t_{ang} = \frac{b_{ang}}{11} = \frac{137}{11} = 12.5\ \text{mm}$$

We select from among the standard shapes the angle 140×12 with the following geometrical characteristics: $A_{ang}=32.5$ cm^2 and $I_0=602$ cm^2.

The distance from the centre of gravity of the angles to the neutral axis

$$c=\frac{150}{2}-3.9=71.1 \text{ cm}$$

The moment of inertia of the web

$$I_w=\frac{t_w h^3}{12}=\frac{1\times 149^3}{12}=276{,}000 \text{ cm}^4$$

The moment of inertia of four angles

$$I_{ang}=4\,(I_0+c^2A_{ang})=4\,(602+71.1^2\times 32.5)=660{,}000 \text{ cm}^4$$

The total moment of inertia of the web and four angles is considerably less than $I_{req\cdot gr}$, namely,

$$I_w+I_{ang}=276{,}000+660{,}000=936{,}000<1{,}600{,}000 \text{ cm}^4$$

For this reason it is necessary to develop the section of the beam flanges by adding horizontal plates.

3. We determine the required moment of inertia of the horizontal plates from equation (6-44)

$$I_{req\cdot pl}=I_{req\cdot gr}-(I_w+I_{ang})=1{,}600{,}000-936{,}000=664{,}000 \text{ cm}^4$$

and then the required area of the horizontal plates of one flange from equation (6-45)

$$A_{req\cdot pl}=\frac{2I_{req\cdot pl}}{h^2}=\frac{2\times 664{,}000}{150^2}=59 \text{ cm}^2$$

We take one plate with a section of 340×18 mm having an area of

$$A_{pl}=34\times 1.8=61.2 \text{ cm}^2$$

4. For the beam section selected we find the exact value of the gross moment of inertia from equation (6-43)

$$I_b=Iw+I_{ang}+I_{pl}=276{,}000+2\times 61.2\times \times 75.9^2=1{,}642{,}000 \text{ cm}^4$$

To determine W_n it is first necessary to compute the moment of inertia I_r of the rivet hole area weakening the beam.

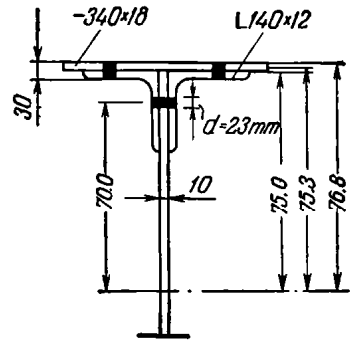


Fig. 6-19. To example 6-6

Figure 6-19 shows the arrangement of the rivet holes in the flange of the beam and the distances from them to the neutral axis of the section. The diameter of the rivets is 23 mm.

We find the moment of inertia of the weakened area from equation (6-48)

$$I_r=2\,(A_{o\cdot h}\,c_h^2+A_{o\cdot v}\,c_v^2)+0.15I_w=2\,(2\times 2.3\times 3.0\times 75.3^2+ +2\times 2.3\times 1.2\times 70^2)+0.15\times 276{,}000=251{,}400 \text{ cm}^4$$

Next we find from equation (6-47)

$$I_n=I_{gr}-I_r=1{,}642{,}000-251{,}400=1{,}390{,}600 \text{ cm}^4$$

while

$$W_n=\frac{1{,}390{,}600}{76.8}=18{,}120 \text{ cm}^3$$

5. We check the strength of the section by means of expression (6-46)

$$\sigma = \frac{M}{W_n} = \frac{38{,}000{,}000}{18{,}120} = 2{,}100 \text{ kg/cm}^2$$

We enter the calculated values into Table 6-5.

TABLE 6-5

Components of section	A, cm²	I_{gr}, cm⁴	W_n, cm³	M, t-m	δ, kg/cm²
2—340×18	2×61.2=122.4	706,000			
4 ⌊s140×12	4×32.5=130	660,000			
—1,490×10	149.0	276,000	18,120	380	2,100
	ΣA =401.4	I_{gr} =1,642,000			
		I_r = 251,400			
		I_n = 1,390,600			

NOTE. The symbol — denotes a plate, ⌊s — angles.

The area of the section obtained in this example and, therefore, the weight of a riveted beam, are 25% greater than those of a similar welded beam computed in example 6-5.

(3) Variation of Section Along Length of Beam

The bending moments near the supports are considerably less than the maximum ones, and therefore there is no necessity of selecting a constant section over the entire length of the beam on the basis of the maximum moment. There are two methods of reducing the cross section and thus of lowering its section modulus.

The first of these methods, used most frequently, consists in reducing the section of the flanges. In welded beams this is done by reducing the width (or more seldom, the thickness) of the flange (Fig. 6-20*a*), and in riveted beams—by reducing the length of the horizontal plates so that the beam at the supports consists only of a web and four angles (Fig. 6-20*b*).

The second method consists in reducing the depth of the vertical web (Fig. 6-20*c*), as a result of which the beam will have a trapezoidal form.

In welded beams withstanding a uniformly distributed load, the most advantageous place for changing the section is located at a distance of $x \cong \frac{1}{6} L$ from the support. The moment M_1 acting at

this point can be found graphically from a moment diagram or analytically from the equation

$$M_1 = \frac{qx(L-x)}{2} \tag{6-49}$$

The computed moment M_1 is used to find the required section modulus and select the new section of the flanges in the usual manner (retaining their thickness, as a rule). At the place where the section is

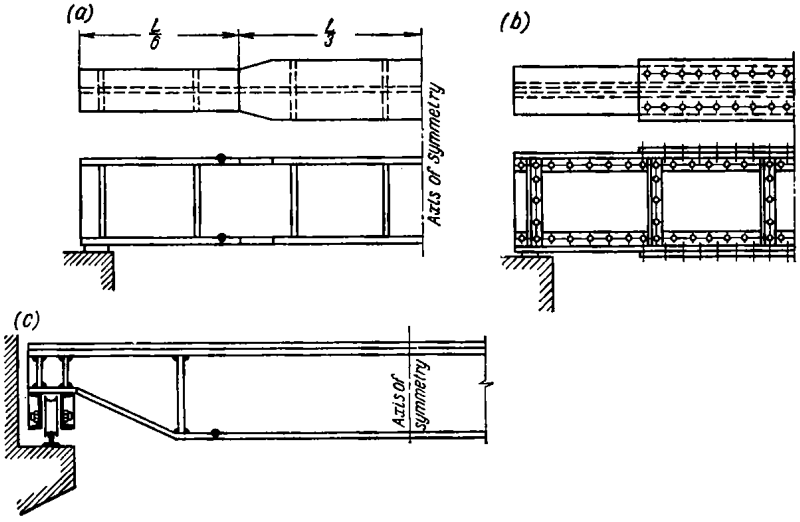


Fig. 6-20. Variation of section along beam length

changed, the flange plates are connected by means of a butt weld (Fig. 6-20*a*), and therefore the stress in the beam should not exceed the design tensile strength of a weld, which for type Э42 electrodes is $R_{w.t}=1{,}800$ kg/cm^2.

Example 6-7. Design a variable section in length for the welded beam selected in example 6-5. The beam with a span of $L=12$ metres resists a uniformly distributed load of $q=21.13$ t/m. The maximum moment $M_{max}=380$ t-m.

Solution. We find the moment M_1 at a distance of $x=\frac{L}{6}=\frac{12}{6}=2$ m from the support from equation (6-49)

$$M_1 = \frac{qx(L-x)}{2} = \frac{21.13\times 2(12-2)}{2} = 211.3 \text{ t-m}$$

The required section modulus

$$W_{req.1} = \frac{M_1}{R_{w.t}} = \frac{21{,}130{,}000}{1{,}800} = 11{,}740 \text{ cm}^3$$

The required area of the flange, from equation (6-33)

$$A_{req\cdot 1}=\frac{W_{req\cdot 1}}{h}-\frac{t_w h}{6}=\frac{11{,}740}{150}-\frac{1\times 150}{6}=53.3\ \text{cm}^2$$

We take the section of the flange to be a plate 300×20 mm in size with an area of $A_{fl\cdot 1}=30\times 2=60\ \text{cm}^2$. Hence for the new section of the beam we shall have

$$I_b=\frac{t_w h_w^3}{12}+2c^2A=\frac{1\times 146^3}{12}+2\times 74^2\times 60=916{,}000\ \text{cm}^4$$

$$W_1=\frac{I_b}{\frac{h}{2}}=\frac{916{,}000}{75}=12{,}200\ \text{cm}^3$$

$$\sigma=\frac{M_1}{W_1}=\frac{21{,}130{,}000}{12{,}200}\cong 1{,}730\ \text{kg/cm}^2<1{,}800\ \text{kg/cm}^2$$

In riveted beams the place of theoretical plate termination is determined by the load-carrying capacity of the remaining part of the beam section with the reduced number of plates, or entirely without plates. Thus, for example, for the beam considered in example 6-6, the gross moment of inertia of the section without horizontal plates is equal to $I_{gr}=936{,}000\ \text{cm}^4$ (see Table 6-5). The net moment of inertia is $I_n=859{,}400\ \text{cm}^4$, whence the net section modulus of this part of the beam, consisting only of a vertical plate and flange angles, will be

$$W_{n\cdot 1}+\frac{859{,}400}{75}=11{,}450\ \text{cm}^3$$

With complete utilization of the material, such a section can resist a moment of

$$M_1=WR=11{,}450\times 2{,}100=24{,}045{,}000\ \text{kg-cm}\cong 240\ \text{t-m}$$

When the moment M_1 is known, the moment diagram can be used to find the point where the plates can be terminated. This point, called the place (line) of theoretical plate termination, can also be found analytically.

For this purpose equation (6-49) is used to find the distance x from the support determining the place of theoretical plate termination. In practice, the horizontal plates should be extended further, beyond the theoretical termination line, in order to ensure their participation in the work of the beam beginning from this line after they have been riveted in place. The number of rivets necessary for connecting a plate beyond the line of theoretical termination is determined from its sectional area, assuming that the plate is subjected to a force of $F=0.5A_pR$ (since the section of the beam when the plate begins to participate in taking the load has a reserve area beyond the line of termination).

At the place in a beam where its section changes, which is ordinarily not far from a support, shearing stresses arise besides the large normal stresses. The resulting combined stressed state will require investigation of the maximum equivalent or reduced stresses [Sec. 2-2 (3)] at the level of the flange welds (Fig. 6-21) or rivets. This investigation is carried out for the elastic stage of behaviour by means of expression (2-8)

$$\sigma_{red} = \sqrt{\sigma_1^2 + 3\tau_1^2} \leqslant R$$

where $\sigma_1 = \sigma \frac{h_w}{h_b}$ and $\tau_1 = \frac{QS_{fl}}{I_b t_w}$ are the normal and shearing stresses

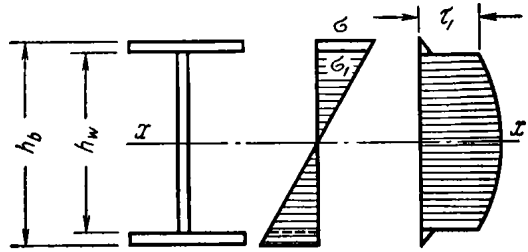


Fig. 6-21. To determination of reduced stresses

in the web at the level of the flange welds or rivets in the section of the beam under consideration at the same load.

Example 6-8. Find the reduced stress in the section of a welded beam that has been selected as computed in example 6-7. The span of the beam is $L=12$ metres, the uniformly distributed load is $q=21.13$ t-m. At a distance of $x=2$ m from the support, at the place where the section changes, the moment in the beam is $M_1=211.3$ t-m. The characteristics of the reduced section of the beam are

$$A_{fl} = 30\times 2 = 60 \text{ cm}^2; \quad I_b = 916{,}000 \text{ cm}^4;$$
$$\sigma = 1{,}730 \text{ kg/cm}^2$$

Solution. We find the shear force Q in the section at a distance of $x=2$ m from the support

$$Q = q\frac{L-2x}{2} = \frac{21.13\,(12-2\times 2)}{2} = 84.5 \text{ tons}$$

The statical moment of the flange with respect to the neutral axis $S_{fl}=30\times \times 2\times 74=4{,}440$ cm^3.

The shearing stress at the level of the flange weld

$$\tau_1 = \frac{QS_{fl}}{I_b t_w} = \frac{84{,}500\times 4{,}440}{916{,}000\times 1}\times 410 \text{ kg/cm}^2$$

The normal stress at the level of the flange weld

$$\sigma_1 = \sigma\frac{h_w}{h_b} = 1{,}730\times \frac{146}{150} = 1{,}680 \text{ kg/cm}^2$$

The reduced stress being sought will be found from expression (2-8)

$$\sigma_{red} = \sqrt{\sigma_1^2 + 3\tau_1^2} = \sqrt{1{,}680^2 + 3 \times 410^2} = 1{,}820 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

(4) Investigation of Flange-to-Web Connection

In welded beams the flanges are connected to the web by means of continuous welds, while in their riveted counterparts, as their name implies, with the aid of rivets. If the flange and the web were not connected together, then in bending they would be displaced in respect to each other (Fig. 6-22*a*). The connection of the flange

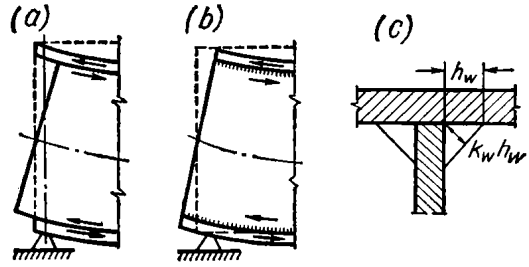


Fig. 6-22. To investigation of welded beam flange connection

to the web does not permit the formation of shear, and as a result shearing stresses appear in the joint which are directed along the axis of the beam (Fig. 6-22*b*). In welded beams the shearing stress in the web along the line of connection of the flange therewith is determined from equation (6-36)

$$\tau = \frac{QS_{fl}}{I_b t_{web}} \text{ kg/cm}^2$$

in which S_{fl} is the statical moment of the flange (the moving part of the section in shear) with respect to the neutral axis.

The shearing force F_{sh} per linear metre of beam length

$$F_{sh} = \tau t_{web} = \frac{QS_{fl}}{I_b} \text{ kg/cm} \qquad (6\text{-}50)$$

This shearing force is resisted by two welds 1 centimetre long having a working area (Fig. 6-22*c*) of

$$A_w = 2k_w h_w$$

where h_w = leg (thickness) of weld

k_w = fillet weld area reduction factor described on page 112.

Thus the expression used to investigate the strength of welds connecting the flange of a beam to its web will be

$$\tau = \frac{QS_{fl}}{2k_w h_w I_b} \leqslant R_{w \cdot f} \tag{6-51}$$

or

$$h_w \geqslant \frac{QS_{fl}}{2k_w I_b R_{w \cdot f}} \tag{6-52}$$

The maximum value of the shearing stresses will be encountered at the support, where the maximum shear force Q acts. The flange welds employed in beams are generally continuous, and of one size.

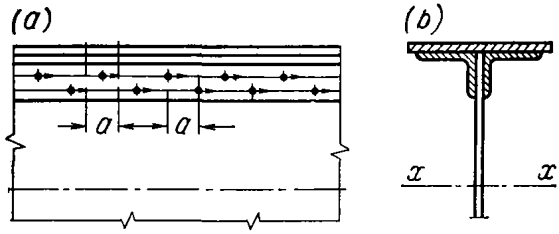


Fig. 6-23. To investigation of riveted beam flange connection

When thick flange plates are used great internal stresses appear in the flange welds. For this reason these welds should be made either by automatic welding, or with the use of type Э42А electrodes that deposit metal having a greater plasticity. The minimum sizes of flange welds are given in Table 6-6.

TABLE 6-6 **Minimum Sizes h_w of Flange Fillet Welds**

Kind of steel used in members	Minimum size h_w of weld in mm for flange thickness, mm, of				
	up to 10	11-20	21-30	31-50	over 50
Carbon	4	6	8	10	12
Low-alloy	6	8	10	12	—

In riveted beams the shearing force F_{sh} is resisted by the flange rivets. Each rivet is subjected to the sum of the unit shearing forces determined from equation (6-50) for a length equal to half the pitch of the rivets (Fig. 6-23*a*). This total force must not exceed the

allowable crushing or double-shear force per rivet F'_r, i. e.,

$$F_{sh}a \leqslant F'_r$$

Hence the half-pitch of the flange rivets will be [see expression (6-46*a*)]

$$a \leqslant \frac{F'_r}{F_{sh}} = \frac{F'_r I_b}{QS_{fl}} \tag{6-53}$$

where S_{fl} = gross statical moment of flange with respect to the neutral axis (the hatched part in Fig. 6-23*b*)

I_b = gross moment of inertia of entire beam section.

To reduce the labour requirements for manufacturing a beam, the number of rivets should be reduced to a minimum and, therefore, their pitch should be as great as possible, but not over 12*d* (Table 5-2).

Ordinarily the pitch of the rivets is found from the magnitude of the shear force acting on the support, retaining a uniform pitch over the whole length of the beam, which simplifies the shop work. In large-span beams it will be excellent practice to increase the pitch of the rivets in the central part of the beam where the shear force has a lower magnitude.

6-4. GENERAL AND LOCAL STABILITY OF STEEL BEAMS

(1) General Stability of Beams

A long and narrow beam that is not stiffened in a lateral direction and that is loaded above a certain limit may lose its stability and buckle a considerable extent in a horizontal direction (Fig. 6-24).

This phenomenon is termed the *loss of general stability* of a beam, while the load and the stresses at which the loss of general stability commences are known as the *critical* ones.

Upon a loss of general stability, twisting of the beam cross section commences. This leads to displacement of the flanges laterally, and the beam, besides bending in a vertical plane, is also subjected to bending in a horizontal plane and torsion. It is obvious that the wider the flanges and the greater the value of I_y, the higher will be the critical stresses and the greater the stability of the beam. The critical stresses can also be considerably raised by fixing certain points of the upper flange of the beam in the span to prevent possible lateral displacement. The magnitude of the critical stresses depends upon the constructive form or layout of the beams, and first of all upon the ratio between the span (or distance between the intermediate constraints of the compression flange) and the width of the flange L/b (Table 6-4).

With high values of the L/b ratio the general stability of the beam should be checked by introducing the buckling factor φ_b into expression (6-14) used for this analysis

$$\sigma = \frac{M}{\varphi_b W_{gr}} \leqslant R$$

For beams with a symmetrical I section the factor φ_b, which is the ratio between the critical stress resulting in the loss of stability

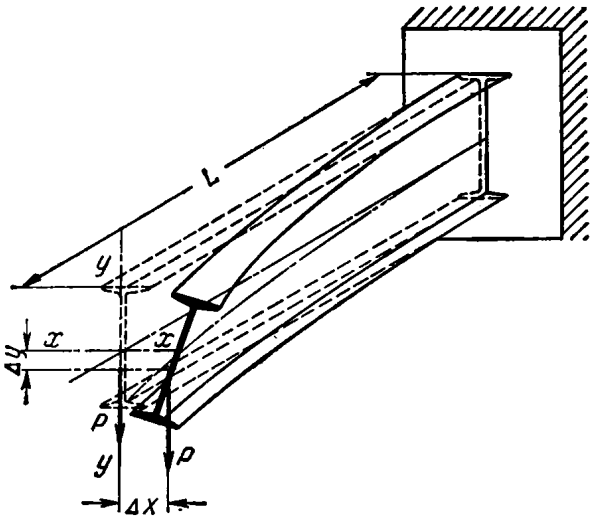


Fig. 6-24. Loss of general stability in a cantilever beam

and the yield point, is determined from the equation

$$\varphi_b = \psi \frac{I_y}{I_x}\left(\frac{h}{L}\right)^2 10^3 \qquad (6\text{-}54)$$

Here ψ is a factor determined from Table 3 (for ordinary beams or girders) and Table 4 (for cantilever beams) of Appendix II, depending on the parameter

$$\alpha = 1.54 \frac{I_t}{I_y}\left(\frac{L}{h}\right)^2 \qquad (6\text{-}55)$$

where I_t is the moment of inertia in torsion, taken for rolled I beams from Table 6-7.

Since by equation (2-22) for built-up beams

$$I_t = 1.3 \frac{ht_w^3 + 2bt_{fl}^3}{3} \quad \text{and} \quad I_y = 2\frac{t_{fl}b^3}{12}$$

TABLE 6-7 **Moments of Inertia in Torsion I_t for Rolled I Beams (According to GOST 8239-56*)**

No of I beam	10	12	14	16	18	18a	20	20a	22	22a	24	24a	27	27a
I_t in cm^4	2.3	2.9	3.6	4.5	5.6	6.5	6.9	7.9	8.6	9.8	11	13	14	17
No. of I beam		30	30a	33	36	40	45	50	55	60	65	70	70a	70b
I_t in cm^4		17	20	24	31	41	55	75	100	135	180	244	352	534

then equation (6-55) for built-up (welded and riveted) beams can be written as follows

$$\alpha = 8\left(\frac{Lt_{fl}}{bh}\right)^2\left(1+\frac{dt_w^3}{bt_{fl}^2}\right) \tag{6-56}$$

in which for welded beams t_w=thickness of beam web, $d=0.5h$, b and t_{fl}=width and thickness of compression flange, h=total depth of beam section; for riveted beams t_w=thickness of web together with vertical legs of angles, t_{fl}= thickness of flange together with horizontal leg of angle, d=depth of vertical leg of angle plus the thickness of the group of horizontal plates.

With a buckling factor φ_b exceeding 0.85, the critical stresses are within the plastic range of behaviour of the material, and in this instance the factor φ_b' taken from Table 6-8 is substituted in expression (6-14) for the factor φ_b.

TABLE 6-8 **Values of Factor φ_b' for Steel Beams**

φ_b	0.85	0.90	0.95	1.00	1.10	1.20	1.30	1.40	1.55
φ_b'	0.850	0.871	0.890	0.904	0.927	0.948	0.964	0.980	1.00

The stability of channel beams is analyzed in the same way as that of I beams, the value of the parameter α being computed from equation (6-56). The values of the buckling factor φ_b obtained on the basis of this parameter are multiplied, however, by 0.5 when the load is applied along the main axis passing through the centre of

gravity of the section, and by 0.7 when the load is applied in the plane of the web.

For beams with a more developed compression flange, the factor φ_b is determined in accordance with the standards for designing steel structures.

(2) Local Stability

Local buckling of separate elements of members under the action of normal (compressive) or shearing stresses is known as the *loss of local stability*. In beams the loss of local stability of a flange or the web is frequently the main cause of a loss of load-carrying capacity. The web of a beam may lose its stability owing to the action of shearing or normal stresses, and also of a combination of these stresses.

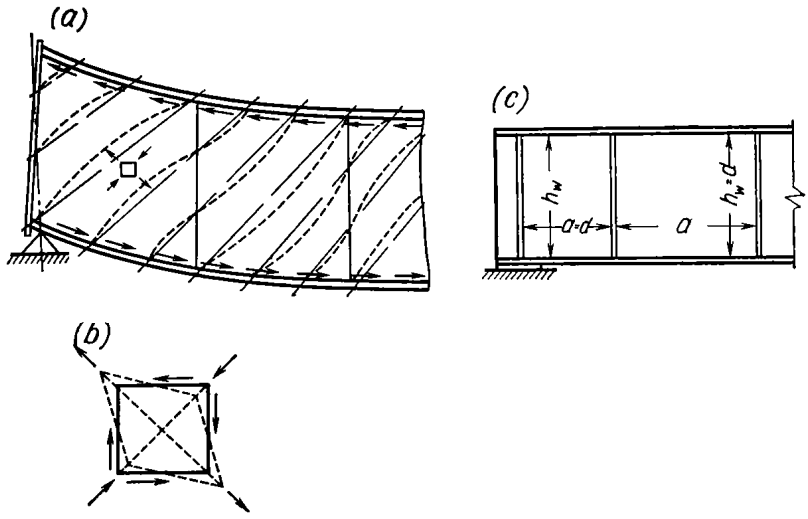


Fig. 6-25. Loss of local stability of web due to shearing stresses

Loss of Stability of Web Due to Shearing Stresses. Near the support, the web of a beam is subjected to the action of shearing stresses under whose influence it is distorted (Fig. 6-25*a*): along the lines of the shortened diagonals the web is compressed, and along those of the extended diagonals it is stretched (Fig. 6-25*b*). Under the action of compression the web may buckle, forming waves inclined at an angle of about 45 deg to the axis. To prevent buckling of the web, vertical (lateral) stiffening ribs (stiffeners) are installed that intersect the possible buckling waves (Fig. 6-25*c*). In this

instance the web will be separated into rectangles limited on four sides by the flanges and the stiffeners. If we designate by a the distance between the axes of the stiffeners, by h_w the depth of the web and by d the smaller side of the rectangle, then the critical shearing stress in the web can be expressed by the equation (with account taken of elastic constraint of the web in the flanges)

$$\tau_{cr} = \left(1.25 + \frac{0.95}{\mu^2}\right)\left(\frac{100t}{d}\right)^2 \text{t/cm}^2 \qquad (6\text{-}57)$$

in which $\mu =$ the ratio of the larger side (a or h_w) to the smaller one d
$t =$ thickness of web.

Stability is generally calculated in tons and centimetres.

If the value of μ or correspondingly a is very large (which in the extreme case denotes the presence of stiffeners only at the supports of the beam), the smaller side of the rectangle d will be equal to the depth of the web h_w and hence

$$\tau_{cr} = 1.25\left(\frac{100t}{h_w}\right)^2 = \frac{1.25\times 100^2}{\lambda_w^2} \text{t/cm}^2 \qquad (6\text{-}58)$$

where $\lambda_w = \frac{h_w}{t}$ is the slenderness ratio of the web.

The expression obtained is similar in structure to Euler's formula (2-13).

Let us determine at what value of the web slenderness ratio $\lambda_w = \frac{h_w}{t}$ the stress τ_{cr} may reach the yield point τ_y, and let us call this ratio the *critical slenderness ratio* λ_{cr}. It was indicated above [Sec. 2-2 (3)] that (for steel 3)

$$\tau_y = \frac{\sigma_y}{\sqrt{3}} \cong 0.6\sigma_y = 0.6\times 2{,}400 = 1{,}440 \text{ kg/cm}^2 = 1.44 \text{ t/cm}^2$$

Substitution of this value for τ in equation (6-58) yields the critical slenderness ratio

$$\lambda_{cr} = \sqrt{\frac{1.25\times 100^2}{1.44}} \cong 90$$

Thus, when the slenderness ratio of the web h_w/t is less than 90, the web upon being loaded will reach the yield point owing to shearing stresses before it loses its stability. In accordance with the standards, however, the maximum critical web slenderness ratio $\lambda_w = h_w/t$, up to which it is not neccessary to strengthen a web with stiffeners, is established with a certain margin of safety, allowing for the possibility of the web plate being bent, and also for a certain influence of the normal stresses.

According to the standards, the web of a beam in bending must be strengthened with stiffeners if $\frac{h_w}{t} > 70\sqrt{\frac{2{,}100}{R}}$, where R is the design strength of steel in kg/cm^2.

The maximum distance between the stiffeners is taken equal to $a_{max} = 2h$ for $h/t > 100$ and $a_{max} = 2.5h$ for $h/t \leqslant 100$.

With such an arrangement of the stiffeners, we shall obtain a critical slenderness ratio of the web, assuming $\mu = 2$ in equation (6-57), equal to

$$\lambda_{cr} = \sqrt{\frac{1.49 \times 100^2}{1.44}} = 102$$

In view of a certain constraining effect of the stiffeners and a number of simplifications, the standards permit doing without an investigation of beam web stability when the values of the web slenderness ratio $\lambda_w = h_w/t$ do not exceed $110\sqrt{\frac{2{,}100}{R}}$, while should there be present local pressure between the stiffeners ($\sigma_{loc} \neq 0$)—when the values of the web slenderness ratio do not exceed $80\sqrt{\frac{2{,}100}{R}}$.

When determining the value of the h_w/t ratio for riveted beams, the design depth of the web h_w is taken between the internal rivet gauge lines of the flange angles.

Loss of Stability of Web and Flange Due to Normal Stresses. At a considerable distance from the supports, nearer to the centre of the beam, the influence of the shearing stresses on the web is not large, and the web at this part of the beam is subjected mainly to the action of normal stresses. These may also be the cause of a loss of stability. The magnitude of the critical normal stresses depends upon the law of distribution of the normal stresses applied to the edges of a rectangular plate-web, which are characterized by the factor

$$k_n = \frac{\sigma_{max} - \sigma_{min}}{\sigma_{max}} \qquad (6\text{-}59)$$

where σ_{max} and σ_{min} are the normal stresses, each taken with its own sign, and applied at the top and bottom boundaries of the plate (tension is considered positive and compression—negative).

Figure 6-26*a* and *b* pictures plates elastically constrained along both horizontal edges with stresses applied to their vertical edges. With uniformly distributed stresses (pure compression, Fig. 6-26*a*) $k_n = 0$, in bending (Fig. 6-26*b*) $k_n = 2$.

Figure 6-26*c* illustrates a uniformly compressed plate constrained along one long side and free along the other one.

The general expression determining the normal critical stress, which is valid within the limits of elastic behaviour of the web, has

the following form

$$\sigma_{cr} = \frac{\zeta 10^4}{\lambda_w^2} \text{ t/cm} \tag{6-60}$$

in which ζ is a coefficient depending upon the value of k_n; the values of ζ are indicated in Table 6-9.

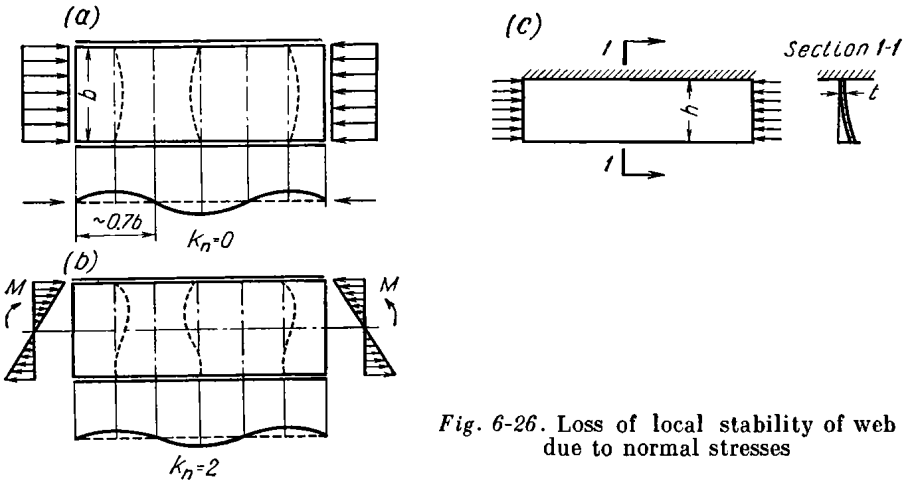


Fig. 6-26. Loss of local stability of web due to normal stresses

The critical normal stress in the web of a beam in bending is

$$\sigma_{cr} = \frac{k_0 10^4}{\lambda_w^2} = k_0 \left(\frac{100t}{h_w}\right)^2 \text{t/cm}^2 \tag{6-61}$$

where k_0 is taken from Table 6-10 depending upon the quantity

TABLE 6-9 **Values of Coefficient ζ in Equation (6-60)**

Stress diagrams	Plate elastically constrained along two long sides			Plate constrained along one long side
	Compression	Compression and bending	Bending	Compression
k_n	0	1	2	—
ζ	1.00	2.10	k_0	0.08

TABLE 6-10 **Values of Coefficient k_0**

	For welded beams							For riveted beams
γ	$\leqslant 0.8$	1.0	2.0	4.0	6.0	10	$\geqslant 30$	at any values of γ
k_0	6.3	6.62	7.0	7.27	7.32	7.37	7.46	7.0

γ, the latter allowing for the elastic constraint of the web in the flanges

$$\gamma = C\frac{b_{fl}}{h_w}\left(\frac{t_{fl}}{t}\right)^3 \qquad (6\text{-}62)$$

Here b_{fl} and t_{fl} = width and thickness of compressed flange of beam, respectively

C = factor taken equal to ∞ (infinity) in the case of continuous bearing of rigid plates or slabs on the compression flange of the beam, and to 0.8 in other cases.

Let us determine the value of the critical slenderness ratio at which the critical normal stress reaches the yield point $\sigma_y = 2.4$ t/cm² (for steel 3).

From equation (6-60) we get

for $k_n = 0$

$$\lambda_w = \frac{h_w}{t} = 100\sqrt{\frac{\zeta}{\sigma_y}} = 100\sqrt{\frac{1}{2.4}} \cong 65$$

for $k_n = 2$ and $y_0 = 6.3$

$$\lambda_w = \frac{h_w}{t} = 100\sqrt{\frac{6.3}{2.4}} \cong 162$$

With one side constrained and the other free

$$\lambda_w = \frac{h_w}{t} = 100\sqrt{\frac{0.08}{2.4}} \cong 18$$

In the case of compression of a plate that is elastically constrained along two long sides (the web of a solid column) with a depth to thickness ratio h_w/t less than 65, the web will reach the limiting stress with respect to strength before it loses its stability.

In the case of bending, the web begins to lose its stability owing to the action of the normal stresses at values of the h_w/t ratio exceeding 162. Taking into account, however, the approximate nature of a number of assumptions, in the deduction of the above equations it

is recommended to take the critical web slenderness ratio (with only normal stresses acting) equal to $h_w/t=160$ for steel 3, and to $h_w/t=$ $=160\sqrt{\frac{2{,}100}{R}}$ for other grades of steel.

A compression plate constrained along one side (half of a beam flange) can lose its stability only with $\frac{h_w}{t}=\frac{b_1}{t_{fl}}>18$. However, owing to certain indeterminateness in the constraint, and for other reasons, the standards prescribe that in this instance a value of $\frac{b_1}{t_{fl}}=15$ must not be exceeded for steel 3 and a value of $\frac{b_1}{t_{fl}}=15\sqrt{\frac{2{,}100}{R}}$ for other grades of steel, which for the main group of low-alloy steels (with $R=2{,}900$ kg/cm^2) gives $b_1/t_{fl}=12.5$.

With understressing of a beam, the limiting value of b_1/t_{fl} may be increased by multiplying it by $\sqrt{\frac{R}{\sigma}}$, but not over 25%.

Loss of Stability of Web Due to Combined Action of Normal and Shearing Stresses. Upon bending of a beam a combined stressed state appears in the web owing to the combined action of normal and shearing stresses, which may produce a loss of local stability of the web.

As has already been indicated, the stability of the web must be checked if the h_w/t ratio exceeds $110\sqrt{\frac{2{,}100}{R}}$. In this instance the web must be strengthened with pairs of lateral stiffeners installed over its entire depth.

It is recommended practice to strengthen thin webs of deep beams with an additional longitudinal twin rib installed in the compression zone of the web, besides the lateral ribs (see Sec. 6-6), when the h_w/t ratio exceeds $160\sqrt{\frac{2{,}100}{R}}$.

The stability of a beam web is analyzed by the separate panels (rectangles) that are formed between the flanges of the beam and the stiffeners (Fig. 6-25*c*). By changing the distance between the stiffeners, such a ratio between the panel sides can be obtained that will ensure stability of the beam web.

After having planned the location of the stiffeners with the maximum possible distances between them, the stability of the web is investigated for the combined action of normal and shearing stresses. Investigations carried out by S. Timoshenko, P. Papkovich, and later by B. Braude, have shown that the stability of a web will be ensured when the following condition is observed

$$\sqrt{\left(\frac{\sigma^2}{\sigma_{cr}}\right)+\left(\frac{\tau}{\tau_{cr}}\right)^2}\leqslant 1 \qquad (6\text{-}63)$$

Here σ and τ = design stresses in web of beam
σ_{cr} and τ_{cr} = critical values of normal and shearing stresses when they act separately, computed from equations (6-61) and (6-57), i. e.,

$$\sigma_{cr} = k_0 \left(\frac{100t}{h_w}\right)^2 \text{t/cm}^2$$

$$\tau_{cr} = \left(1.25 + \frac{0.95}{\mu^2}\right)\left(\frac{100t}{d}\right)^2 \text{t/cm}^2$$

The design stress σ is computed for the gross section, without introducing the buckling factor φ_b and, as prescribed in the standards, is determined for the mean value of the bending moment within the limits of the panel if its length does not exceed the depth of the beam; otherwise the value of σ is computed from the mean value of the moment for the most stressed part with a length equal to the depth of the panel. The mean shearing stress in the web will be

$$\tau = \frac{Q}{h_w t} \tag{6-64}$$

where Q = mean value of vertical shear force within the limits of the panel
h_w = total depth of web.

Should the section of the beam vary within the limits of the panel under consideration, the stability of the web is checked for the place of change in section according to the stresses computed for the reduced section.

When concentrated loads are applied at places not strengthened with stiffeners, the stability of the web must be investigated by means of expression (6-84). Here the local stress σ_{loc} is determined from expression (6-13) with a view to the remark concerning the application of this expression on page 179.

Example 6-9. Investigate the stability of the welded beam web considered in examples 6-5 and 6-7 and locate the stiffeners. The span of the beam is $L=12$ m, the uniformly distributed design load is $q=21.13$ t/m. The selected (reduced) section of the beam consists of a web $1{,}460\times 10$ mm in size and 300×20 mm flanges. The moment and force acting at the place of the change in the beam section are $M_1=211.3$ t-m and $Q=84.5$ tons.

Solution. 1. We check the necessity of installing stiffeners. Seeing that $\frac{h_w}{t}=\frac{146}{1}=146$ exceeds 70, such stiffeners are essential, and since h_w/t exceeds 110, the stability of the web must be checked.

2. We find the design normal and shearing stresses σ and τ in the web at the place of the change in its section.

The normal compressive stress in the extreme fibres of the web for the reduced beam section will be

$$\sigma = \frac{M_1}{W_1}\frac{h_w}{h} = \frac{21{,}130\times 146}{12{,}200\times 150} = 1.68 \text{ t/cm}^2$$

(the value of $W_1=12{,}200$ cm^3 has been taken from example 6-7).

The mean shearing stress is determined from equation (6-64)

$$\tau=\frac{Q}{h_w t}=\frac{84.5}{146\times 1}=0.58 \text{ t/cm}^2$$

3. We find the critical normal stress, after having determined the factor γ characterizing the constraint of the web. From equation (6-62)

$$\gamma=C\frac{b_{fl}}{h_w}\left(\frac{t_{fl}}{t}\right)^3=\frac{0.8\times 30}{146}\left(\frac{2}{1}\right)^3=1.31$$

and from Table 6-10 we find the coefficient $k_0=0.74$ (by interpolation). Now we can compute the critical normal stress from equation (6-61)

$$\sigma_{cr}=k_0\left(\frac{100t}{h_w}\right)^2=6.74\left(\frac{100\times 1}{146}\right)^2=3.16 \text{ t/cm}^2$$

4. We plan the location of the stiffeners with a maximum distance between them of $a=2h=2\times 150=300$ cm, i.e., we assume $\mu=\frac{a}{h_w}=\frac{3}{1.46}=2.05$.

5. The critical shearing stress is found from equation (6-57)

$$\tau_{cr}=\left(1.25+\frac{0.95}{\mu^2}\right)\left(\frac{100t}{h_w}\right)^2=\left(1.25+\frac{0.95}{2.05^2}\right)\left(\frac{100\times 1}{146}\right)^2=$$
$$=0.695 \text{ t/cm}^2$$

6. We use expression (6-63) to check the stability of the web

$$\sqrt{\left(\frac{\sigma}{\sigma_{cr}}\right)^2+\left(\frac{\tau}{\tau_{cr}}\right)^2}=\sqrt{\left(\frac{1.68}{3.16}\right)^2+\left(\frac{0.58}{0.695}\right)^2}=0.97<1$$

Hence, the planned location of the stiffeners, i.e., spaced 3 metres apart, is satisfactory.

(3) Design of Stiffeners

In welded beams the stiffening ribs are made from strips having a width of b_{rib}, which is determined from the empirical formula

$$b_{rib}\geqslant\frac{h_w}{30}+40 \text{ mm} \qquad (6\text{-}65)$$

where h_w is the depth of the web, mm.

In riveted beams the ribs are made from angles, the width of whose projecting leg is found from the same formula (6-65).

The thickness of the ribs is taken equal to at least $\frac{1}{15}b_{rib}$. All the ribs are designed with a symmetrical arrangement with respect to the axis of the beam, and they are located opposite each other.

The ribs (except supporting ones) are welded to the web by means of continuous welds (Fig. 6-27*a*) with a minimum size (h_{weld}=4-5 mm). At both ends the ribs are cut away, the dimensions of the legs of the cut-away part being 40 mm in width and 60 mm in depth.

In riveted beams the ribs made from angles are extended onto the flange angles, up to the fillet of the angle legs (Fig. 6-27*b*). For this purpose a filler plate whose thickness is equal to that of the flange angles is placed between the stiffening angles and the web of the beam. The stiffening angles are riveted in place with the minimum number of rivets, with a pitch approaching 12*d*.

At the spot where the beam rests on its support, there will exist a reaction that is distributed over a comparatively small length of the beam. The reaction of the support is transmitted as an active concentrated force through the supporting stiffeners that are tightly fitted to the lower flange of the beam (Fig. 6-27*a* and *b*), or through a plate welded to the edge of the beam (Fig. 6-27*c*).

In this instance the surface of the protruding lower end of the bearing plate is subjected to crushing.

The sectional area of the bearing plate or of the supporting ribs is determined from the equation

$$A = \frac{F_{r \cdot d}}{R_{cr \cdot e}} \tag{6-66}$$

in which $F_{r \cdot d}$ = design reaction of support
$R_{cr \cdot e}$ = design resistance of steel to crushing of end surface.

In riveted beams, equation (6-66) is used to determine the area of that part of the protruding angle legs which must be tightly fitted to the flange, since the leg of the angle adhering to the web and part of the protruding leg are cut away to prevent their getting onto the fillet of the flange angle (Fig. 6-27*d*).

The connection of the supporting ribs to the web of a beam must also be checked for transmission of the support reaction F_r.

Besides, the supporting ribs or angles should be investigated for buckling out of the plane of the beam as verticals loaded with the support reaction. The sectional area A of such a conditional vertical includes the stiffeners and a strip of the web up to $15t$ at each side of the web, a cross-shaped section thus being obtained. The design length of the vertical L_d is taken equal to the depth of the web. The following expression is used for checking

$$\sigma = \frac{F_r}{\varphi_A} \leqslant R \tag{6-67}$$

The connection of the supporting ribs or angles to the web is investigated for shear caused by the support reaction F_r.

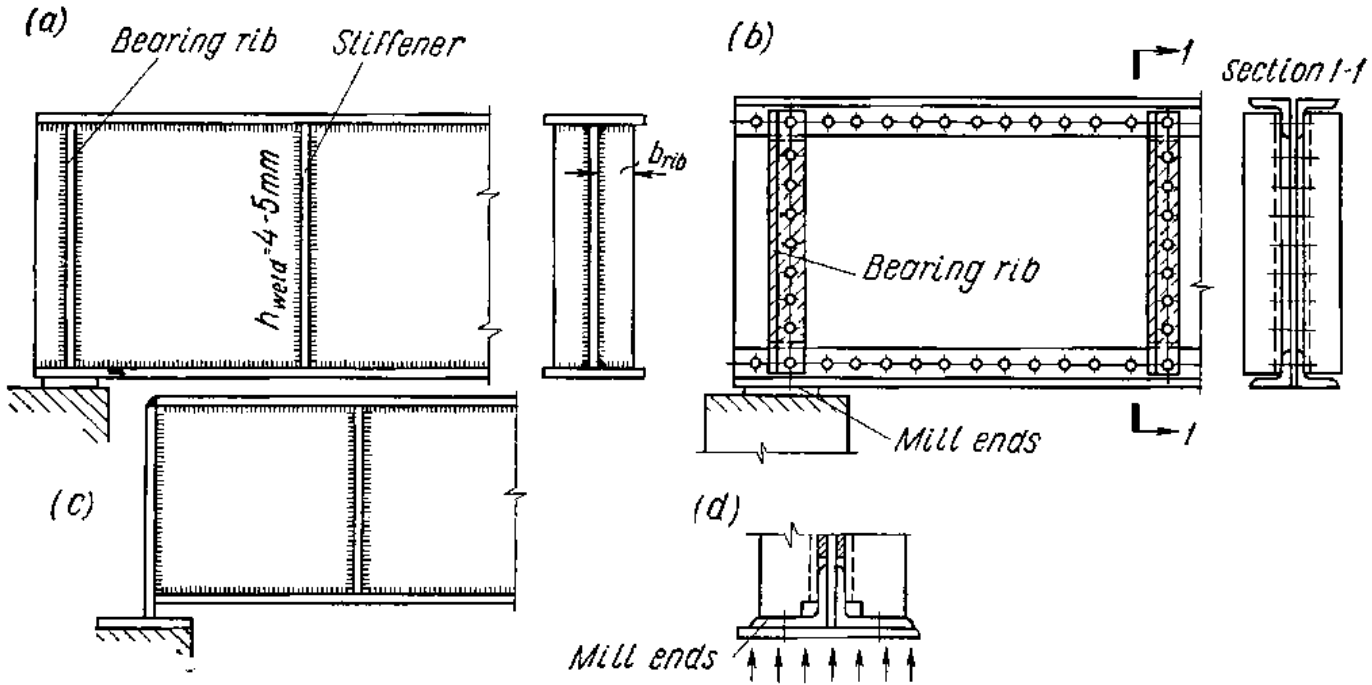


Fig. 6-27. Design of stiffeners

6-5. SPLICES AND CONNECTIONS OF BEAMS

(1) Splices

Two types of beam splices are distinguished, namely, shop and field ones.

Shop splices, made at a plant, are connections of separate rolled shapes or plates, forming a section, which are essential to obtain the required length. The splices of the separate shapes or plates may be located at various places along the length of the beam, depending upon considerations of design.

Field splices are connections of separate parts of members (shipping elements) made necessary by the conditions of transportation of the members from the plant to the place of erection. The most convenient field splice is one in which all the rolled elements are joined together in one cross section of the member. Such a splice is referred to as a *universal* one.

The length of a separate shipping element when hauled on one four-wheel 20-ton flat car should not exceed 9.1 metres, and when hauled on one eight-wheel 50- or 60-ton flat car—13 metres. Members can also be shipped on two flat cars coupled together. In this case the length of a shipping element may correspondingly reach 19 and 27 metres.

Field splices are considerably more costly than shop ones, and for this reason their number should be as small as possible.

When planning the work, every effort should be made to utilize to the highest possible degree the capacity of the railway cars used for shipping the structural members, since this will cut transportation costs.

Design of Welded Splices. Figure 6-28 pictures the splices of a welded built-up beam. Figure 6-28*a* shows staggered shop splices of flange and web elements, while Fig. 6-28*b* illustrates a field splice. The square butt-weld web splice used here may be made by manual welding. The ordinary methods of welding control may be employed in the section of the beam where the stress at point *C* of the butt weld (Fig. 6—28*a*) will not exceed the design tensile strength of the weld. A square butt-weld web splice (Fig. 6-28*a* and *b*) may also be used at the place where the maximum moment is observed, but on condition of employing improved methods of weld-quality control or, in some instances, when the most stressed parts of the butt weld are strengthened with rhombic straps (Fig. 6-28*c*).

The flanges of beams are connected by means of equistrength welds, which may be skew or square.

Of considerable importance in the manufacture of beams is the sequence of welding operations. First the elements that have trans-

verse splices must be assembled, and they should be welded separately (the splices of flange plates, web splices, welding on of stiffeners), since transverse welds cause the greatest shrinkage. After assembly of the beam, the longitudinal flange welds are made. In automatic welding the stiffeners are put in place after making the flange welds, since they hinder the passage of the welding machine head. The reference numbers *1* through *5* in Fig. 6-28*d* show the sequence of welding a field splice. First the transverse welds are

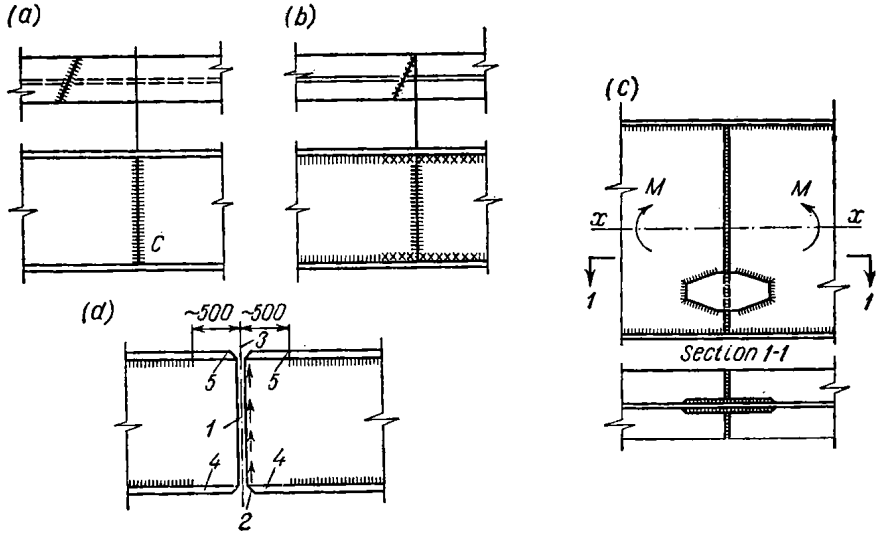


Fig. 6-28. Splices of welded beams

made, and to provide for free shrinkage of welds *2* and *3*, which depends upon possible deformation of the flange plates, a section of the flange welds about 500 mm long at each side of the joint is left unwelded. The last to be made are the longitudinal welds on sections *4* and *5*, which have an insignificant longitudinal shrinkage.

Analysis of Welded Splices. The splice of each element forming part of the section of a beam is investigated separately. The bending moment is distributed between the elements of the beam in proportion to their moments of inertia. The moment to be taken by the web of the beam can be found from the equation

$$M_w = M_b \frac{I_w}{I_b} \tag{6-68}$$

in which M_b = total design moment in splice of beam
I_w = moment of inertia of beam web
I_b = total moment of inertia of entire beam.

The design shear force Q acting in the splice is completely transmitted to the web. The strength of a butt-weld web splice (Fig. 6-28*a*) is investigated with the aid of the usual bending formulas.

The force at a flange splice is found from the equation

$$F_{fl} = \frac{M_b - M_w}{h} \tag{6-69}$$

The web splice illustrated in Fig. 6-28*c* is investigated as a single section consisting of a butt weld and two straps. The maximum design tensile stress in this section must not exceed $R_{w \cdot t}$.

Design of Riveted Splices. Figure 6-29*a* shows a shop splice of a riveted beam web covered with straps on both sides over the entire depth of the web. Figure 6-29*b* and *c* shows shop splices of

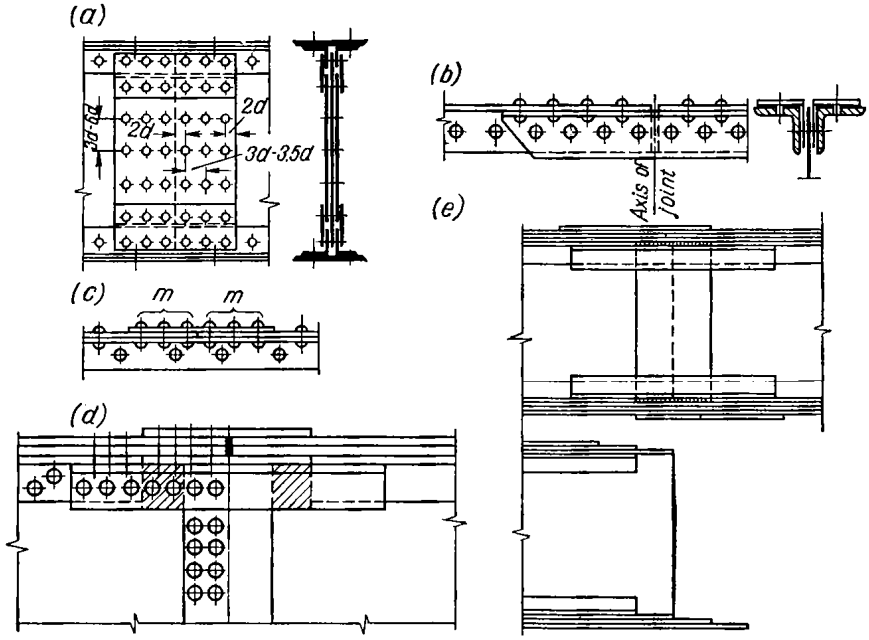


Fig. 6-29. Splices of riveted beams

flange angles and a flange plate. The main rule for designing a butt splice consists in covering it with a strap whose cross-sectional area is not less than that of the elements being spliced. Figure 6-29*d* pictures an example of a field beam splice. Figure 6-29*e* shows field staggered splices of horizontal plates used in a multiplate member.

A minimum rivet pitch (3-3.5d) should be used at the splices so as not to increase the weight of the straps. In case jig-drilled field rivet holes are used a pitch should be adopted that is a multiple of the module (30, 40 or 50 mm) established by the manufacturer. It is also desirable that the maximum pitches of flange rivets outside the splice be a multiple of the module.

The field splices of welded beams may sometimes be made using rivets or high-strength bolts, with straps on both sides. It should be remembered that the holes for the rivets or bolts weaken the section of the beam.

Analysis of Riveted Splices. The riveted splice of each element is investigated separately. The number of rivets necessary for a splice of horizontal plates and flange angles is determined from the area

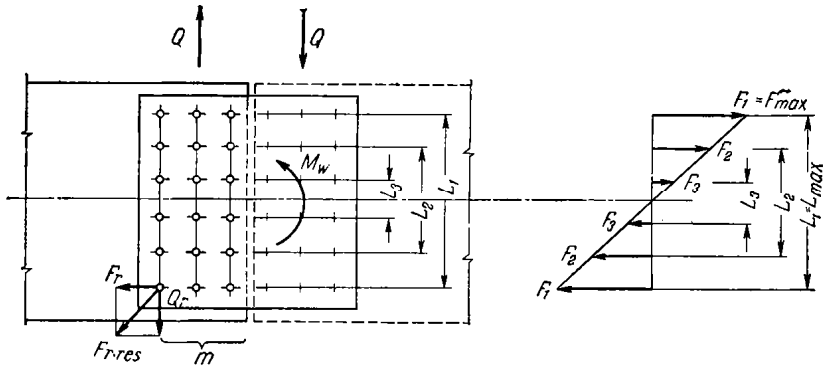


Fig. 6-30 To investigation of web splice for bending

of the element, on the assumption that the latter is used completely for resisting compression or tension, i. e.,

$$N = \frac{F}{F'_r} \tag{6-70}$$

where N = number of rivets per half-strap (at one side of the joint)
F = design force determined in accordance with the sectional area of the element
F'_r = allowable force per rivet.

When investigating the riveted splice of a web in bending, it is assumed that the external moment applied to the web is balanced by the sum of the internal pairs formed by the forces acting along the horizontal rows of rivets on the splice half-strap, with arms symmetrically arranged with respect to the neutral axis (Fig. 6-30)

$$M_w = \sum F_i L_i = F_1 L_1 + F_2 L_2 + F_3 L_3 + \ldots$$

Since all the forces F_i can be expressed through the maximum force F_1, namely, $F_2 = F_1 \frac{L_2}{L_1}$, $F_3 = F_1 \frac{L_3}{L_1}$, etc., then we get

$$M_w = \frac{F_1}{L_1} (L_1^2 + L_2^2 + L_3^2 + \ldots)$$

Hence the maximum force acting in the extreme, most heavily loaded, row of rivets will be

$$F_{max} = \frac{M_w L_{max}}{\sum L_i^2} \tag{6-71}$$

The horizontal force acting on one rivet will be

$$F_r = \frac{F_{max}}{m}$$

where m is the number of vertical rows of rivets on a half-strap.

Besides the moment, a splice may be subjected to the action of a shear force Q, which we assume to be uniformly transmitted to all the rivets located on the half-strap. Hence the vertical force acting on one extreme rivet will be

$$Q_r = \frac{Q}{N}$$

in which N is the number of rivets on a half-strap.

The resultant force acting on one extreme rivet will be

$$F_{r.res} = \sqrt{F_r^2 + Q_r^2} = \sqrt{\frac{M_w L_{max}}{m \sum L_i^2} + \left(\frac{Q}{N}\right)^2} \leqslant F_r' \tag{6-72}$$

Example 6-10. Investigate the splice in the beam selected in example 6-6, located at the middle of the span. The initial data are $L=12$ metres, $q=21.13$ t/m, $M_{max}=380$ t-m, the shear force at the middle of the span $Q=0$. The section of the beam consists of a vertical plate (web) $1{,}490 \times 10$ mm in size, flange angles 140×12 mm in size with a cross-sectional area of $A_{ang}=32.5$ cm^2 and horizontal plates 340×18 mm in size with an area of $A=61.2$ cm^2.

Solution. 1. We design the splice as pictured in Fig. 6-31 and select a rivet diameter of $d=23$ mm.

The horizontal plate is covered by two straps, of which one is on top and the other under the plate, in the space between the splice angles and the horizontal plate. We take the thickness of each strap equal to that of the flange angles, i.e., $t=12$ mm, which gives a total of 24 mm and is greater than the thickness of the horizontal plate, equal to $t=18$ mm.

2. We determine the number of rivets required for the splice of the horizontal plates.

The force which a horizontal plate can withstand is

$$F = A_n R = (61.2 - 2 \times 2.3 \times 1.8)\, 2{,}100 = 111{,}000 \text{ kg}$$

The allowable force per rivet will be: in crushing over a thickness of 18 mm (1.8 cm) with drilled holes

$$F'_{r\cdot cr} = dtR_{r\cdot cr} = 2.3\times 1.8\times 4{,}200 = 17{,}400 \text{ kg}$$

in double shear

$$F'_{r\cdot sh} = n_{sh}\frac{\pi d^2}{4} R_{r\cdot sh} = 2\times \frac{3.14\times 2.3^2}{4}\times 1{,}800 = 14{,}940 \text{ kg}$$

Thus the number of rivets required per half-strap will be

$$N = \frac{F}{F'_{r\cdot sh}} = \frac{111{,}000}{14{,}940} \cong 8$$

A staggered rivet pattern will be used.

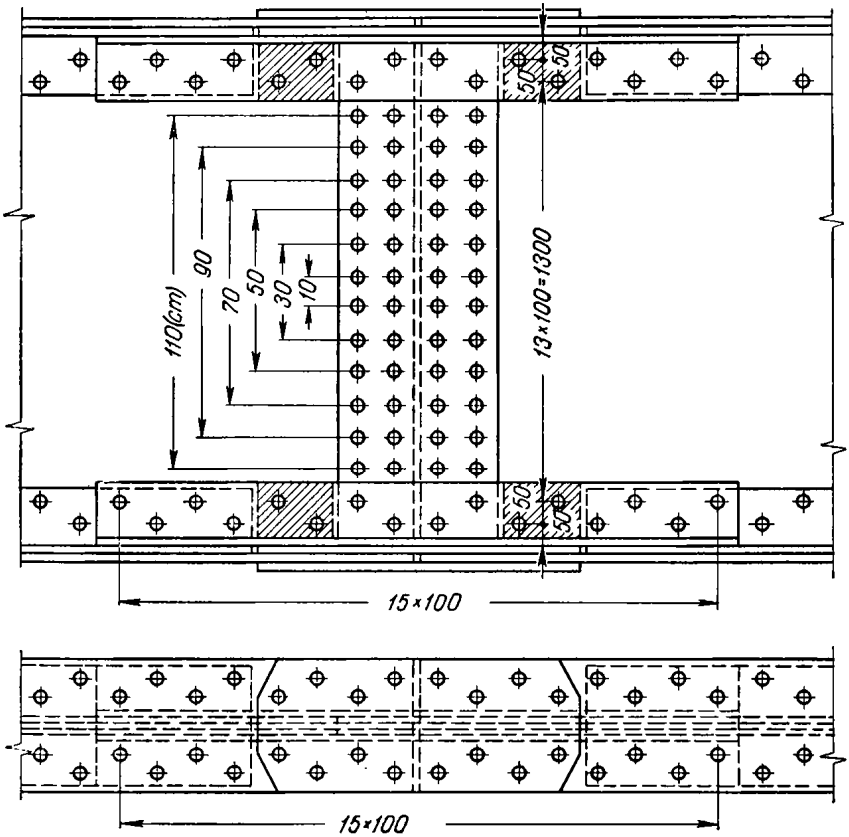


Fig. 6-31. To example 6-10

3. We find the number of rivets necessary for the angle splice. The force which one angle can resist (see Figs. 6-18*a* and 6-19) is

$$F = A_n R\frac{2c}{h_0} = (32.5 - 2\times 2.3\times 1.2)\, 2{,}100\, \frac{2\times 71.1}{2\times 76.8} = 52{,}600$$

The force which can be taken by one rivet with $d=23$ mm, for the condition of single shear, is

$$F'_{r\cdot sh}=\frac{\pi d^2}{4}R_{r\cdot sh}=\frac{3.14\times 2.3^2}{4}\times 1,800=7,470 \text{ kg}$$

The number of rivets required in an angle will be

$$N=\frac{F}{F'_{r\cdot sh}}=\frac{52,600}{7,470}=7.1$$

We shall use 8 rivets and locate them on the horizontal and vertical legs of the angle.

4. We investigate the rivets connecting the splice straps to the web. Splice straps with a thickness of $t=12$ mm will be used, as otherwise it will be impossible to put the splice angle in place. We plan two vertical rows of rivets, i.e., $m=2$ (more than two rows are used on a half-strap only in very heavy beams).

We assume the vertical arrangement of the rivets shown in Fig. 6-31.

The following moment will act on the web of the beam

$$M_w=\frac{M_b I_w}{I_b}=\frac{380\times 276,000}{1,642,000}=63.8 \text{ t-m}$$

We determine the maximum horizontal force per rivet of the extreme row

$$F_{max}=\frac{M_w L_{max}}{m\sum L_i^2}=\frac{6,380,000\times 130}{2\times 45,500}=9,100 \text{ kg}$$

where

$$L_{max}=130 \text{ cm}$$

$$\sum L_i^2=10^2(1^2+3^2+5^2+7^2+9^2+11^2+13^2)=45,500 \text{ cm}^2$$

The allowable force per rivet (in crushing)

$$F'_{r\cdot cr}=dtR_{r\cdot cr}=2.3\times 1.2\times 4,200=11,600 \text{ kg}$$

Since $Q=0$, then $F_{r.res}=9,100$ kg$<11,600$ kg.

Thus two vertical rows of rivets per half-strap are sufficient.

(2) Connections

Beam-to-column connections are divided as regards their design into top and side connections (flexible or rigid). Side connections can be either of the framed or the seated type. A flexible connection transmits only the reaction of the support, whereas a rigid one, besides this reaction, also transmits the moment at the support.

Examples of top beam-to-column connections are shown in Fig. 6-32. Generally, the support pressure in beams with a span of up to 25-30 metres is transmitted to the column through a flat bearing plate. In Fig. 6-32*a* the support stiffeners (support plates) are located vertically along the ends of the beams and protrude 10-15 mm downward. The milled ends of these stiffeners ensure the central transmission of the support pressure. The bottom flange of the beams does not touch the column, but is pulled toward it by means of bolts.

In Fig. 6-32*b*, on the contrary, the stiffeners ensure the transmission of the support pressures through the bearing plate onto the branches of the column (made of channels). The thickness of the bearing plate is usually designed equal to 16-20 mm (if only the plate is not subjected to bending).

Figure 6-33*a* illustrates a flexible (framed) side bolted beam-to-column connection. The bolts in this connection are investigated for shear under the action of the support reaction F_r increased by 20%.

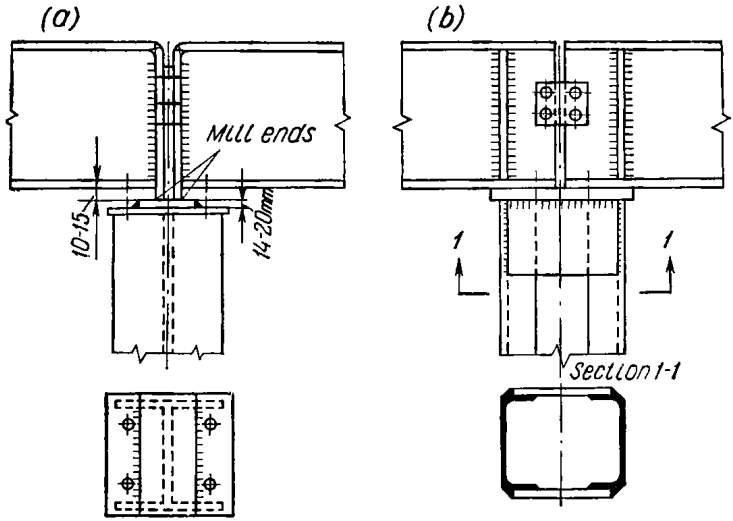


Fig. 6-32. Top beam-to-column connections

Such a connection is a flexible one owing to possible motion of the entire connection (bending back of the angle legs, play of the nuts, etc.). For providing a rigid connection, it is necessary to design a strong joint between the flange of the beam and the bearing member. Figure 6-33*b* shows an example of a rigid connection having a different design for the bottom and the top flanges of the beam in order to avoid overhead field welding.

The connections shown in Fig. 6-33 can be employed only for a statical load, since they have openings around which stresses will concentrate which are dangerous under a dynamic load.

Figure 6-34*a* pictures a flexible side-seated beam-to-column connection employing an unequal leg seat angle obtained by cutting off part of a leg. This connection is convenient for erection. The seat resists the entire support pressure F_r of the beam, which is transmitted to the column through the welds. However, the design length of the weld L_w at either side of the seat is generally determined

for a load equal to $\frac{2}{3}F_r$, owing to possible overloading of one of the sides because of inaccuracy in manufacture. The angles welded to the beam web serve for constructive purposes, each of them being secured to the column with two bolts.

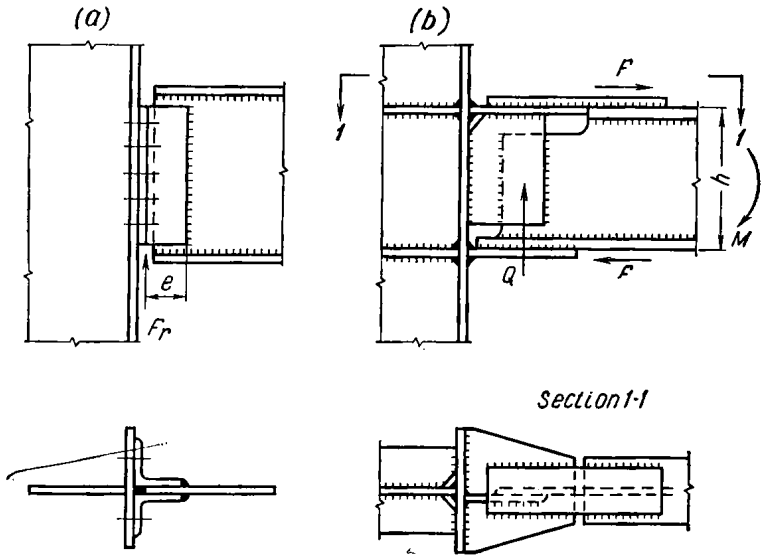


Fig. 6-33. Side beam-to-column connections

The seats are frequently made from a thick plate (t=25-30 mm). Figure 6-34*b* shows a rigid beam-to-column connection with a seat made from a thick plate. This connection is capable of resisting not only the support pressure transmitted to the seat, but also the moment

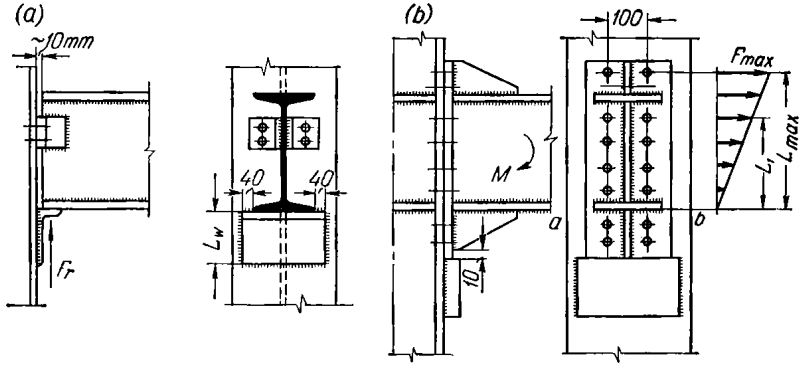


Fig. 6-34. Side seated beam-to-column connections

transmitted from the beam flanges to the bearing plate (flange) bolted to the column. The line *a-b* of the axis of flexible rotation of the connection (the neutral line), as investigations have shown, is located approximately at the level of the bottom beam flange. The maximum force in the two upper bolts, arranged on one horizontal line and resisting tension, is determined from the equation (Fig. 6-34*b*)

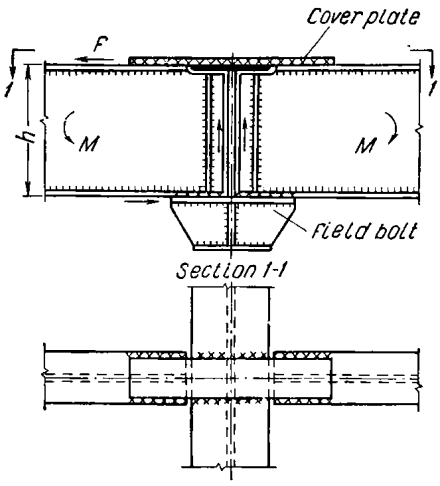


Fig. 6-35. Rigid connection of welded beams

$$F_{max} = \frac{ML_{max}}{\sum L_i^2} \qquad (6\text{-}71)$$

The lower edge of the bearing plate, which is made to protrude by 10 mm, should be milled, as should the upper edge of the seat. To ensure complete transmission of the support pressure to the seat, the diameter of the holes in the bearing plate should be designed 2-3 mm greater than the diameter of the bolts, thus preventing the bolts from taking shear. Since the bearing plate resists not only compression, but also bending, it should be made sufficiently thick (about 16-20 mm).

Figure 6-35 shows an example of a rigid connection between secondary welded beams and a main beam or girder. The support moment is transmitted here along the top flange through a plate (referred to as a cover or fish plate) designed to take the total force $F = \frac{M}{h}$, and along the bottom flange through the seat.

6-6. CRANE GIRDERS

(1) Design and Loads of Crane Girders

Crane beams and girders, which for convenience will all be termed herein "crane girders", are members used as runways for overhead cranes (also called bridge cranes, travelling cranes, shop travellers, etc.) serving shops and other premises of industrial buildings. The crane girders are usually supported on columns. The cranes run along rails laid on the top flange of the girder.

Crane girders may be either solid (simple or continuous) or, less frequently, lattice ones.

The main features characterizing the behaviour of crane girders are:

1. The withstanding of a vertical live load of the crane, which has a dynamic action on the girder.

2. The action of comparatively large concentrated loads applied by the wheels of the crane and transmitted through the flange connections (welds or rivets) to the web of the girder, causing its crushing.

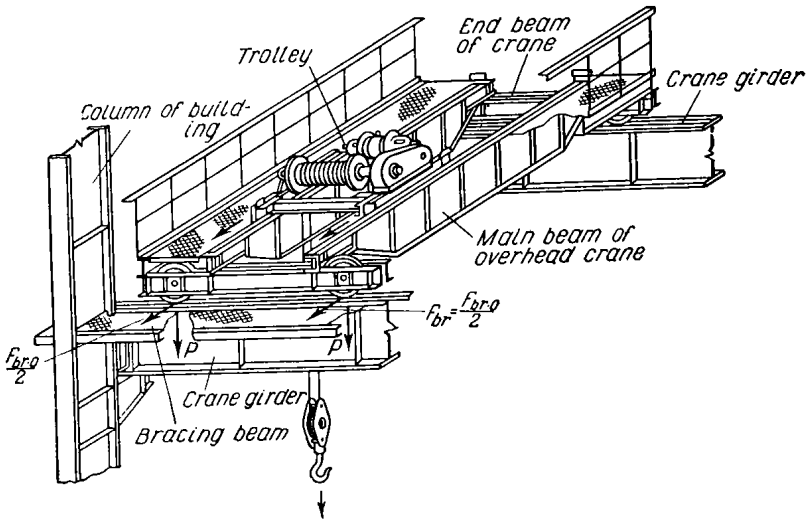


Fig. 6-36. Diagrammatic view of overhead crane

3. The presence of lateral braking forces that induce bending of the top beam flange in a horizontal plane.

An overhead crane consists of two main girders (trusses), along which the crane trolley with its load runs (Fig. 6-36). The load being handled, as well as the weight of the crane and the trolley, is transmitted to the crane girders through the crane wheels.

Depending upon the location of the trolley, the crane wheel load may have a maximum or minimum value. The maximum service (working) loads of crane wheels, as well as the arrangement of the wheels with respect to each other, are indicated in the standards relating to cranes. To obtain the design loads, the service ones should be multiplied by a load factor, which is taken equal to $n=1.2$ for cranes (except for cranes with a lifting capacity of less than 5 tons, for which a load factor of $n=1.3$ is taken). Besides, in view of the possibility of sharp changes in the speed of hoisting the load, unevenness of the crane runway and other reasons, the crane load is multi-

plied by a dynamic factor equal to 1.1. Thus the design crane wheel load will be

$$P = 1.1 \times 1.2 P_{max} \tag{6-73}$$

As regards the nature of their service conditions, cranes with a light, medium, heavy, very heavy and very heavy continuous duties are distinguished. Cranes with a light duty include cranes that are used seldom, for instance in the erection of equipment. Cranes with a medium duty cover, for example, the cranes of machine and assembly shops with a duty cycle factor ranging from 15 to 40%. Heavy, very heavy and very heavy continuous duty cranes are characterized by a duty cycle factor varying from 40 to 60 and even to 80%, with a high utilization factor both as regards lifting capacity and time.

Owing to braking of the trolley, lateral horizontal braking forces appear. As a result, a lateral braking force is transmitted to the crane bridge that is determined from the equation

$$F_{br \cdot o} = \frac{1}{10}(Q + g)\frac{2}{4} = \frac{Q+g}{20} \tag{6-74}$$

where Q = lifting capacity of crane, tons

g = weight of trolley, tons, taken in accordance with the standards for cranes; if no data are available it may be assumed that $g = 0.3Q$

$\frac{1}{10}$ = coefficient of friction

$\frac{2}{4}$ = fraction whose numerator indicates the number of brake wheels and denominator—the total number of trolley wheels (since the force of friction appears only under those trolley wheels which are outfitted with brakes).

The braking force $F_{br \cdot o}$ is transmitted to one crane girder and is distributed uniformly between the wheels of the crane. The design braking forces, the same as the vertical loads of the wheels, are obtained by multiplying them by the same load factors. In buildings with heavy service conditions (see page 372) the horizontal braking forces must be increased, in accordance with the Building Standards and Regulations, by multiplying them by the factor k_{br} (Table 6-11) that allows for possible misalignments, impacts and other lateral forces which are transmitted to the top flange of the girder.

It is also recommended to investigate the connections of the bracing members to the crane girders and columns in heavy and medium-duty cranes for lateral forces. These forces are computed by multiplying the horizontal braking forces by the factor $k_{br}/2$.

When designing crane girders for cranes with a heavy, very heavy and very heavy continuous duty, a service condition factor of $k_s = 0.9$ should be taken into account.

TABLE 6-11 **Factors k_{br} for Computing Lateral Forces Caused by Crane Bridges**

Type of crane	Values of factor k_{br} for investigation of	
	top flange of crane girders and bracing members	connections of bracing members to crane girders and columns
Cranes with flexible hoist suspension (on ropes) with a lifting capacity, tons, of:		
5-10	2.5	5
15-20	2	4
30-150	1.5	3
175-275	1.3	2.6
300-350	1.1	2.2
Cranes with rigid hoist suspension	1.5	3

(2) Solid Crane Girders

Types of Sections. Crane girders having a span of up to six metres, with a low lifting capacity of the cranes (up to 3-5 tons inclusively) are generally designed of a rolled I beam strengthened with a plate or angles (Fig. 6-37*a* and *b*). Girders with a span of six metres in 5 to 30 ton light and medium duty cranes are, as a rule, of a built-up design with a solid unsymmetrical section (Fig 6-37*c*) having a developed top flange for resisting the lateral braking forces. In the remaining cases special horizontal bracing members—beams or trusses, are provided to withstand the lateral braking forces (Fig. 6-37*d*).

Crane girders to be used outdoors or in unheated premises at a low temperature, as well as girders designed for heavy service conditions under heavy-duty cranes, must be made from killed steel of grade BCт.3 or from low-alloy steel.

Grade BCт.3пс semikilled steel may be employed for crane girders used by medium or light-duty cranes.

Welded I girders should be built up of three plates, and the tendency should be to secure the most economical distribution of the material between the flanges and the web of the girder [a material distribution factor of $k_d=0.5$ with the optimal depth of the girder, see Sec. 6-3 (1)]. The ratio between the thicknesses of the flange and the web (t_{fl}/t_w) should not exceed 2.5 to 3. Besides, it will be good practice to limit the thickness of the flanges to ensure complete

utilization of the design strength [40 mm for steel 3 and 32 mm for certain grades of low-alloy steel, see Sec. 2-7 (1)]. As has been previously indicated, the width of the flange should be within the limit of $b \leqslant 30 t_{fl} \sqrt{\frac{2{,}100}{R}}$.

In large-span girders under heavy cranes, however, it will not be good to employ flange plates whose width exceeds 0.8-1 metre. In these cases the flanges may be designed in the form of a group

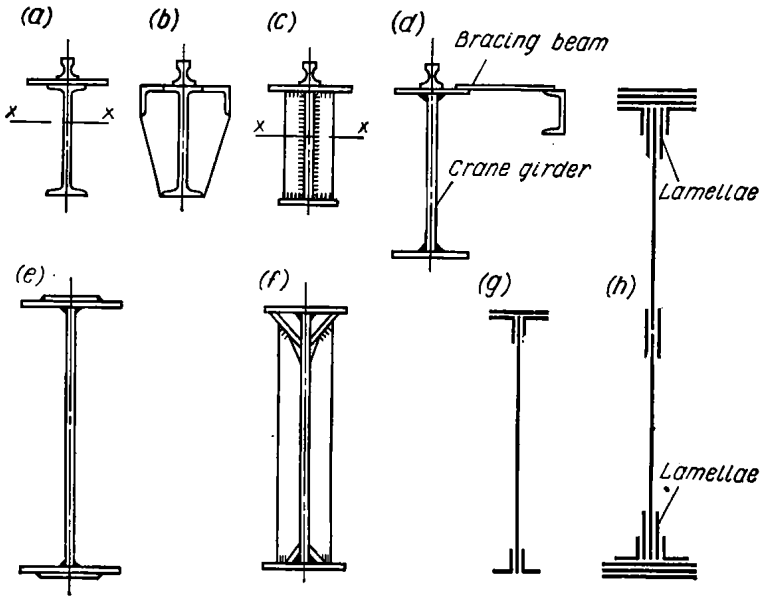


Fig. 6-37. Types of solid crane girders

of two horizontal plates symmetrically arranged with respect to the web. The difference in the width of these plates must not be less than 40 mm (Fig. 6-37*e*). It is also possible to develop the section of the top flange (in light or medium-duty cranes) by welding on two inclined plates arranged at an angle of 45 deg (Fig. 6-37*f*).

Riveted girders are employed in shops with heavy service conditions, where they are subjected to heavy dynamic loads.

The flanges of such girders are ordinarily made from angles and horizontal plates (Fig. 6-37*g*).

The sections of crane girders for heavy cranes (with a lifting capacity of 150 tons and more) are also sometimes designed of the riveted type, because of the difficulties encountered in welding heavy flange sections. The depth of such girders sometimes reaches

4-5 metres, and for this reason the vertical web is made up in depth of two plates connected together by means of a butt splice, with one or two rows of rivets (Fig. 6-37*h*). The flanges are developed not only by means of horizontal plates, but also by using so-called lamellae secured to the vertical web.

It is desirable to use not more than three horizontal plates in the flanges of riveted girders, the area of the angles, as has already been mentioned, being taken not less than 30% of the entire area of a flange.

Features of Solid Crane Girder Analysis. *Determination of Design Forces.* Solid crane girders are investigated in the same way as solid beams or girders carrying a statical load, but with regard to a number of special features.

The design moments and shear forces originated by the crane load may be computed either by using the influence lines plotted for two cranes installed on the runway, or by plotting a diagram of moments and shear forces for the most disadvantageous arrangement of the loads. For finding the maximum bending moment, the loads should be so arranged that the middle of the girder will be at equal distances from the resultant of all the loads and from the nearest load. Under the latter the maximum moment will be observed.

To determine the maximum shear force (support reaction) it is necessary to place one of the loads above a support and the remaining ones as near as possible to it. The location of the cranes for determining the stresses induced by the vertical and the horizontal forces should be identical.

The influence of the own weight of the crane girder and of the live load on the bracing beam is allowed for by increasing the values of the bending moments and the shear forces originated by the cranes (M_c and Q_c). For this purpose they should be multiplied by the factors k_M and k_Q contained in Table 6-12.

TABLE 6-12 **Values of Factors k_M and k_Q**

Factor	Span of girder in metres		
	6	12	18 and more
k_M (for M_c)	1.03	1.05	1.08
k_Q (for Q_c)	1.02	1.04	1.07

Selection of Sections. The sections of a solid crane girder are selected in the same way as those of built-up beams designed for a statical load, with attention paid to the following features.

Owing to the presence of a moving concentrated load that acts on the web through the top flange at spots not reinforced with stiffeners, the web is subjected to a local load (Fig. 6-38). The stress in the web σ_{loc} caused by this local load, when investigating for strength, should not exceed the design strength

$$\sigma_{loc} = \frac{n_1 P_1}{tz} \leqslant R \qquad (6\text{-}75)$$

Here P_1 = value of design concentrated load (without account of the dynamic factor 1.1 (see page 214), but with regard to the load factor)

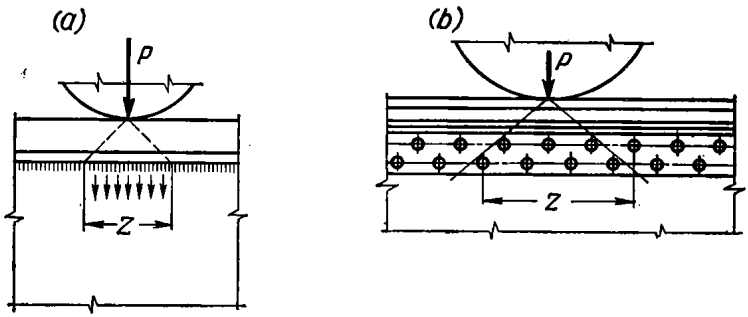


Fig. 6-38. Distribution of concentrated load over web and flange connections

n_1 = factor, taken for crane girders in buildings and structures with heavy service conditions equal to 1.5 for cranes with a rigid hoist suspension, 1.3 for cranes with a flexible suspension and 1.1 for other crane girders

t = thickness of web

z = conditional length of distribution of pressure induced by the concentrated load (Fig. 6-38), determined from the equation

$$z = c_1 \sqrt[3]{\frac{I_{fl}}{t}} \qquad (6\text{-}76)$$

where c_1 = factor taken equal to 3.25 for welded and rolled girders and 3.75 for riveted ones

I_{fl} = sum of moments of inertia of girder flange and crane rail with respect to their own axes; when the rail is welded to the flange by means of welds that ensure joint behaviour of the rail and the girder, I_{fl} is the total moment of inertia of the flange and the rail.

The stress induced by the local load σ_{loc} must be taken into consideration when establishing the thickness of the crane girder web, as also the condition of ensuring adequate resistance of the web to shear.

From expressions (6-75) and (6-76), and assuming in the first approximation that I_{fl} is equal to the moment of inertia of only the crane rail (in cm^4), we shall obtain an equation for the minimum thickness of a crane girder web

$$t_{min} = \frac{n_1 P_1}{3.25R} \sqrt{\frac{n_1 P_1}{3.25 R I_{fl}}} \text{ cm} \qquad (6\text{-}77)$$

in which $P_1 = P'n$ is the design load of a crane wheel on the girder in tons, equal to the product of the service wheel load and the load factor $n = 1.2$.

The required section modulus (W_{gr} for welded girders and W_n for riveted ones) is determined on the basis of the design strength reduced by 150-250 kg/cm². This is done because in the top flange of the girder, which is simultaneously subjected to horizontal braking forces, additional stresses induced by these forces appear.

Having planned the cross-sectional dimensions of the crane girders and the bracing beams, the geometrical characteristics of the section (the moments of inertia, the section moduli, and so on) are determined with respect to the horizontal and the vertical axes after which the strength of the beam is investigated.

The strength of a solid crane girder is checked (when a solid bracing beam will be used) by means of the following expressions:
for the top fibre of the girder

$$\sigma = \frac{M}{W_{n \cdot t}} + \frac{M_{br}}{W_{br}} \leqslant R \qquad (6\text{-}78)$$

for the bottom fibre of the girder

$$\sigma = \frac{M}{W_{n \cdot b}} \leqslant R \qquad (6\text{-}79)$$

Here M = design moment induced by vertical crane load
M_{br} = design moment originated by horizontal lateral braking forces
$W_{n \cdot t}$ = net section modulus for top fibre of girder (in welded girders there are also sometimes encountered holes for the bolts securing the crane rail to the girder with the aid of claws)
$W_{n \cdot b}$ = net section modulus for the bottom fibre of the girder (for welded girders W_{gr} is taken)

W_{br} = section modulus of the bracing beam comprising the top flange of the crane girder, a horizontal plate and an end framing flange (Fig. 6-37*d*) with respect to the vertical axis *y-y*, while if there is no bracing beam—of only the top flange of the girder.

The strength of a solid crane girder is checked (when a bracing member in the form of a truss is employed) as follows:
for the top fibre of the girder according to the expression

$$\sigma = \frac{M}{W_{n \cdot t}} + \frac{F_{br}}{\varphi A_{fl \cdot gr}} + \frac{M_{br \cdot loc}}{W_{fl}} \leqslant R \qquad (6\text{-}80)$$

for the bottom fibre of the girder—by means of expression (6-79).

Here $F_{br} = \frac{M_{br}}{h_{br}}$ = design longitudinal force in top flange of a girder with a sectional area of $A_{fl \cdot gr}$, as in the flange of a horizontal truss (with a depth of h_{br}) induced by the lateral braking forces

φ = buckling factor in a horizontal plane for the length of the panel d

$M_{br \cdot loc} = \frac{F_{br} d}{5}$ = local moment in the top flange of the girder in a horizontal plane, induced by the lateral braking force F_{br}

W_{fl} = section modulus of top flange of crane girder with respect to its vertical axis.

The general stability of crane girders with a reinforced top flange (without a bracing beam) is investigated by means of expression (6-14), i. e.,

$$\sigma = \frac{M}{\varphi_b W_{gr}} \leqslant R$$

When determining the factor φ_b, the width b is taken equal to that of the reinforced top flange.

The endurance of crane girders should be investigated for the service load of one crane from among those operating in the given span, without regard to the dynamic factor. The crane having the highest lifting capacity is taken for these calculations.

The endurance of simple crane girders with a solid section designed from grade ВСт.3 steel (welded) and grade ВСт.3пс steel (riveted) or from low-alloy steels may be left unchecked on condition that the flange welds be made with full penetration (Fig. 6-39) or by means of automatic welding.

Additional Checking of Crane Girder. Having selected the section and investigated its strength, the following should be analyzed:

1. The deflection of the girder.
2. The strength of the web in local crushing induced by the load of the crane wheels [by means of equation (6-7)].

3. The flange welds or rivets connecting the flanges of the girder to the web.

4. The local stability of the girder web in accordance with the contemplated arrangement of the stiffeners.

The deflection of the girder can be checked by means of the equation

$$\delta = \frac{M_s L^2}{10EI} \tag{6-81}$$

in which M_s=moment produced by the vertical service loads (without introducing the load and the dynamic factors).

This deflection should not exceed the allowable value (see Table 3 of Appendix I).

The flange welds or rivets connecting the top flange of the crane girder to the web are analyzed for the action of not only the horizontally directed shearing stresses due to bending τ_h but also of the local vertically directed stresses σ_{loc} [see expression (6-75)] caused by the concentrated load of the wheel. Here it is assumed that in welded girders the concentrated load P is resisted by the welds over a length of z (Fig. 6-38*a*), found from equation (6-76).

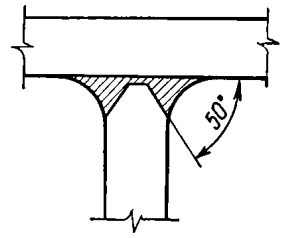


Fig. 6-39. Top flange weld of crane girder with penetration over whole thickness of web

The resultant of the shearing and local stresses, which is conditionally determined by geometrically adding them, must not exceed the design shear strength of the fillet welds

$$\tau = \sqrt{\tau_h^2 + \sigma_{loc}^2} = \frac{1}{2k_w h_w} \sqrt{\left(\frac{QS_f l}{I_{gr}}\right)^2 + \left(\frac{n_1 P_1}{z}\right)^2} \leqslant R_{w \cdot f}$$

whence

$$h_w \geqslant \frac{1}{2k_w R_{w \cdot f}} \sqrt{\left(\frac{QS_f l}{I_{gr}}\right)^2 + \left(\frac{n_1 P_1}{z}\right)^2} \tag{6-82}$$

For the values of the factor k_w see page 112, and of the factor n_1—page 219.

In crane girders the welds securing the top flange to the web should be made with penetration through the whole thickness of the web. To ensure such a penetration, the edge of the girder web, when it has a thickness of 10 mm or above, is milled to a double-bevel shape (Fig. 6-39); with automatic welding edges are milled in this manner when their thickness is 14 mm and more. When welding is performed with penetration over the whole thickness of the web, the standards permit designers to consider that the weld has the same strength as the web.

In riveted crane girders it will be very good practice to mill the upper edge of the web flush with the back ends of the flange angles. For this purpose they are first made to protrude five millimetres beyond the angles. It is assumed here that the concentrated loads are partly transmitted directly to the web (60%) and partly to the angles of the flange riveted to the web (40%).

When investigating the top flange rivets, it is assumed that the concentrated load P is uniformly distributed between the rivets located along the length z (Fig. 6-38*b*) determined from equation (6-76).

The pitch of the flange rivets in this case is determined from the expression

$$a \leqslant \frac{F_r'}{\sqrt{\left(\frac{QS_{fl}}{I_{gr}}\right)^2 + \left(\frac{\alpha n_1 P_1}{z}\right)^2}} \qquad (6\text{-}83)$$

where $\alpha = 0.4$ if the upper edge of the web is milled flush with the back ends of the angles

$\alpha = 1$ if the web is not milled

F_r' = maximum design force allowed per rivet in crushing or double shear.

The local stability of crane girder webs is checked if $\frac{h_w}{t} > 80\sqrt{\frac{2{,}100}{R}}$ (see page 195). To ensure stability of the web there are installed pairs of main stiffeners, whose spacing, as has already been indicated, should not exceed $2h$ when $\frac{h}{t} > 100$ and $2.5h$ when $\frac{h}{t} \leqslant 100$.

Having planned the arrangement of the stiffeners, the dimensions of the panels between them being investigated are established. The stability of the web is checked for the combined action of the normal, shearing and local stresses σ, τ and σ_{loc}. When the web is being braced only with transverse stiffeners and when $\frac{a}{h_w} \leqslant 0.8$ (a is the spacing of the stiffeners and h_w is the design depth of the web), the stability is checked by means of the expression

$$\sqrt{\left(\frac{\sigma}{\sigma_{cr}} + \frac{\sigma_{loc}}{\sigma_{loc \cdot cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} \leqslant 0.9 \qquad (6\text{-}84)$$

Here the values of the normal and shearing stresses σ and τ are determined in the same way as in expression (6-63). The values of the critical stresses σ_{cr} and τ_{cr} are taken in accordance with equations (6-61) and (6-57). The local stress σ_{loc} is determined from expression (6-75), but the factor n_1 is taken equal to 1.1 for all crane girders.

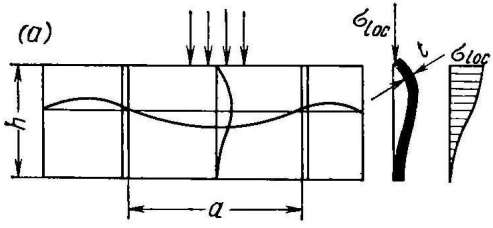
Figure 6-40 pictures the loss of stability of a web owing to local stresses. The critical local stress is found from the equation

$$\sigma_{loc \cdot cr} = \frac{k_g 10^4}{\lambda_{loc}^2} = k_g \left(\frac{100t}{a}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}85)$$

where $\lambda_{loc} = \frac{a}{t}$ = slenderness ratio of web between stiffeners

a = spacing (distance between centre lines) of transverse stiffeners

k_g = factor depending upon the depth of the web and the value of γ [see equation (6-62)] and taken in accordance with Table 6-13. When computing γ from equation (6-62) the factor C is taken equal to 2.



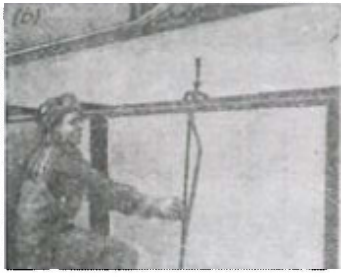


Fig. 6-40. Loss of stability of crane girder web due to local stresses:
a — diagram; *b* — general view

TABLE 6-13 **Values of Factor k_g for Welded Girders**

γ	Factor k_g for a/h_w equal to								
	⩽0.5	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
⩽0.8	2.04	2.39	3.03	3.67	4.11	4.67	5.32	6.04	6.88
1.0	2.07	2.44	3.13	3.80	4.29	4.90	5.61	6.39	7.29
2.0	2.13	2.54	3.33	4.17	4.85	5.67	6.59	7.59	8.72
4.0	2.17	2.60	3.47	4.46	5.33	6.38	7.58	8.89	10.35
6.0	2.18	2.62	3.52	4.58	5.55	6.73	8.07	9.54	11.19
10.0	2.19	2.64	3.57	4.68	5.76	7.07	8.56	10.21	12.06
⩾30.0	2.21	2.67	3.65	4.86	6.15	7.74	9.56	11.58	13.86

If $a > 2h_w$, then in determining $\sigma_{loc \cdot cr}$ it is assumed that $a = 2h_w$. For riveted girders k_g is determined from Table 6-13 for $\gamma = 10$.

When $\frac{a}{h_w} > 0.8$ there may be two forms of buckling of the crane girder web, one with a ratio of the sides of a/h_w having one half-wave over the length of the panel, and the other with a ratio of the sides of $\frac{0.5\,a}{h_w}$ having correspondingly two half-waves.

For this reason, with $\frac{a}{h_w} > 0.8$ two cases are distinguished when using expression (6-84) for investigating the web of the girder.

1. Should σ_{loc}/σ be greater than the values indicated in Table 6-15, then the critical stress σ_{cr} used in expression (6-84) is found not from equation (6-61), but from the equation

$$\sigma_{cr} = k'_g \left(\frac{100t}{h_w}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}86)$$

in which the values of k'_g are taken from Table 6-14, while those of the other symbols remain the same as above.

TABLE 6-14 **Values of Factor k'_g**

$\frac{a}{h_w}$	$\leqslant 0.8$	0.9	1.0	1.2	1.4	1.6	1.8	$\geqslant 2.0$
k'_g	According to Table 6-10	7.78	8.23	9.50	11.10	13.02	15.25	17.79

2. If $\frac{\sigma_{loc}}{\sigma}$ does not exceed the values indicated in Table 6-15, then σ_{cr} is determined from equation (6-61), and $\sigma_{loc \cdot cr}$ from equation

TABLE 6-15 **Boundary Values of $\frac{\sigma_{loc}}{\sigma}$**

Type of girder	γ	Values of σ_{loc}/σ for a/h_w equal to			
		$\leqslant 0.8$	1.2	1.6	$\geqslant 2$
Welded	$\leqslant 1$	0	0.24	0.29	0.33
	4	0	0.24	0.40	0.50
	10	0	0.27	0.49	0.67
	$\geqslant 30$	0	0.29	0.62	0.94
Riveted	—	0	0.33	0.55	0.73

(6-85), but with $\frac{a}{2}$ substituted for a both in equation (6-85) and in Table 6-13.

In both cases τ_{cr} is calculated from equation (6-57). A possible way of designing riveted girders to prevent buckling of the web under the action of the local stresses is the installation in the upper third part of the web of short stiffeners. The latter are located between the main stiffeners extending over the entire depth of the web (Fig. 6-41). According to the standards, in this instance the analysis is carried out twice:

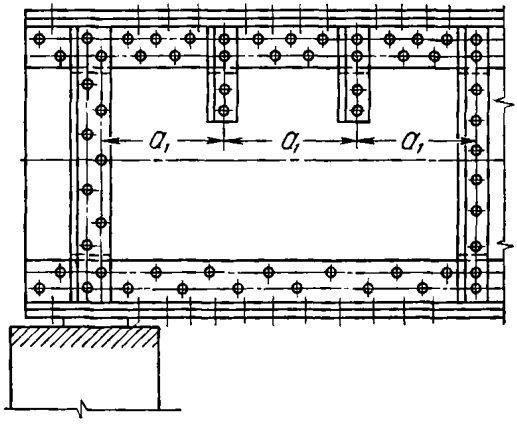


Fig. 6-41. Bracing of riveted crane girder web with short stiffeners

1. With the aid of expression (6-84), on the assumption that $\sigma_{loc}=0$ and that there are no short stiffeners.
2. By means of the expression

$$1.5\sigma_{loc}\left(\frac{a_1}{100t}\right)^3 \leqslant 1 \qquad (6\text{-}87)$$

In the above expression σ_{loc} is taken in t/cm².

As has already been indicated, it will be good to install a longitudinal stiffener (Fig. 6-42) in heavily loaded girders with a large span, and having a web slenderness ratio of $\frac{h_w}{t} > 160\sqrt{\frac{2{,}100}{R}}$. Such a stiffener, generally located at a distance of $b_1=(0.2 \text{ to } 0.25)h$ from the extreme compression fibre of the panel, divides it into two parts, namely, the top and the bottom ones.

The top panel, located between the compression flange and the longitudinal stiffener, is under conditions of non-uniform com-

pression. It is checked by means of expression (6-88) obtained from consideration of the boundary curve separating the limiting areas of the load-carrying capacity of the panels

$$\frac{\sigma}{\sigma_{cr_1}}+\frac{\sigma_{loc}}{\sigma_{loc\cdot cr_1}}+\left(\frac{\tau}{\tau_{cr_1}}\right)^2\leqslant 0.9 \qquad (6\text{-}88)$$

When $\sigma_{loc}=0$, the critical stress σ_{cr_1} is found from the equation

$$\sigma_{cr_1}=\frac{1}{1-\frac{b_1}{h_w}}\left(\frac{100t}{b_1}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}89)$$

This equation has been obtained from the general equation (6-60), where the coefficient ζ in accordance with Table 6-9 has the value

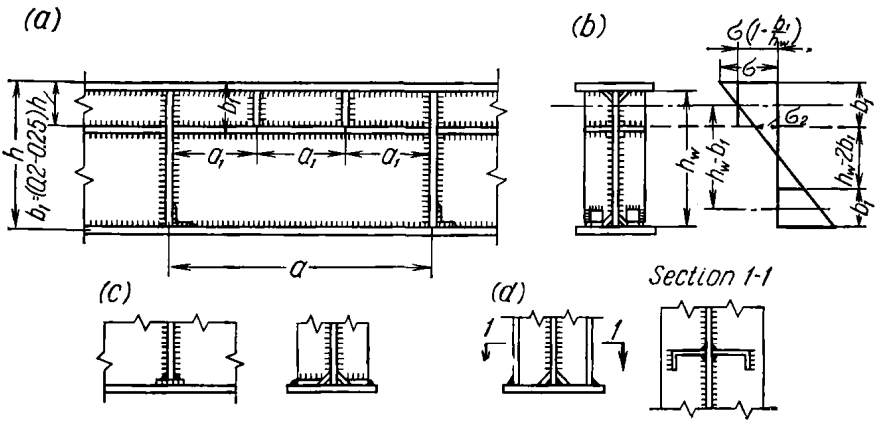


Fig. 6-42. Bracing of crane girder web with longitudinal stiffener, and details of ribs

of $\zeta=1$ (with $k_n=0$) for uniform compression; here the mean uniform compressive stress of the top panel is taken equal to $\sigma_m=\sigma\left(1-\frac{b_1}{h_w}\right)$ (Fig. 6-42*b*), whence $\sigma=\frac{\sigma_m}{1-\frac{b_1}{h_w}}$

When $\sigma_{loc}\neq 0$ we have

$$\sigma_{cr_1}=\frac{0.25}{1-\frac{b_1}{h_w}}\frac{(1+\mu_1^2)^2}{\mu_1^2}\left(\frac{100t}{b_1}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}90a)$$

and

$$\sigma_{loc \cdot cr_1} = k' \frac{(1+\mu_1^2)^2}{\mu_1^2} \left(\frac{100t}{a}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}90b)$$

where $\mu_1 = \frac{a}{b_1} \leqslant 2$

k' = factor whose values are given in Table 6-16

σ, σ_{loc} and τ = design stresses; σ_{cr_1} is determined from equation (6-57).

TABLE 6-16 **Values of Factor k'**

$\mu_1 = \frac{a}{b_1}$	1.0	1.5	2.0
k'	0.36	0.42	0.45

The bottom panel, located between the tension flange and the longitudinal stiffener, is in non-uniform tension. The stability of this part is higher than that of a girder web with a conditional depth of $h_w - 2b_1$, the latter being used for analysis. The stress in the extreme fibre of the conditionally separated girder (Fig. 6-42*b*) is equal to $\sigma_2 = \sigma\left(1 - \frac{2b_1}{h_w}\right)$.

The boundary curve of the stable and unstable areas in this case is assumed to have the form of

$$\sqrt{\left[\frac{\sigma\left(1-2\frac{b_1}{h_w}\right)}{\sigma_{cr_2}} + \frac{\sigma_{loc_2}}{\sigma_{loc \cdot cr_2}}\right]^2 + \left(\frac{\tau}{\tau_{cr_2}}\right)^2} \leqslant 1 \qquad (6\text{-}91)$$

where $\sigma_{cr_2} = \frac{1.14}{\left(0.5 - \frac{b_1}{h_w}\right)^2} \left(\frac{100t}{h_w}\right)^2 \text{ t/cm}^2$ has been obtained from the general equation (6-60) for the value of $\zeta = 4.54$, which corresponds to a panel in bending (with $k_n = 2$) not fixed at its ends; $\sigma_{loc \cdot cr_2}$ is determined from equation (6-85) and Table 6-13 for $\gamma = 0.8$, with substitution of the expression $\frac{a}{h_w - b_1}$ for $\frac{a}{h_w}$; when a load is applied to the compression flange, the value of σ_{loc_2} is taken equal to $\sigma_{loc_2} = 0.4\ \sigma_{loc}$.

In continuous girders, when the load is applied to the tension flange and the horizontal stiffener is correspondingly located in the bottom compression part of the girder, $\sigma_{loc_2} = \sigma_{loc}$.

When a longitudinal stiffener is employed in a welded girder, the compression zone of the web may be braced with short stiffeners welded to the longitudinal one (Fig. 6-42*a*).

The web stability of girders with an unsymmetrical section (with a more developed compression flange) is investigated for webs braced only with transverse stiffeners by means of expressions (6-84), (6-61) and (6-86), in which h_w is taken equal to the double distance from the neutral axis to the compression boundary of the panel.

Transverse stiffeners must be welded to the top flange of the girder by means of small-size welds (6-8 mm).

The transverse welds connecting the stiffeners to the bottom flange must also have a minimum size, since shrinkage of these transverse welds leads to the appearance of internal tensile stresses (see Chapter 4). In the present case the latter will be added to the tensile stresses induced by the external load.

It is not good policy to leave a space between the stiffeners and the bottom flange, since this will greatly reduce the stiffness in twisting of the girder, and the bottom flange will be easily deformed.

The stiffeners can be welded to the bottom flange with the aid of short plates or angles that are connected to the flange only with longitudinal welds (Fig. 6-42*a* and *c*).

For heavy crane girders in shops with heavy service conditions, stiffeners made from unequal leg angles welded to the web along their edge may be used (Fig. 6-42*d*).

The stiffeners should be welded to the web by means of continuous welds of minimum size. When located near welded web splices, the stiffeners must be arranged at a distance of at least $10t_{web}$ from the welds. To reduce the influence of the heat-affected zone of the welds, the ends of vertical stiffeners should be bevelled 60 mm in depth and 40 mm in width. For the same reason at the intersections of girder web splice welds with stiffeners, the welds connecting the stiffeners to the web should be terminated at a distance of 40-50 mm before the splice weld.

The lower ends of supporting ribs can be designed with bevels 30 mm in depth and 20 mm in width, but with the weld connecting the stiffener to the flange terminating 40 mm before the web.

The dimensions of the main transverse stiffeners are established in the same way as in ordinary beams, i. e., the width of the stiffener $b_{rib} \geqslant \frac{h}{30} + 40$ mm and the thickness thereof $t_{rib} \geqslant \frac{1}{15} b_{rib}$.

Example 6-11. Design a crane girder with a span of $L=12$ metres for two cranes with a lifting capacity of $Q=50/10$ tons with flexible suspension of the load, and with a very heavy duty. The span of the crane $L_c=28.5$ metres. The maximum load on a crane wheel is $P_{max}=50.5$ tons, the weight of the trolley $g=18.5$ tons, the type of rail used is KP80 (to GOST 3332-54); the arrangement of the wheels is shown diagrammatically in Fig. 6-43*a*. The beam is welded, type Э42 electrodes being used for this purpose, the material of the beam is grade ВСт.3 steel.

Solution. 1. Statical Analysis. We find the values of the design loads. The vertical loads are determined from equation (6-73)

$$P = 1.1 \times 1.2 P_{max} = 1.1 \times 1.2 \times 50.5 = 66.6 \text{ tons}$$

the horizontal (braking) loads are found with account taken of equation (6-74) and Table 6-11

$$F_{br} = \frac{F_{br \cdot o}}{2} n k_{br} = \frac{Q+g}{2 \times 20} n k_{br} = \frac{50+18.5}{2 \times 20} \times 1.2 \times 1.5 = 3.1 \text{ tons}$$

We place the cranes in such a position that will permit us to determine the maximum bending moments induced by the vertical and the braking forces (Fig. 6-43*b*, point 2).

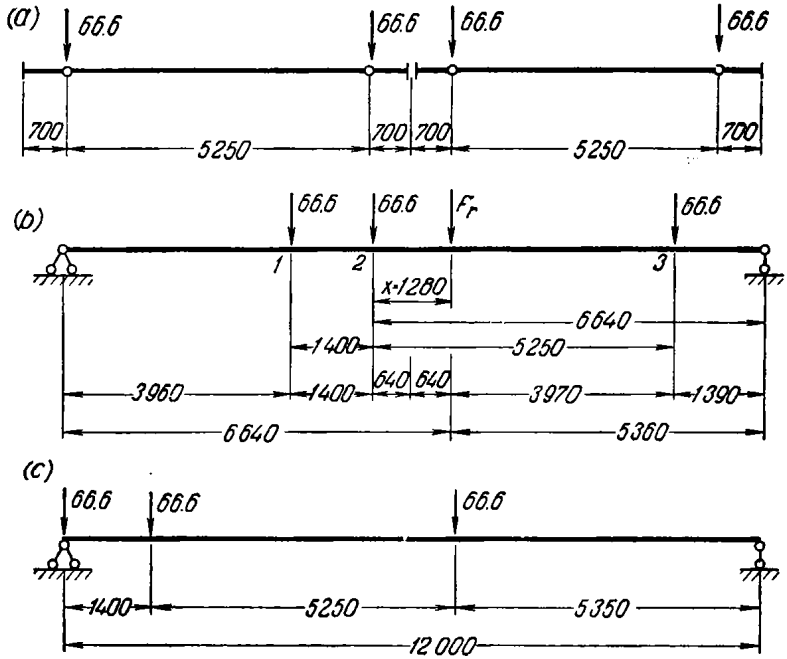


Fig. 6-43. To example 6-11:
a—diagram of crane load; *b* and *c*—diagrams showing positioning of cranes for determination of maximum moments and shear force

The position of the resultant force F_{res} is determined by the value of

$$x = \frac{66.6\ (5.25 - 1.4)}{3 \times 66.6} = 1.28 \text{ m}$$

The bending moments

$$M_c = \frac{3 \times 66.6 \times 6.64^2}{12} - 66.6 \times 5.25 = 385 \text{ t-m}$$

$$M_{br} = M_c \frac{F_{br}}{P} = 385 \times \frac{3.1}{66.6} = 17.9 \text{ t-m}$$

The corresponding shear force induced by the vertical forces will be

$$Q_c = \frac{3 \times 66.6 \times 6.64}{12} - 66.6 = 43.9 \text{ tons}$$

We place the cranes in a position that will allow us to determine the maximum shear force on a support caused by the vertical forces (Fig. 6-43*c*)

$$Q_{c \cdot s} = \frac{66.6}{12}(5.35 + 10.6 + 12) = 155 \text{ tons}$$

The weight and the live load on the bracing beam are taken into account with the aid of the factors k_M and k_Q (see Table 6-12).

Thus, the final design values of the moments and shear forces will be as follows:

at the middle of the girder (at point 2)

$$M = k_M M_c = 1.05 \times 385 = 405 \text{ t-m}$$

at the support

$$Q_s = k_Q Q_{c \cdot s} = 1.04 \times 155 = 161 \text{ tons}$$

2. Selection of Section and Analysis of Strength. *We determine the minimum allowable depth of the girder* to ensure its adequate stiffness from equation (6-4), based on an allowable unit deflection of $\frac{1}{r_0} = \frac{1}{750}$, and assuming that the load factor $n_q = 1.2$, viz.,

$$h_{min} = \frac{Lr_0}{4{,}800} \frac{1}{n_q} = \frac{1{,}200 \times 750}{4{,}800 \times 1.2} = 156 \text{ cm}$$

We find the approximate optimal depth of the girder from equation (6-22), for which purpose we first select the slenderness ratio of the web from Table 6-3, namely, $\lambda_w = \frac{h_w}{t} = 125$, and find the required section modulus

$$W_{req} \cong \frac{M}{k_s R - 200} = \frac{40{,}500{,}000}{0.9 \times 2{,}100 - 200} = 24{,}000 \text{ cm}^3$$

Here a service condition factor $k_s = 0.9$ has been introduced that reduces the design strength R, seeing that the crane has a very heavy duty (see Table 2 of Appendix I). Besides, the stress has been reduced in view of the action of horizontal braking forces on the top flange of the girder (see page 219).

The optimal depth

$$h_{opt} = \sqrt[3]{\frac{3}{2} \lambda_w W_{req}} = \sqrt[3]{\frac{3}{2} \times 125 \times 24{,}000} = 165 \text{ cm}$$

We take the depth of the girder equal to 160 cm and establish a depth of the web h_w of 156 cm, leaving 2 cm for each flange.

We select the thickness of the web. The minimum thickness of the web when investigating its strength at the support for resisting shearing stresses is found from equation (6-26)

$$t_{min} = \frac{1.5Q}{k_s R_{sh} h_w} = \frac{1.5 \times 161{,}000}{0.9 \times 1{,}300 \times 156} = 13.2 \text{ mm}$$

The minimum thickness of the web when checked for strength to resist local pressure is determined from equation (6-77)

$$t_{min}=\frac{n_1P_1}{3.25k_sR}\sqrt{\frac{n_1P_1}{3.25k_sRI_{fl}}}=\frac{1.3\times60.6}{3.25\times0.9\times2.1}\sqrt{\frac{1.3\times60.6}{3.25\times0.9\times2.1\times1,547}}=1.16\text{ cm}$$

($P_1=P'n=50.5\times1.2=60.6$ tons, $n_1=1.3$, $k_s=0.9$, a type KP80 rail is used with $I_{fl}=1,547$ cm⁴).

We establish a web thickness of 1.4 cm or 14 mm.

The flange cross-sectional areas are preliminarily determined from equation (6-34)

$$2A_{fl}=\frac{3W_{req}}{2h_w}=\frac{3}{2}\times\frac{24,000}{156}=231\text{ cm}^2$$

We plan the final section of the girder, namely, web—1,556×14, top flange plate—600×22, bottom flange plate—460×22 (Fig. 6-44), after which we find the geometrical characteristics and investigate the strength of the girder section.

We find the moment of inertia (with regard to the weakening of the top flange by the holes for securing the rail).

The displacement of the centre of gravity with respect to the middle of the girder will be

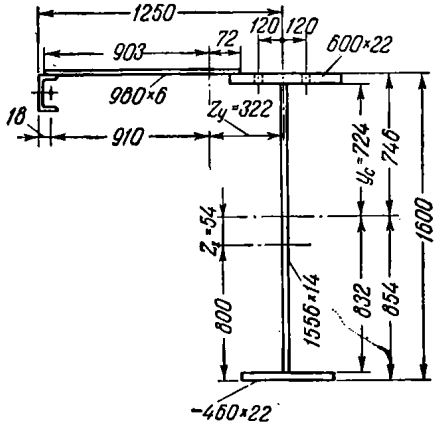


Fig. 6-44. To example 6-11. Section of girder

$$z_x=\frac{60\times2.2\times78.9-46\times2.2\times78.9}{60\times2.2+46\times2.2+155.6\times1.4}=5.4\text{ cm}$$

$$I_{gr\cdot x}=\frac{1.4\times72.4^3}{3}+\frac{1.4\times83.2^3}{3}+60\times2.2\times73.5^2+46\times2.2\times84.3^2=$$
$$=1,878,000\text{ cm}^4$$

$$I_{n\cdot x}=\frac{1.4\times72.4^3}{3}+\frac{1.4\times83.2^3}{3}+(60-4)\,2.2\times73.5^2+$$
$$+46\times2.2\times84.3^2=1,829,000\text{ cm}^4$$

The section moduli will be

$$W_{gr\cdot t}=\frac{1,878,000}{74.6}=25,200\text{ cm}^3\quad W_{gr\cdot b}=\frac{1,878,000}{85.4}=$$
$$=22,000\text{ cm}^3\quad W_{n\cdot t}=\frac{1,829,000}{74.6}=24,500\text{ cm}^3$$
$$W_{n\cdot b}=\frac{1,829,000}{85.4}=21,400\text{ cm}^3$$

The statical moment of a half-section will be

$$S_x=60\times2.2\times73.5+72.4\times1.4\times36.2=13,370.\text{cm}^3$$

The section of the bracing beam (Fig. 6-44) will be made up of a channel No. 16 ($A=18.1$ cm^2), a horizontal plate with a thickness of $t=6$ mm and the top flange of the girder made from a plate 600×22 mm in size. The distance from the centre of gravity to the axis of the girder is

$$z_y=\frac{18.1\times123.2+98\times0.6\times74}{18.1+98\times0.6+60\times2.2}=32.2 \text{ cm}$$

The moment of inertia of the bracing beam with respect to axis y-y will be

$$I_{n\cdot y}=18.1\times91^2+\frac{0.6\times90.8^3}{3}+\frac{0.6\times7.2^3}{3}+\left(\frac{2.2\times60^3}{12}-2\times2.2\times2\times12^2\right)+$$
$$+2.2\,(60-4)\,32.2^2=467{,}760 \text{ cm}^4$$

The corresponding section modulus is

$$W_{n\cdot br}=\frac{467{,}760}{32.2+30}=7{,}540 \text{ cm}^3$$

We investigate the strength of the girder, for which purpose we find: the normal stresses in the top flange from equation (6-78)

$$\sigma_t=\frac{M}{W_{n\cdot t}}+\frac{M_{br}}{W_{n\cdot br}}=\frac{40{,}500{,}000}{24{,}500}+\frac{1{,}790{,}000}{7{,}540}=$$
$$=1{,}650+238=1{,}888 \text{ kg/cm}^2<$$
$$>0.9\times2{,}100 \text{ kg/cm}^2=1{,}890 \text{ kg/cm}^2$$

the normal stresses in the bottom flange from equation (6-79)

$$\sigma_b=\frac{M}{W_{n\cdot b}}=\frac{40{,}500{,}000}{21{,}400}=1{,}890 \text{ kg/cm}^2=0.9\times2{,}100 \text{ kg/cm}^2$$

the shearing stresses on the support from expression (6-36)

$$\tau=\frac{Q_s S_s}{I_{gr\cdot x}t}=\frac{161{,}000\times13{,}370}{1{,}878{,}000\times1.4}=820 \text{ kg/cm}^2<0.9\times$$
$$\times1{,}300 \text{ kg/cm}^2=1{,}170 \text{ kg/cm}^2$$

We check the deflection of the girder caused by the vertical service loads (without the dynamic factor)

$$M_{ser}=\frac{405}{1.1\times1.2}=306 \text{ t-m}$$

and from equation (6-81)

$$\delta_{max}=\frac{M_{ser}L^2}{10EI_{gr\cdot x}}=\frac{30{,}600{,}000\times1{,}200^2}{10\times2{,}100{,}000\times1{,}878{,}000}=1.12 \text{ cm}$$

The unit deflection will be

$$\frac{\delta_{max}}{L}=\frac{1.12}{1{,}200}=\frac{1}{1{,}070}<\frac{1}{750}$$

We check whether the strength of the web is sufficient to take the local load of the crane wheel, by means of expression (6-75). For this purpose we first compute the sum of the moments of inertia of the top flange and the type KP80 rail (to GOST 4121-62)

$$I_{fl}=1{,}547+\frac{60\times2.2^3}{12}=1{,}600 \text{ cm}^4$$

and the conditional length over which the load of the wheel is distributed, from equation (6-76)

$$z = c_1 \sqrt[3]{\frac{I_{fl}}{t}} = 3.25 \sqrt[3]{\frac{1,600}{1.4}} = 34 \text{ cm}$$

$$P_1 = 50.5 \times 1.2 = 60.6 \text{ tons } (60,000 \text{ kg})$$

$$\delta_{loc} = \frac{n_1 P_1}{tz} = \frac{1.3 \times 60,600}{1.4 \times 34} =$$

$$= 1,655 \text{ kg/cm}^2 < 0.9 \times 2,100 \text{ kg/cm}^2 = 1,890 \text{ kg/cm}^2$$

3. Investigation of Local Stability of Web. The necessity of installing stiffeners and checking the stability follows from the ratio

$$\frac{h_w}{t} = \frac{155.6}{1.4} = 111 > 80$$

We space the stiffeners 1.5 metres apart and check two panels, namely, at the middle of the girder and at the support (Fig. 6-45*b*).

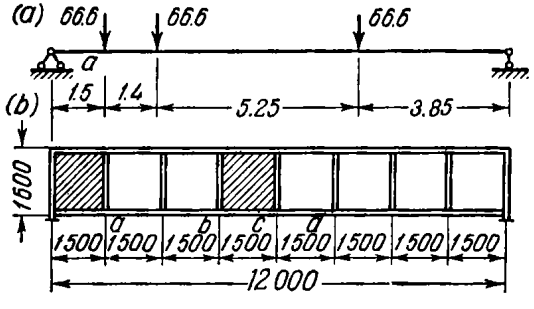


Fig. 6-45. To example 6-11. Arrangement of crane load for investigations

We investigate the middle panel when the crane is arranged as shown in Fig. 6-43*b*. We determine the bending moments at points *b*, *c* and *d*

$$M_b = \left(\frac{3 \times 66.6 \times 5.36 \times 4.5}{12} - 66.6 \times 0.54 \right) 1.05 = 385 \text{ t-m}$$

$$M_c \cong 385 \text{ t-m (see above)}$$

$$M_d = \left(\frac{3 \times 66.6 \times 6.64 \times 6}{12} - 66.6 \times 4.61 \right) 1.05 = 375 \text{ t-m}$$

$$M_m = \frac{385 + 385 + 375}{3} = 382 \text{ t-m}$$

We find the stresses in the middle (hatched) panel (Fig. 6-45*b*)

$$\sigma = \frac{M_m y_c}{I_{gr \cdot x}} = \frac{38{,}200 \times 72.4}{1{,}878{,}000} = 1.47 \text{ t/cm}^2$$

$$\tau = \frac{Q}{h_w t} = \frac{43.9}{155.6 \times 1.4} = 0.2 \text{ t/cm}^2$$

$$\sigma_{loc} = \frac{n_1 P_1}{tz} = \frac{1.1 \times 60.6}{1.4 \times 34} = 1.41 \text{ t/cm}^2$$

(when checking the stability of the web $n_1 = 1.1$).

Since the value of $\frac{a}{h_w} = \frac{1.5}{1.556} = 0.965 > 0.8$

while the ratio $\frac{\sigma_{loc}}{\sigma} = \frac{1.41}{1.47} \times 0.96 > 0.12$

then the critical normal stress is found from equation (6-86)

$$\sigma_{cr} = k'_g \left(\frac{100t}{h_w}\right)^2 = 8.07 \left(\frac{100 \times 1.4}{155.6}\right) = 6.55 \text{ t/cm}^2$$

where the factor k'_g has been taken by interpolation from Table 6-14.

The quantity 0.12 has been taken from Table 6-15 by interpolation for $\frac{a}{h_w} = 0.965$, with the value of γ determined from equation (6-62) equal to

$$\gamma = C \frac{b_{fl}}{h_w} \left(\frac{t_{fl}}{t}\right)^3 = \frac{2 \times 60}{155.6} \left(\frac{2.2}{1.4}\right)^3 = 3$$

The critical shearing stress is determined from equation (6-57)

$$\tau_{cr} = \left(1.25 + \frac{0.95}{2}\right) \left(\frac{100t}{d}\right)^2 = \left(1.25 + \frac{0.95}{1.035^2}\right) \left(\frac{100 \times 1.4}{150}\right)^2 = 1.86 \text{ t/cm}^2$$

where $\mu = \frac{155.6}{150} = 1.035$.

Next we find the critical stress for the local load from equation (6-85)

$$\sigma_{loc \cdot cr} = k_g \left(\frac{100t}{a}\right)^2 = 4.21 \left(\frac{100 \times 1.4}{150}\right)^2 = 3.67 \text{ t/cm}^2$$

in which the factor k_g has been taken from Table 6-13 by interpolation.

We investigate the stability of the web by means of expression (6-84)

$$\sqrt{\left(\frac{\sigma}{\sigma_{cr}} + \frac{\sigma_{loc}}{\sigma_{loc \cdot cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} = \sqrt{\left(\frac{1.47}{6.55} + \frac{1.41}{3.67}\right)^2} + \\ + \sqrt{\left(\frac{0.2}{1.86}\right)^2} = 0.618 < 0.9$$

We do not investigate the resultant stresses in the web caused by the combined action of M and Q, since

$$\tau = 0.20 \text{ t/cm}^2 < 0.4 \times 0.9R = 0.756 \text{ t/cm}^2$$

In a similar way we check the stability of the web at the support, in the first panel, when the crane is arranged as illustrated in Fig. 6-45*a*

$$Q=\frac{66.6}{12}(3.85+9.1+10.5)\,1.04=137 \text{ tons}$$

$$M_a=137\times1.5=206 \text{ t-m } M_m=\frac{206+0}{2}=103 \text{ t-m}$$

$$\sigma=\frac{10.300\times72.4}{1,878,000}=0.4 \text{ t/cm}^2 \quad \tau=\frac{137}{155.6\times1.4}=0.632 \text{ t/cm}^2$$

Investigation of the web stability by means of expression (6-84) gives

$$\sqrt{\left(\frac{0.4}{6.55}+\frac{1.41}{3.67}\right)^2+\left(\frac{0.632}{1.86}\right)^2}=0.565<0.9$$

In the second panel from the support $M_m=\frac{206+312}{2}=259$ t-m

$$\sigma=\frac{25,900\times72.4}{1,878,000}=1.0 \text{ t/cm}^2$$

$$\tau=\frac{(137-66.6)}{155.6\times1.4}=0.323 \text{ t/cm}^2$$

Investigation of the web stability gives

$$\sqrt{\left(\frac{1.0}{6.55}+\frac{1.41}{3.67}\right)^2+\left(\frac{0.323}{1.86}\right)^2}=0.572<0.9$$

Thus the results of analyzing the stability of the web show that the thickness of the web used and the spacing of the stiffeners at $a=1.5$ m are satisfactory.

4. Analysis of Flange Connections. The top flange welds in buildings with heavy service conditions are made with penetration over the entire thickness of the web. Since these welds have a strength equal to that of the base metal, they do not have to be investigated.

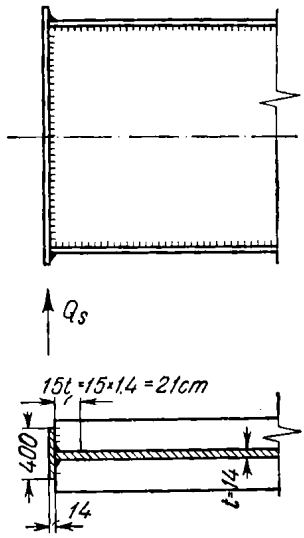


Fig. 6-46. To example 6-11. Support section of girder

The bottom flange welds are taken in accordance with Table 6-6 with a size of $h_w=8$ mm, and we check them for shear (when manual welding is used)

$$\tau=\frac{Q_sS_{fl}}{1.4h_wI_{gr}}=\frac{161,000\times8,540}{1.4\times0.8\times1,878,000}=654 \text{ kg/cm}^2<$$
$$>0.9\times1,500 \text{ kg/cm}^2=1,350 \text{ kg/cm}^2$$

where $S_{fl}=46\times2.2\times84.3=8,531\cong8,540$ cm^3.

5. Investigation of Support Rib. We take the support rib made from a strip 400×14 mm in size (Fig. 6-46).

The rib crushing area is

$$A_{cr}=40\times1.4=56 \text{ cm}^2$$

The crushing stress will be

$$\sigma_{cr}=\frac{Q_s}{A_{cr}}=\frac{161,000}{56}=2,880 \text{ kg/cm}^2=0.9\times3,200 \text{ kg/cm}^2$$

We find the area, radius of gyration and the slenderness ratio of the support vertical section (see page 201).

$$A_c = 40 \times 1.4 + 21 \times 1.4 = 85.4 \text{ cm}^2$$

$$I = \frac{1.4 \times 40^3}{12} = 7{,}470 \text{ cm}^4 \quad r = \sqrt{\frac{7{,}470}{85.4}} = 9.36 \text{ cm}$$

The slenderness ratio of the support vertical

$$\lambda_x = \frac{155.6}{9.36} = 16.6; \text{ the buckling factor } \varphi = 0.976$$

We check the stability of the support vertical against buckling perpendicular to the plane of the girder by means of expression (6-67)

$$\sigma_c = \frac{Q_s}{\varphi A_c} = \frac{161{,}000}{0.976 \times 85.4} = 1{,}930 \text{ kg/cm}^2 \cong 0.9 \times 2{,}100 \text{ kg/cm}^2$$

(3) Lattice Crane Girders (Trusses)

With spans of 18 metres or more, and with cranes having a lifting capacity of $Q = 10$-20 tons, it will be good practice to employ lattice crane girders (Fig. 6-47*a*).

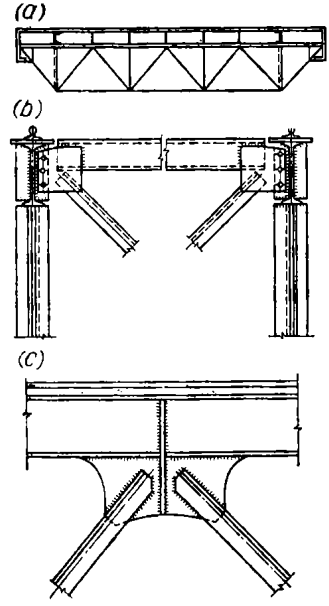


Fig. 6-47. Lattice crane girder

The top flange or chord of such girders is made from a rigid section (I beam) subjected not only to compression as part of the girder, but also to local bending under the load of the crane wheel P. The local bending moment can be found from the equation

$$M_b = \frac{Pd}{3} \qquad (6\text{-}92)$$

in which d is the distance between panel points of the top chord.

The remaining sections of the elements, as a rule, are designed of two angles.

The elements of the lattice are ordinarily centred on the bottom edge of the chord. All the bars of the lattice girder, except for the top chord, resist axial loads and are analyzed in the same way as the elements of trusses (Chapter 7).

The expression used for investigating the top chord is

$$\sigma = \frac{F + F_{br}}{\varphi A_{gr}} + \frac{M_b}{W_x} + \frac{M_{br}}{W_y} \leqslant R \qquad (6\text{-}93)$$

where F = design force induced by vertical loads
F_{br} and M_{br} = design compressive force and local bending moment in horizontal plane induced by lateral braking forces (when a horizontal bracing truss is used)
φ = buckling factor with respect to the vertical axis (with respect to the horizontal axis when a solid bracing beam is used).

The limiting slenderness ratios of elements are given in Table 7-1 (see p. 286). It is necessary to brace in a horizontal plane not only the top chord (Fig. 6-47*b*), but also the bottom one, seeing that the bottom chord may be displaced sideways as a result of deflection of the girder owing to the influence of even the most insignificant eccentricity. For this reason the standards establish a limiting slenderness ratio for the bottom chord of lattice crane girders equal to $\lambda = 150$. For the same reason ribs should be installed on the gussets of the top chord (Fig. 6-47*c*). The welds connecting the compression diagonals should be terminated at a distance of 40-50 mm before the nearest welds on the gusset.

(4) Horizontal Bracing Beams

Bracing beams are mainly designed of corrugated steel sheets from 6 to 10 mm thick, with one flange made from a channel (Fig. 6-48*a*) or an angle. The top flange of the crane girder serves as the second flange. With a girder span of 12 metres or more the outer flange of the bracing beam is usually suspended from members lo-

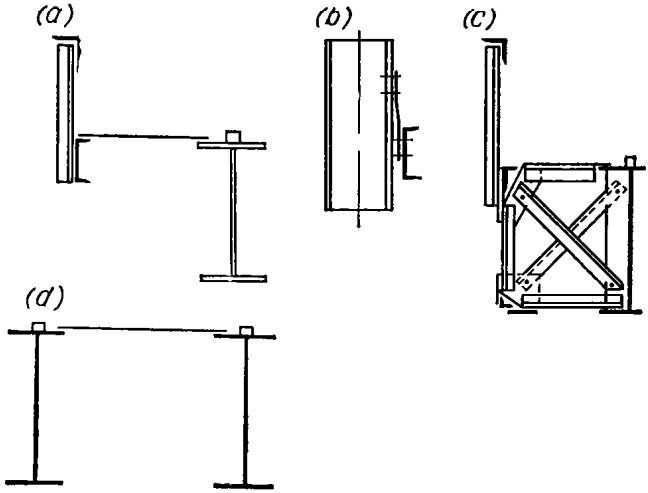


Fig. 6-48. Bracing beams

cated above it. It will not be good practice to support it by means of a strut connected to the bottom flange of the girder, as this will result in oscillation in a horizontal plane. The flange of the bracing beam may also be secured in a span between columns to a vertical of the wall framework, with the aid of a plate hinge 4-6 mm thick (Fig. 6-48*b*). Finally, a light vertical truss may be installed in the plane of the bracing beam flange with braces also located along the bottom flange of the crane girder. Such a design is especially good in open trestles, where it considerably increases the lateral stiffness of the girder, forming a space beam (Fig. 6-48*c*).

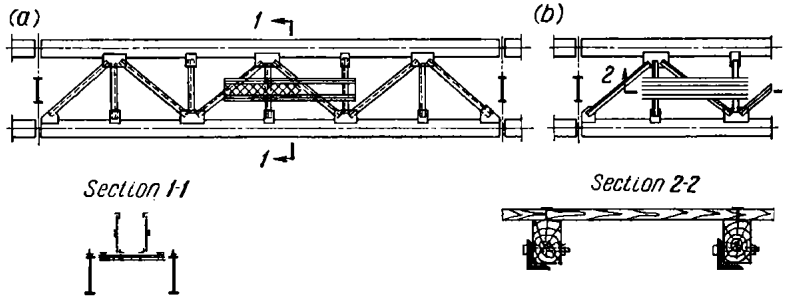


Fig. 6-49. Bracing truss

To provide a convenient passageway, the width of the bracing beam should be at least 750 mm. With two crane girders, a bracing plate connects the top flanges of both girders (Fig. 6-48*d*).

If the distance between the girders exceeds 1.2 m, it will be good to install a bracing truss, replacing the plate with a lattice of angles (Fig. 6-49*a*) on top of which a wooden flooring is laid (Fig. 6-49*b*). In hot shops a corrugated flooring at least 600 mm wide is used (Fig. 6-49*a*).

The design of a bracing beam connection is depicted in Fig. 6-50*a*. Should the plate of the bracing beam have an h/t ratio exceeding 100, transverse ribs made from an angle 75×6 or a strip 100×8 are installed on it from below with a spacing of $2h$. The plate of the bracing beam is connected to the top flange of the crane girder in erection by means of a continuous weld over the entire thickness of the plate. Here the use of type Э42А electrodes is obligatory, since the weld, while participating in the resistance of the whole girder to bending, is also subjected to the thrust of the plate.

At connections of bracing beams to columns, the plates should not be continued into the column owing to the difficulties that may arise in erection (Fig. 6-50*a*). These plates must also be reliably

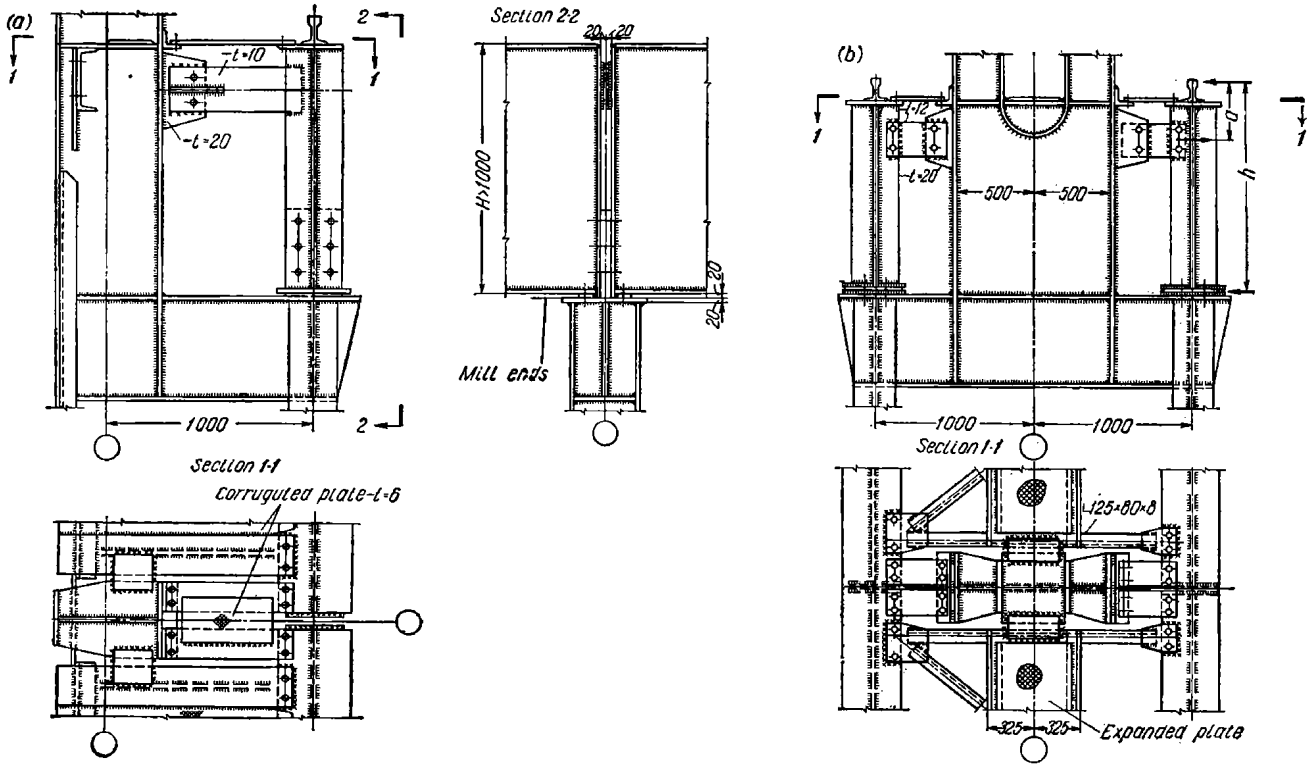


Fig. 6-50. Connection of crane girders and bracing members to columns

welded or riveted to the ribs of the column in order to transmit the braking and lateral forces to the column web.

Figure 6-50*b* pictures the design of a bracing truss-to-column connection.

(5) Crane Girder-Column Connections

Crane girders are generally supported on columns by means of top-seated connections in accordance with one of the following two alternatives:

1. The milled edges of the end bearing plates of adjacent girders, which are bolted together, rest on the axis of the column (Figs. 6-50 and 6-51*a*).

2. The girders are supported on the crane I-shape branch of the column with the support stiffeners of the girders located opposite the flanges of the column branch (Fig. 6-51*b*), or displaced a small distance inward toward the axis of the column.

Crane girders of different depth are supported in the ways illustrated in Fig. 6-52*a-d*. It is essential to check the web in the smaller section of the girder for shearing stresses induced by the reaction of the support.

The length of the horizontal bearing plate (Fig. 6-52*a*) is determined to satisfy the condition of adequate connection strength for the total shearing force acting over the length where the web has been cut away.

Crane girders are ordinarily connected to steel columns in the way shown in Fig. 6-50. For girders with a depth exceeding 1.2 metres it is recommended, while in shops with heavy service conditions it is obligatory, to install bearing membranes (Fig. 6-50*b*). In buildings with heavy service conditions, these membranes are connected by welding, using type Э52А electrodes or by means of rivets, the connection being investigated for the action of the force computed from the equation

$$R_c = \frac{Q_{br}h}{h-a} \tag{6-94}$$

where Q_{br} = reaction of bracing beam multiplied by the factor k_{br} found in Table 6-11

h = depth of girder

a = distance from top of rail to centre of riveted connection (Fig. 6-50*b*).

The vertical bolts connecting crane girders to columns are analyzed for shear induced by the force developed in longitudinal braking of the cranes, this force being

$$F_{br\cdot 1} = 0.1 \sum P \tag{6-95}$$

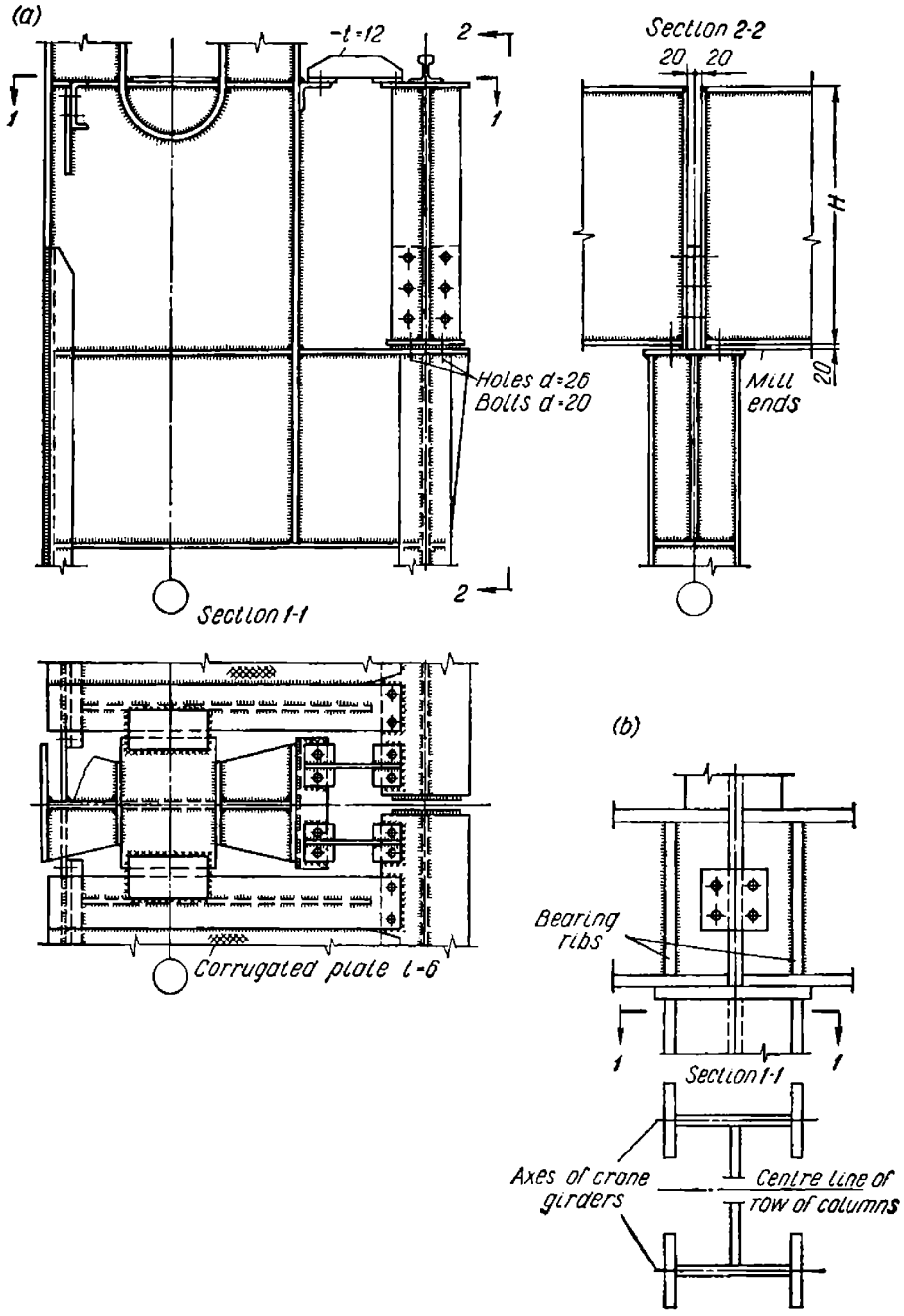


Fig. 6-51. Connection to columns of crane girders having the same depth

in which $\sum P$ is the sum of the loads of all the crane braking wheels on the girder (the number of braking wheels is generally equal to half of the total number of crane wheels).

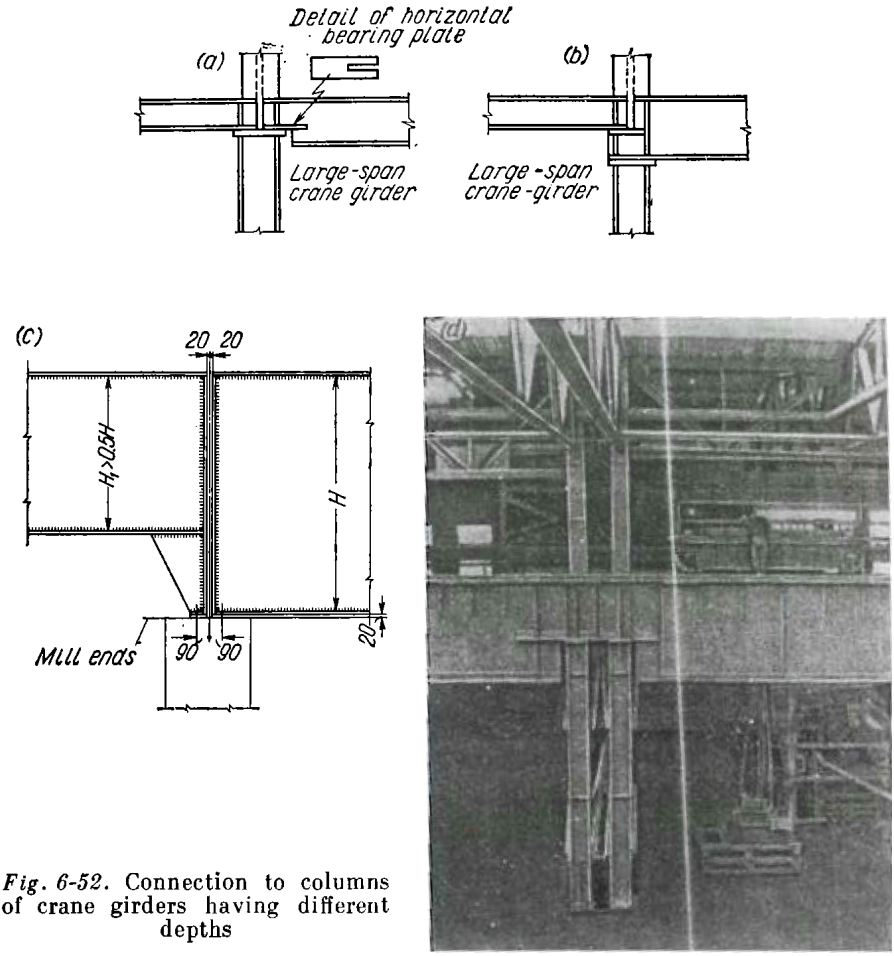


Fig. 6-52. Connection to columns of crane girders having different depths

The diameter of the bolt holes in the bearing plate is made 3-5 mm greater than the diameter of the bolts, while the washers are tightly fitted onto the bolts and welded in place during erection. Steel crane girders are connected to reinforced concrete columns in the way depicted in Fig. 6-53*a* for 6-metre girders and in Fig. 6-53*b* for 12-metre ones.

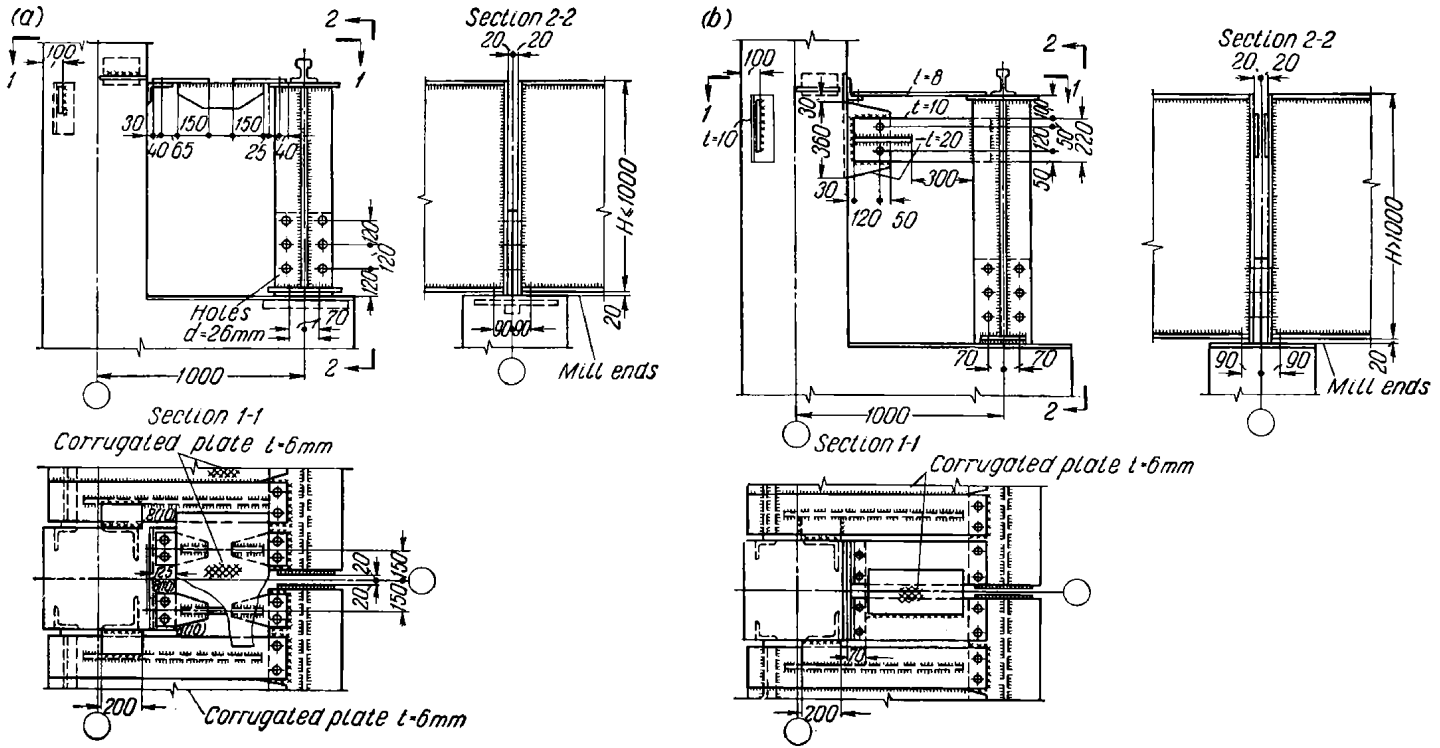


Fig. 6-53. Connection of steel crane girders to reinforced concrete columns

(6) Types of Crane Rail Fastenings

The capacity of crane rails is filled by type KP rails having a special section to GOST 4121-62, by ordinary railway rails, or by rectangular bars. The type of rail and fastening to be employed is selected depending upon the capacity of the crane, its duty and the type of crane wheels (cylindrical or tapered).

The required rail head width is indicated in the standards for cranes and varies from 50 to 140 millimetres.

The fastening of the rails to the crane girders may be either rigid or movable, i. e., permitting lining (straightening) of the track.

Reliable fastening of the rails is very important for normal service. Rapid deterioration of the track and flange of the girder owing to poor initial lining and poor fastening of the rails is sometimes observed.

Fixed attachment, i. e., welding of the rail to the girder, is permitted only for cranes with a light duty, and in all other cases a movable fastening should be used. With a small width of the girder flange, railway rails can be secured in place with hooks (spaced 500-700 mm apart) directed to both sides (Fig. 6-54*a*). The special type KP crane rails are fastened in place by means of batten plates (Fig. 6-54*b*).

Rectangular-section rails (plates or bars) are fastened with the aid of batten plates inserted into longitudinal milled grooves in the rail (Fig. 6-54*c*). Oval holes are made for the bolts in the batten plates, and round ones in the crane girders and baseplates.

A clearance of 10-20 mm is left between the baseplates and the sides of the rail to permit lining of the latter, while the thickness of the baseplates is made 2-3 mm less than the distance from the groove of the rail to its bottom, to ensure a spring action when the batten plates are bolted in place.

After installation of the batten plates into the grooves, they are welded to the baseplates.

As a rule, each batten plate is bolted with at least two bolts.

When selecting the section of the crane girders, account should be taken of weakening of the section by the holes for these bolts.

6-7. ALUMINIUM ALLOY BEAMS

The section of aluminium beams (or girders) is selected and their optimal depth is determined in the same way as for steel beams. Since the modulus of elasticity of aluminium alloys E_{al} is only about a third of that for steel, however, considerably greater requirements must be met in determining the minimum depth necessary to ensure adequate stiffness of the beam, i. e., to keep within the

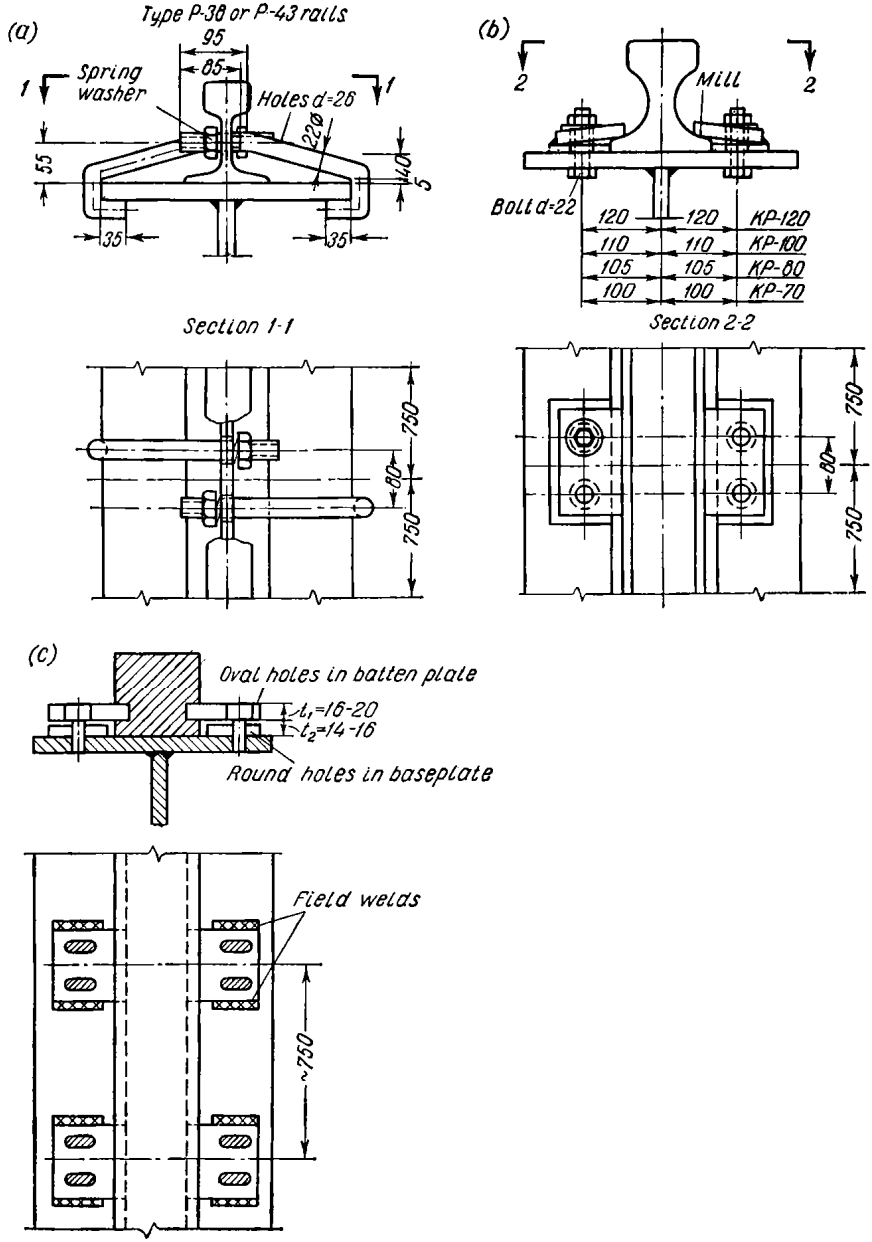


Fig. 6-54. Fastening of rails to crane girder

allowable deflection, which is characterized by the quantity $\frac{\delta}{L}=\frac{1}{r_0}$ (see Appendix I, Table 3). Greater attention must also be paid to problems of stability.

The equation for computing the minimum essential depth of an aluminium beam will be obtained for complete utilization of the material ($\sigma=R$), assuming in equation (6-3) that $n_p=n_q$

$$h_{min}=\frac{5}{24}\frac{\sigma L r_o}{E_{al}n_q}=\frac{RLr_0}{3.4\times10^6 n_q}\text{ cm} \tag{6-96}$$

where R=design strength of aluminium alloys in kg/cm², taken from Table 2-15 or 2-16

L=span of beam, cm

$r_0=\frac{L}{\delta}$= radius of curvature of beam, i. e., reciprocal of the established unit deflection

$E_{al}=710{,}000$=modulus of elasticity of aluminium alloys, kg/cm²

n_q=load factor of live load.

The optimal depth of aluminium beams is also determined from equation (6-22) or (6-27). If the value of h_{min} is found to be greater than that of h_{opt}, this indicates that the requirement of stiffness predominates for the beam, and that the moment of inertia of the beam is of greater importance. In this instance the optimal depth is found from equation (6-30)

$$h_{opt}=\sqrt[3]{3W\lambda_w}=1.73\sqrt{\frac{W}{t}}$$

In view of the insignificant change in weight accompanying a small deviation from the optimal depth, the following equation can be recommended for computing the optimal depth of aluminium beams

$$h_{opt}=1.6\sqrt{\frac{W}{t}} \tag{6-97}$$

Here the material distribution factor $k_d=3/4$, i. e., 75% of the material should be concentrated in the web of the beam and only 25% in both flanges.

Owing to the small width of aluminium beam flanges, the investigation of the general stability of these beams acquires great importance.

The strength of aluminium beams is checked by means of the first of the expression (6-12), while the general stability is analyzed by means of expression (6-14)

$$\sigma=\frac{M}{\varphi_b W}\leqslant R$$

The buckling factor φ_b is determined, similarly to steel beams, from equation (6-54), where the factor ψ is taken from Tables 5 and 6 of Appendix II depending upon the value of the parameter α [see equations (6-55) and (6-56)]. The maximum ratios between the design length L of the compression flange of a beam and its width b for which no checking of the stability of welded and extruded beams is required by the standards, are given in Table 6-17.

TABLE 6-17 **Maximum Ratios $\frac{L}{b}$ for Which No Checking of Aluminium Beam Stability is Required**

$\frac{h}{b}$	Name of alloy	Maximum value of L/b ratio					
		for load applied to				with intermediate constraints of top flange regardless of place of load application	
		top flange of beam		bottom flange of beam			
		Welded and extruded	Riveted	Welded and extruded	Riveted	Welded and extruded	Riveted
≤5	АМц-М, АМг-М and АД31-Т	12.8	15.0	20.5	23.4	15.7	17.7
	АВ-Т, АД31-Т1 and АМц-П	10.5	11.8	17.0	18.3	13.0	13.9
	АВ-Т1 and АД33-Т1	10.5	10.6	17.0	16.5	13.0	12.5
	АМг6-М and АМг-П	9.1	10.6	14.8	16.5	11.2	12.5
	АМг61 and В92 (plate)	8.3	9.8	13.5	15.2	10.2	11.5
	Д1-Т	7.6	8.8	12.2	13.8	9.3	10.4
	Д16-Т and В92 (shape)	6.6	7.6	10.6	11.9	8.0	9.0

NOTE. It is not recommended to employ beams with a depth h exceeding $5b$.

If the value of the buckling factor φ_b determined from equation (6-54) exceeds 0.667, then the factor φ_b' found from Table 6-18 is substituted in expression (6-14) for φ_b.

The maximum permissible ratios of the unframed overhang of the compression flange to its width b/t_{fl} for welded and extruded aluminium beams are given in Table 6-19.

TABLE 6-18 **Values of Factor φ_b' for Aluminium Beams**

φ_b	0.667	0.7	0.8	1.0	1.2	1.4	1.6	1.8	2.0
φ_b'	0.667	0.698	0.747	0.820	0.876	0.917	0.949	0.975	1.0

TABLE 6-19 **Maximum Permissible Values of b/t_{fl} for Welded and Extruded Aluminium Beams**

Grades of alloys						
АМц-М, АМг-М, АВТ-1, АД31-Т	АМг6-М, АД31-Т1, АВ-Т, АМц-П, АМг-П	АМг61	АД33-Т1	Д16-Т	В 92	
					Plate	Shape
11	10	9	12	7.5	9	7.5

The maximum permissible ratios b/t_{fl} for the unframed legs of angles in the compression flanges of riveted beams without horizontal plates are given in Table 6-20.

TABLE 6-20 **Maximum Permissible Values of b/t_{fl} for Riveted Beam Angles**

Grades of alloys			
АМц-М, АМг-М, АМг6-М АВ-Т, АД31-Т, АД31-Т1	АВ-Т1, АД33-Т1	Д1-Т, АМг61	Д16-Т, В92
15	14	12	11

When the free overhangs are strengthened by designing them with bulbs, the limiting values of the b/t_{fl} ratio increase.

Local stability of the beam webs, in the absence of a live load applied to the top flange, is considered to be adequate and not requiring analysis when the slenderness ratio of the web $\lambda_w = \frac{h_w}{t}$ does not exceed the values indicated in Table 6-21.

When the values of $h_w/t \leqslant 60$ the installation of stiffeners is not required. Otherwise they are spaced not more than $2h$ apart.

TABLE 6-21 **Values of $\frac{h_w}{t}$ for Which No Checking of Aluminium Beam Web Stability is Required**

Type of beam	Grade of alloy						
	АМц-М, АМг-М, АД31-Т	АМг6-М, АД31-Т1, АВ-Т, АМц-П, АМгП	АМг61, В92	АВ-Т1	АД33-Т1	Д1-Т	Д16-Т
Welded or extruded . . .	80	60	55	70	75	55	45
Riveted	—	80	70	75	80	65	55

The local stability of the web (in the absence of a live concentrated load) is investigated by means of the expression

$$\sqrt{\left(\frac{\sigma}{\sigma_{cr}}\right)^2+\left(\frac{\tau}{\tau_{cr}}\right)^2}\leqslant\theta \qquad (6\text{-}98)$$

in which σ and τ = design stresses in beam
σ_{cr} and τ_{cr} = corresponding critical stresses

$$\sigma_{cr}=2.1\left(\frac{100t}{h_w}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}99)$$

$$\tau_{cr}=\left(0.42+\frac{0.32}{\mu^2}\right)\left(\frac{100t}{d}\right)^2 \text{ t/cm}^2 \qquad (6\text{-}100)$$

In the latter equation d is the smaller side of a panel and μ is the ratio between its large and small sides (see page 194).

The value of the parameter θ is taken equal to unity for webs made from grade АВ-Т1 and АД33-Т1 alloys. In other cases the value of θ is determined from Table 6-22 depending on the ratio

TABLE 6-22 **Values of θ**

σ_{red}/R	2/3	0.7	0.75	0.8	0.85	0.9	0.95	1
θ	1	0.974	0.922	0.86	0.788	0.703	0.607	0.5

between the reduced stress and the design strength $\frac{\sigma_{red}}{R}$ within the interval

$$\frac{2}{3}\leqslant\frac{\sigma_{red}}{R}=\frac{1}{R}\sqrt{\left(\frac{2}{3}\sigma\right)^2+3\tau^2}\leqslant 1$$

In the webs of beams a ratio of $\frac{\sigma_{red}}{R} > 1$ is not allowed.

The width of the protruding part of the stiffening ribs must be at least $b_{rib} \geqslant \frac{h_w}{30} + 40$ mm, and the thickness of a rib should be not less than $\frac{1}{12} b_{rib}$.

The support rib should be tightly fitted to the bottom flange of the beam. When investigating the support vertical of a beam for transmission of the support reaction, the design section of the vertical, besides the support ribs, also includes strips of the web up to $12t_w$ wide at each side of the rib.

6-8. PRESTRESSED BEAMS

Prestressing is employed in metal beams and girders in order to reduce the amount of metal required and the deflections of a member, and also to regulate the stresses in a member for obtaining the most rational shape. Prestressing can be achieved in various ways; the main ones are:

1. Tensioning with a high-strength element (a wire cable or a steel rope) located in the tension zone of the member.
2. Regulating the bending moments and deflections by changing the relative level of the supports in continuous beams, the tension of the overhanging ends in cantilever beams, and so on.

In the first case, owing to eccentric compression of the member (as the result of the prestress of a cable of high-strength wires) stresses of the reverse sign to those which will be induced by the live load develop in it. When a live load is applied to the beam, the initial stresses caused by prestressing are first neutralized, and for this reason a higher total service load may be applied to the beam. Here the wire rope or cable located in the tension zone is additionally loaded (self-tensioned). Owing to the high strength of the wires, however, a comparatively small amount of metal is required for them. The total saving in the weight of metal in this case reaches 10-18%, the saving in costs reaching 5-15%.

In continuous beams it is also good practice to employ prestressing by locating the stressing elements above the supports in the tension zone. Each stressing element adds one redundant force to the scheme used for analysis.

With a large dead load on a beam, it is good policy to employ repeated prestressing by gradually loading the beam, bringing up the prestress to the maximum value each time.

In the second case (regulation of moments) it is possible to reduce the bending moments in a span by increasing them on the supports,

for example by tensioning the overhangs or arms, where, as a rule, it is easier to design sections with a large depth. This will also reduce the deflection at the middle of the span. It is good in such cases to make use of composite beams, in which a reinforced concrete flooring included as part of the compression flange of the member takes the load together with the steel beams.

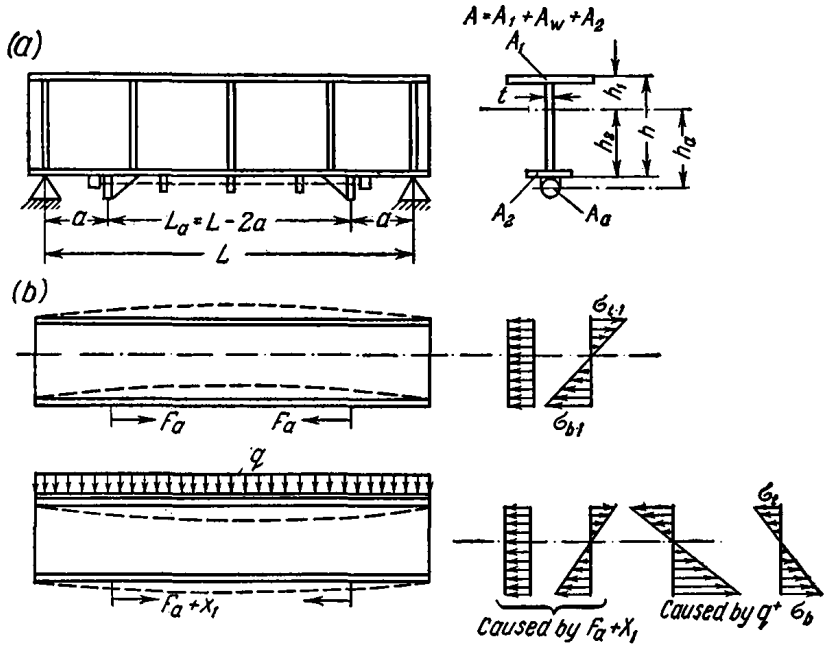


Fig. 6-55. Prestressed steel beam

Let us consider the behaviour of a beam that has been prestressed once (Fig. 6-55). The high-strength element (generally known as a tie rod or cable) in the form of a bundle of wires with a diameter of 3-6 mm (or a rope) is located, as a rule, in the lower part near the bottom flange (better below it). It is freely passed through guide carriers having the form of half-rings welded to the flange at about 1.5-2 metres from each other (Fig. 6-55*a*). These carriers are necessary to prevent the bottom flange, which will be subjected to compression, from losing its stability in the process of prestressing of the cable. Ordinarily the cable does not occupy the whole length of the beam, and its ends are secured in special anchorages.

When analyzing a prestressed simple one-span beam, the sequence of its behaviour step-by-step is taken into consideration. First, when the cable is prestressed with a force of F_{ps}, the beam is sub-

jected to eccentrical compression, which induces in the top flange of the beam a tensile stress (Fig. 6-55*b*)

$$\sigma_{t\cdot 1} = -\frac{n_2 F_{ps}}{A} + \frac{n_2 F_{ps} h_a}{W_t} \tag{6-101}$$

and in the bottom flange—a compressive stress

$$\sigma_{b\cdot 1} = -\frac{n_2 F_{ps}}{A} - \frac{n_2 F_{ps} h_a}{W_b} \tag{6-102}$$

where $n_2 = 1.1$ = load factor for the prestressing force

W_t and W_b = section moduli of the beam for the top and bottom fibres, respectively.

The remaining notation will be clear from Fig. 6-55*a*.

In the next stage, when the beam is subjected to a live service load, it begins to behave as a statically indeterminate trussed beam with one redundant element. We take as the unknown quantity the additional force in the cable X_1, which begins to develop under the action of the external load. This force, which can be termed a self-tensioning one, induces additional stresses in the beam.

The total stresses resulting from the combined action of the prestress and the external load will be:

in the top flange

$$\sigma_t = -\frac{n_1 F_{ps} + X_1}{A} - \frac{M_p - (n_1 F_{ps} + X_1) h_a}{W_t} \tag{6-103}$$

in the bottom flange

$$\sigma_b = -\frac{n_1 F_{ps} + X_1}{A} + \frac{M_p - (n_1 F_{ps} + X_1) h_a}{W_b} \tag{6-104}$$

Here the load factor for the prestressing force F_{ps} is taken equal to $n_1 = 0.9$, i. e., less than unity, since in the present case a reduction of this force increases the stress.

The self-stressing force X_1 is determined from the general equation

$$X_1 = -\frac{\Delta_{1p}}{t_{11}} = \frac{\int_a^{L-a} \frac{M_1 M_p}{EI} dx}{\int_a^{L-a} \frac{M_1^2 dx}{EI} + \frac{L_a}{E_a A_a} + \frac{L_a}{EA}} \tag{6-105}$$

Here M_p = moment in the main system-beam induced by the external load

M_1 = moment induced by the force $X_1 = 1$

EI = stiffness of beam in bending

$E_a A_a$ = stiffness of tensioning element (cable)

EA = stiffness of beam in longitudinal straining

a = length of part of the beam having no prestressing cable.

Hence for a beam with a cable located in accordance with Fig. 6-55*a*, the self-stressing force X_1 will be found as follows:

for a beam which a uniformly distributed load is applied to

$$X_1 = \frac{2M_p h_a}{3\left(h_a^2 + \frac{I}{A} + \frac{EI}{E_a A_a}\right)} \gamma_1 \qquad (6\text{-}106)$$

for a beam which a system of identical concentrated loads is applied to

$$X_1 = \frac{Ph_a\left[L(x_1 + x_2 + \ldots + x_i) - (x_1^2 + x_2^2 + \ldots + x_i^2)\right]}{2L\left(h_a^2 + \frac{I}{A} + \frac{EI}{E_a A_a}\right)} \gamma^1 \qquad (6\text{-}107)$$

Here

$$\gamma_1 = \frac{L^3 - 6La^2 + 4a^3}{L(L-2a)} \cong \left(2 - \frac{L_a}{L}\right)$$

$$a = \frac{L - L_a}{2}$$

h_a = distance from neutral axis to axis of cable

P = magnitude of concentrated load

$x_1, x_2, \ldots, x_i$ = distances to 1st, 2nd, ..., i-th load from left-hand support.

The optimal section of the beam is obtained with such ratios between the sectional areas of the flanges and the web, with which the limiting stresses attained in the flanges of the beam are due not only to the load (σ_t and σ_b), but also to prestressing in the bottom flange ($\sigma_{b\cdot 1}$). It is this that limits the increase in the load-bearing capacity of prestressed beams. Thus, for the optimal section, the following conditions must be observed

$$\sigma_t = R \quad \sigma_b = R \quad \sigma_{b\cdot 1} = R \qquad (6\text{-}108)$$

It is obviously good practice to design beams with an unsymmetrical section, and to select such dimensions of the bottom flange that will just ensure its stability when prestressed (Fig. 6-55).

If we retain the notation for the geometrical characteristics used for ordinary beams in equations (6-18) and (6-19), namely, the web slenderness ratio $\lambda_w = \frac{h_w}{t}$, the material distribution or shape factor $k_d = \frac{A_w}{A}$, i. e., the ratio of the web sectional area to that of the entire beam, and introduce the conception of a factor of asymmetry

$$k_{as} = \frac{h_2}{h_1} = \frac{W_t}{W_b} \qquad (6\text{-}109)$$

then the optimal geometrical characteristics of the section can be expressed as follows

$$A_w = k_d A \quad A_1 = A\left(\frac{k_{as}}{k_{as}+1} - \frac{k_d}{2}\right) \quad A_2 = A\left(\frac{1}{k_{as}-1} - \frac{k_d}{2}\right)$$

$$A = A_1 + A_2 + A_w \tag{6-110}$$

By satisfying the optimal condition (6-108), it is possible to express the moment taken by the beam as a function of the geometrical parameters and the self-stressing factor

$$M = R\sqrt{A^3 \lambda_w C} \tag{6-111}$$

where

$$C = (1-\beta)\sqrt{\frac{6k_{as}^3(1-k_{as})^2[k_{as}-(1+\beta)]}{(k_{as}+1)^3[k_{as}(1-\beta)-(1+\beta)]^3}} \tag{6-112}$$

$$\beta = \frac{n_1 F_{ps} + X_1}{n_1 F_{ps}} = \text{self-stressing factor}$$

The quantity C can be called a parameter of the advantageousness of the section. For practical values of the self-stressing factor β within the limits of 1 to 3, the factor k changes very slightly, and can be assumed to be constant (k_d=0.55) within a quite wide range of values of the asymmetry factor k_{as}.

For the three characteristical types of loading indicated in Table 6-23 it is possible to find the self-stressing force X_1 and establish the relation between k_{as} and C for the optimal conditions. Here it is assumed that the length of the stressing element is established so as to satisfy the condition of attaining the limiting stress ($\sigma = R$) in the beam at the beginning and end of the prestressing cable, while the sectional area of the stressing element is determined from the condition that the sum of the projections of all the forces on the horizontal axis $(A_1 - A_2)R = A_a R_a$ equals zero, by the equation

$$A_a = A\frac{R}{R_a}\frac{k_{as}-1}{k_{as}+1} \tag{6-113}$$

Hence we directly obtain the required magnitude of prestressing F_{ps} if the kind of loading of the beam is known, i. e., if the force X_1 is known

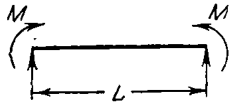
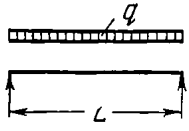
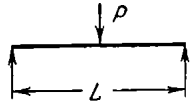
$$F_{ps} n_2 + X_1 = A_a R_a$$

whence

$$F_{ps} = \frac{F_a R_a - X_1}{n_2} \tag{6-114}$$

Table 6-23 gives the values of k_{as} and C depending on the selected ratio μ_a between the unit strain of the beam and that of the prestress-

TABLE 6-23 **Values of Quantities k_{as} and C**

Kind of beam loading	μ_a	$n_1=0.9$	$n_2=1.1$	$n_1=1$	$n_2=1$	Rational length of stressing element
		k_{as}	C	k_{as}	C	
	0.1	1.58	0.347	1.87	0.348	$L_a=L$
	0.2	1.75	0.359	2.11	0.369	
	0.3	1.99	0.381	2.56	0.399	
	0.4	2.40	0.415	3.60	0.446	
	0.1	1.69	0.329	1.83	0.344	$L_a=L\sqrt{\eta}$
	0.2	1.80	0.341	1.98	0.357	
	0.3	1.95	0.354	2.16	0.371	
	0.4	2.12	0.367	2.36	0.384	
	0.1	1.72	0.323	1.82	0.342	$L_a=\eta L$
	0.2	1.88	0.328	1.94	0.353	
	0.3	2.07	0.332	2.06	0.363	
	0.4	2.27	0.336	2.19	0.373	

ing cable materials, and on the kind of beam loading

$$\mu_a=\frac{\varepsilon}{\varepsilon_a}=\frac{RE_a}{R_aE} \tag{6-115}$$

The load factors $n_1=0.9$ and $n_2=1.1$ are used with indirect control of the prestressing force F_{ps} (by noting the deflection of the beam, the force used to tighten the bolts, driving in wedges, etc.).

When determining the magnitude of the prestressing force by means of instruments (a pressure gauge in the hydraulic system, by measuring the stress with a tensometer, etc.) the values of the load factors may be taken equal to $n_1=n_2=1$.

The values of the rational length of the stressing element for the various kinds of loading are indicated in the last column of Table 6-23, where the coefficient η is equal to

$$\eta=1-\frac{\sqrt{0.55}}{C}\left[\frac{6k_{as}-0.55(k_{as}+1)^2}{k_{as}(k_{as}+1)}\right] \tag{6-116}$$

Thus the optimal section of a beam or girder with a straight prestressing cable located at the level of the bottom flange can be selected in the following sequence.

1. The desired slenderness ratio of the web $\lambda_w = \frac{h_w}{t}$ is selected (see Table 6-3) or the minimum possible web thickness t is determined, with a view to the web resisting the action of the shear force Q obtained from expression (6-26), or the action of the local load from equation (6-77).

2. The material of the prestressing cable (a bundle of wires or a steel rope) is selected and the unit strain factor μ_a is found from equation (6-115).

3. The values of the asymmetry factor k_{as} and the parameter C are found in Table 6-23 depending on the value of μ_a and the kind of beam loading.

4. In accordance with the values of M, C, R and λ_w or t, the geometrical characteristics of the beam are determined from equations (6-110), assuming that $k_d = 0.55$, (6-113) and (6-117)

$$A = \sqrt[3]{\frac{M^2}{C^2 R^2 \lambda_w}} \quad \text{or} \quad A = \sqrt{\frac{M}{CR}} \sqrt[4]{\frac{t}{k_d}} = 1.16 \sqrt[4]{t} \sqrt{\frac{M}{CR}}$$

$$h_{opt} = \frac{A_w}{t} \tag{6-117}$$

5. The rational length of the prestressing cable is found for the given type of loading from Table 6-23.

6. By means of equation (6-106) or (6-107) the self-stressing force X_1 in the cable is found for the given kind of loading, and then the prestressing force is found by equation (6-114).

7. The stresses in the beam are checked by means of equations (6-103) and (6-104). In the process of beam fabrication, the force in the cable when it is being tensioned is controlled by referring to the force

$$F_{con} = \frac{F_{ps}}{0.95} + \Delta_a \frac{E_a A_a}{L_a} \tag{6-118}$$

Here 0.95 = relaxation factor for cables made from bundles of high-strength wires and ropes

Δ_a = magnitude of yield or pliancy of the anchorages securing the cable to the beam, taken equal to 0.1 cm when the anchorages have the form of tightly screwed on nuts or wedge-shaped blocks, and to 0.2 cm when anchorages with gaskets or washers are employed.

It should be kept in mind that the controlled force in the prestressing cable F_{con} should not be greater than the force that can be resisted by the bottom flange of the beam when it is compressed in the process of prestressing (over the distance between the guide carriers). This limiting force is checked by means of the equation

$$F_{ps.l} = \frac{R\varphi_{b.y} A W_b}{W_b + h_a A} \tag{6-119}$$

where $\varphi_{b.y}$ is the buckling factor for the bottom flange of the beam.

Thus the requirement must be met that

$$F_{con} \leqslant F_{ps.l} \tag{6-120}$$

The stressing element can be made from a bundle of parallel high-strength wires, of wire ropes (seven-strand ropes) or of stiff high-strength steel rods.

The design strength of a bundle of high-strength wires (according to GOST's 7348-55, 7372-55 and 8480-57) is taken from Table 6-24.

TABLE 6-24 **Design Strength in kg/cm² for Various Wire Diameters**

Diameter of wire, mm	2.5	3	4	5	6	7	8
Design strength, kg/cm²	11,200	10,600	10,100	9,500	9,000	8,400	7,800

The design strength of seven-strand ropes (according to GOST's 3065-55 through 3068-55, 7675-55, 7676-55 and 7680-55) is taken equal to 65% of the breaking force indicated in the standard, and for galvanized ropes equal to 60% of the breaking force.

The use of high-strength wire prestressing cables has the advantage that a sufficiently high value of their modulus of elasticity can be taken ($E_a = 2.0 \times 10^6$ kg/cm²). A drawback of these cables is that the bundle of wires is manufactured manually. Steel rope cables are manufactured by industrial methods, but have a reduced modulus of elasticity ($E_a = 1.6 \times 10^6$ to 1.8×10^6 kg/cm²), as a result of which the member will be more liable to deformation, and the effect of prestressing will be utilized less rationally.

The prestressing cables (bundle of wires or wire rope) can be anchored in various ways. With small forces and small bundles (which is seldom the case) use can be made of a double-row wedge anchorage similar to those used in reinforced concrete prestressed members. With larger forces, a steel sleeve with a tapered wall is used. The wire or unbraided rope end is introduced into the widening sleeve, and the latter is filled with babbitt metal or some other alloy, for instance grade ЦАМ9-1.5 alloy to GOST 7117-54 (Fig. 6-56*a*). When using a cable made from a bundle of wires, a plug can be pressed in of a soft metal (grade Ст.2 steel), or a sufficient number of various metal wedges (with their ends brazed or welded together) can be driven in. This will create a large spreading force, which will prevent pulling out of the bundle from the sleeve (Fig.

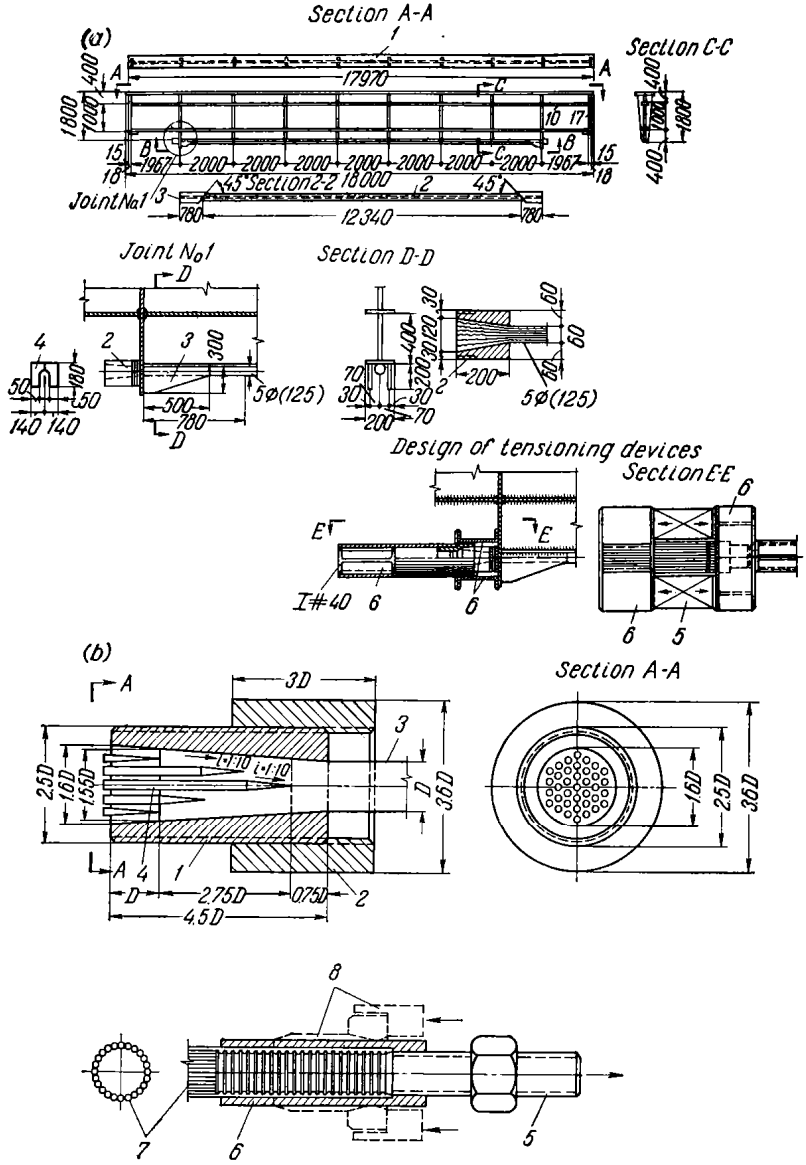


Fig. 6-56. Anchoring of prestressing cable to beam:

Anchorage (*a*): *1*—bundle of wires (cable); *2*—anchorage sleeve; *3*—ribs; *4*—anchoring washer; *5*—hydraulic jacks; *6*—auxiliary beams for tensioning.

Anchorage (*b*): *1*—sleeve; *2*—coupling; *3*—bundle of wires; *4*—wedges; *5*—rod of anchorage with nut; *6*—casing of anchorage; *7*—high-strength wire; *8*—casing of anchorage and ring for compression

6-56*b*). Such a design has the advantage of not requiring the labour-consuming process of pouring in the metal. The bundles with the anchorage sleeves are manufactured separately from the beam and are installed on the latter in a finished state. For prestressing of the cable, the sleeve is provided with a thread. Onto the latter is screwed a coupling which auxiliary beams bearing against hydraulic jacks are connected to. After tensioning of the cable, controlled by means of a pressure gauge installed on the hydraulic jack, fork-shaped washers are installed between the anchorage sleeve and the support. It should be borne in mind that the web of the beam must be reinforced with stiffeners at the support of the anchorage.

With appropriate control, the cable may also be preliminarily tensioned by lateral displacement of the bundle of wires or rope, or by driving in metal wedges between the anchorage device and the supporting member.

Example 6-12. Investigate, using the data of examples 6-4 and 6-5, a welded prestressed beam made from grade BCт.3 steel with a 12-metre span subjected to a uniformly distributed load $q=21.13$ t/m. The design moment $M=380$ t-m, the shear force $Q=127$ tons.

Solution. 1. We select a tensioning element in the form of a bundle of 3-mm wires to GOST 7348-55. The design strength of the wires from Table 6-24 is $R_a=10{,}600$ kg/cm^2, and $E_a=2\times10^6$ kg/cm^2.

We determine the unit strain factor

$$\mu_a=\frac{\varepsilon}{\varepsilon_a}=\frac{RE_a}{R_aE}=\frac{2{,}100\times2\times10^6}{10{,}600\times2.1\times10^6}=0.189$$

For the case when the beam is subjected to a uniformly distributed load, by interpolation of the data contained in Table 6-23, we find the values of the quantities k_{as} and C for the load factors $n_1=0.9$, $n_2=1.1$ and for $\mu=0.189$, namely, $k_{as}=1.79$ and $C=0.34$.

2. We select the slenderness ratio of the web λ_w or take the minimum possible thickness of the web. Assuming the same thickness of the beam web as in example 6-4 and 6-5, i.e., $t=1$ cm, we find the minimum essential geometrical characteristics from equations (6-117) and (6-110)

$$A=1.16\sqrt[4]{t}\sqrt{\frac{M}{CR}}=1.16\sqrt[4]{1}\sqrt{\frac{38{,}000{,}000}{0.34\times2{,}100}}=268\text{ cm}^2$$

$$A_1=A\left(\frac{k_{as}}{k_{as}+1}-0.275\right)=268\left(\frac{1.79}{2.79}-0.275\right)=99\text{ cm}^2$$

$$A_2=A\left(\frac{1}{A_{ks}+1}-0.275\right)=268\left(\frac{1}{2.79}-0.275\right)=22.5\text{ cm}^2$$

$$A_w=0.55\times268=147\text{ cm}\qquad h_{opt}=\frac{A_w}{t}=\frac{147}{1}=147\text{ cm}$$

$$A_a=A\frac{R}{R_a}\frac{k_{as}-1}{k_{as}+1}=268\times\frac{2{,}100\times0.79}{10{,}600\times2.79}=15.1\text{ cm}^2$$

The rational length of the tensioning element will be [see Table 6-2 tion (6-116)]

$$L_a = L\sqrt{\eta} = 1{,}200\sqrt{0.535} = 880 \text{ cm, where}$$

$$\eta = 1 - \frac{\sqrt{0.55}}{C}\left[\frac{6k_{as} - 0.55(k_{as}+1)^2}{6k_{as}(k_{as}+1)}\right] = 1 - \frac{0.74}{0.34}\times$$

$$\times\left[\frac{6\times 1.79 - 0.55\times 2.79^2}{6\times 1.79\times 2.79}\right] = 0.535$$

3. We select the section and check the stress in the beam. We ta tion of the top flange made from a plate 520×20 mm in size and the from a plate 300×10 mm in si made from a plate 1,470×10 (Fig. 6-57)

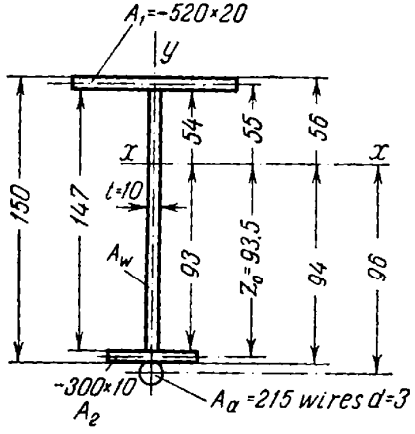


Fig. 6-57. To example 6-12. Section of beam

$$A = A_1 + A_2 + A_w = 52\times 2 + 3(\quad + 147\times$$

We find the distance from of gravity of the beam to the bottom plate

$$z_0 = \frac{147\times 74 + 104\times 148.5}{281} =$$

after which we determine: the moment of inertia of

$$I = 104\times 55^2 + 30\times 93.5^2 + \frac{1\times}{\quad} + \frac{1\times 93^3}{3} = 8($$

and the section moduli

$$W_b = \frac{897{,}000}{94} = 9{,}570 \text{ cm}^3 \quad W_t = \frac{897{,}000}{56} = 16{,}000 \text{ cm}^3$$

Next we find the moment of inertia of the bottom flange with res *y-y*

$$I_y = \frac{1\times 30^3}{12} = 2{,}240 \text{ cm}^4 \text{ and } r_y = \sqrt{\frac{2{,}240}{30}} = 9 \text{ cm}$$

The self-stressing force in the cable, from equation (6-106), will

$$X_1 = \frac{2M_{ha}}{3\left(h_a^2 + \frac{I}{A} + \frac{EI}{E_a A_a}\right)}\left(2 - \frac{L_a}{L}\right) =$$

$$= \frac{2\times 38{,}000{,}000\times 96}{3\left(96^2 + \frac{897{,}000}{281} + \frac{2.1\times 10^6\times 897{,}000}{2\times 10^6\times 15.1}\right)}\left(2 - \frac{880}{1{,}200}\right) = 40{,}3($$

The prestressing force in the cable from equation (6-114) is

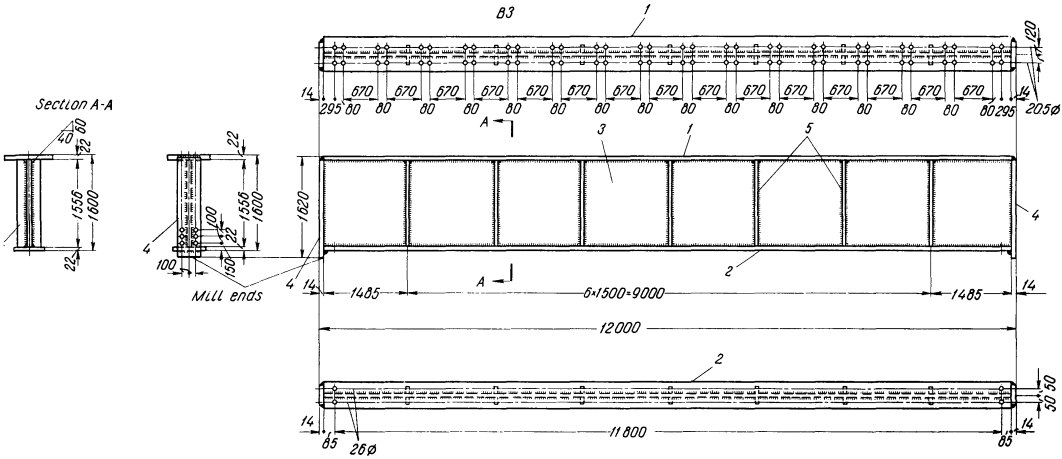


Fig. 6-58. Working drawing of crane girder

Specification of Standard Element Б3. Material Grade ВСт.3 Steel to GOST 380-60

No. of part	Section	Length in mm	Q-ty	Weight in kg		Remarks
				of one part	of all parts	
1	—580×22	11,970	1	1,200	1,00	—
2	—460×22	11,970	1	950	950	—
3	—1,556×14	11,970	1	2,050	2,050	Machining of edges for welding
4	—400×14	1,556	2	68.4	137	Mill ends
5	—80×6	1,556	14	5.9	82	—
					66 *	
		Total			4,485	

* Weight of deposited metal 1.5 per cent.

Manufacture

Standard element	Quantity	Weight in kg	
		of element	total
Б3	24	4,485	107,640

NOTES. 1. Use type Э42 electrodes for all welds, except where otherwise specified.
2. Use automatic machine for flange welds.
3. Sizes of welds $h=6$ mm, except where otherwise specified.
4. All holes with diameter $d=20$ mm, except where otherwise specified.

Normal stresses under service conditions: in top flange of beam, from equa 6-103)

$$= -\frac{(n_1F_{ps}+X_1)}{A} - \frac{M-(n_1F_{ps}+X_1)h_a}{W_t} = -\frac{(0.9\times108,500+40,300)}{281} -$$

$$-\frac{38,000,000-(0.9\times108,500+40,300)96}{16,000} = -492-1,560 = -$$

$$-2,050 \text{ kg/cm}^2 < 2,100 \text{ kg/cm}^2$$

tom flange of beam, from equation (6-104)

$$= -\frac{(n_1F_{ps}+X_1)}{A} + \frac{M-(n_1F_{ps}+X_1)h_a}{W_b} = -\frac{(0.9\times108,500+40,300)}{281} +$$

$$\frac{3,000,000-(0.9\times108,500+40,300)96}{9,570} = -492+2,600 \cong 2,100 \text{ kg/cm}^2$$

Normal stress in bottom flange of beam during the prestressing of the cable. equation (6-102)

$$\sigma_{b\cdot1} = -\frac{n_2F_{ps}}{A} - \frac{n_2F_{ps}h_a}{W_b} = -\frac{1.1\times108,500}{281} - \frac{1.1\times108,500\times96}{9,570} =$$

$$= -425-1,200 = -1,625 \text{ kg/cm}^2 < 2,100 \text{ kg/cm}^2$$

Maximum stress in cable (consisting of 215 wires with diameter $d=3$ mm ea $A_a=15.1$ cm^2)

$$\sigma_a = \frac{n_2F_{ps}+X_1}{A_a} = \frac{1.1\times108,500+40,300}{15.1} = 10,600 \text{ kg/cm}^2$$

controlled force when tensioning the cable, by equation (6-118), will

$$P_{con} = \frac{F_{ps}}{0.95} + \Delta_a\frac{E_aA_a}{L_a} = \frac{108,500}{0.95} + \frac{0.2\times2\times10^6\times15.1}{880} = 121,400 \text{ kg}$$

h the cable guide carriers spaced $L_{gc}=2$ metres (200 centimetres) apart he bottom flange of the beam, we find the maximum permissible force in le satisfying the condition of stability of the bottom flange when com in the process of prestressing

$$P = \frac{R\varphi_{b\cdot y}AW_b}{W_b+h_aA} = \frac{2,100\times0.96\times281\times9,570}{9,570+96\times281} = 149,000 \text{ kg} > 121,400 \text{ kg}$$

$\varphi_{b\cdot y}=0.96$ with $\lambda_{b.fl}=\frac{L_{gc}}{r_y}=\frac{200}{9}=23.$

prestressed beam designed in this example is lighter than that considered iples 6-4 and 6-5, with an area of $A_{beam}=346$ cm^2, by 14.5%, namely

$$\frac{A_{beam}-(A+A_a)}{A_{beam}} = \frac{346-(281+15.1)}{346}\times100 = 14.5\,\%$$

ire 6-58 contains an example of a working drawing of a welane girder. This drawing illustrates the material contained present chapter relating to ordinary simple welded beams rders, which are the most widespread ones at present.

CHAPTER SEVEN

Trusses

7-1. ROOF TRUSSES

(1) General Characteristics of Trusses

A *truss* is a latticed member designed, the same as a beam or girder, to withstand mainly bending. In contrast to a beam or girder, a truss is formed of separate straight bars or elements connected to each other at multiple joints (theoretically pinned or hinged) to form a geometrically unchangeable system. The load, as a rule, is applied to a truss only at the joints. Owing to such transmission of the load, the elements of a truss are subjected only to the axial action of tensile or compressive forces, which leads to better utilization of the material than in a solid beam or girder. Trusses are especially advantageous in members where a great depth is required to satisfy the condition of adequate stiffness. With large loads and relatively small spans, trusses become cumbersome and require a large expenditure of labour for their fabrication. In this respect they are inferior to solid beams. The advantages of changing over from solid beams or girders to trusses grow with an increase of the span of the member and a reduction of the load applied to it.

Trusses have a multitude of applications. They can be classified according to several aspects:

1. Application—bridge trusses, roof trusses, crane trusses, the trusses used in the masts and towers of power transmission lines, etc.
2. Design—light single-plane and heavy double-plane trusses.
3. The direction of the support reactions and the design of the support members—girder trusses (simple, cantilever and continuous), strutted arch trusses, etc.

Besides, trusses can be of the plane or space types.

The present chapter is devoted to light single-plane trusses used in roofs—roof trusses.

Heavy trusses with double-plane sections of their elements are generally employed when the forces in the chords exceed 350-400 tons. These include, for example, bridge trusses, and the large-span trusses of aircraft assembly shops or ship-building slips from which cranes are suspended.

(2) Elements of Roof

Roof trusses serve to support the roof covering (cladding) and resist the loads acting on it. The roof covering together with the roof trusses and the various ties and braces form the *roof*. The main

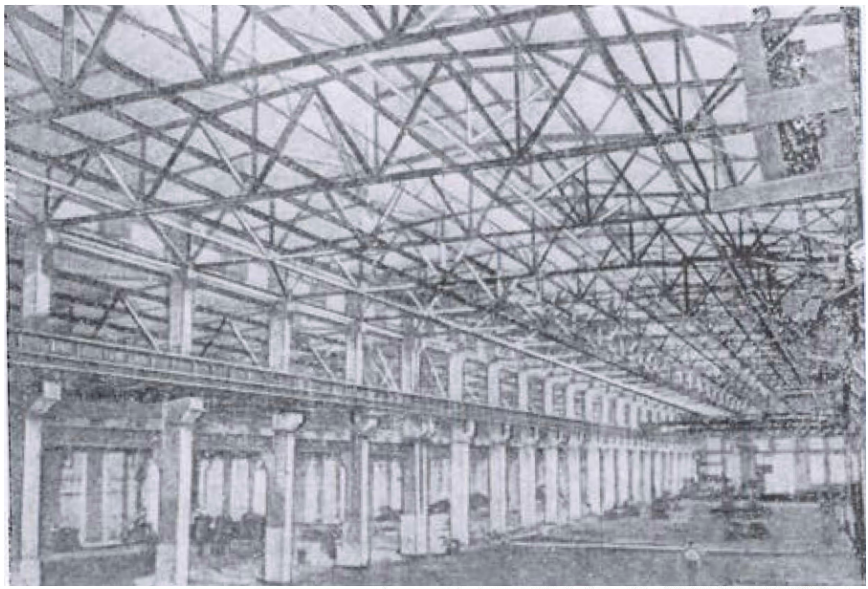


Fig. 7-1. Roof trusses on reinforced concrete columns

designation of the roof is to protect the premises against the weather (snow, rain, cold, etc.). Roof trusses are mainly supported on steel or reinforced concrete columns (Fig. 7-1).

In industrial structures there are distinguished "warm" (heat-insulated) and "cold"(not insulated) roofings. Warm roofings consist of load-bearing slabs, a heat-insulating material, an asphalt binding course and a dampproof course made from a roll material such as roofing felt or Ruberoid (Figs. 7-2 and 7-3).

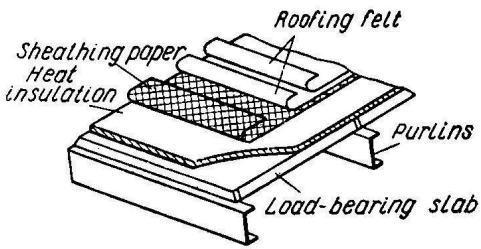


Fig. 7-2. Heat-insulated roofing on purlins

Two types of roof are distinguished, namely, with or without purlins. In the first type the load-bearing elements may be standard precast reinforced concrete slabs or reinforced foam concrete

and reinforced foam silicate ones (combining the functions of a load-bearing element and a heat-insulating material), etc. These slabs are laid on the purlins (Fig. 7-2), the latter, in turn, being supported on the roof trusses at the joints (panel points), transferring the load to them. The most widely used length of a slab, which determines the size of the truss panels, is three metres.

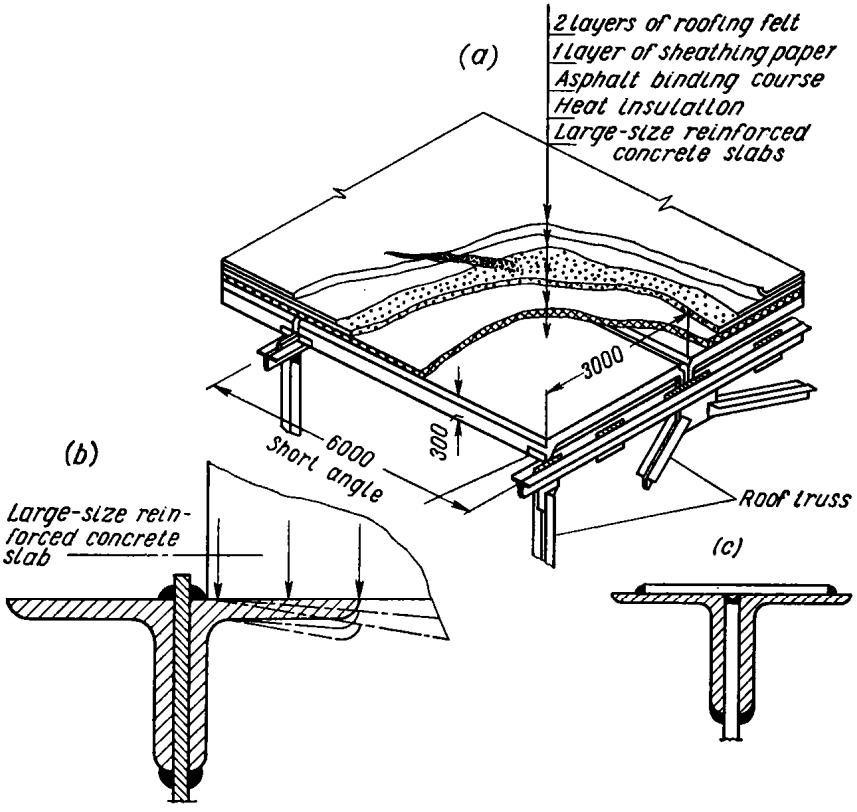


Fig. 7-3. Heat-insulated roofing without purlins

The second, more widespread type of roofing (without purlins) consists of large-size standard reinforced concrete slabs with the following dimensions: 3×6 m (ПКЖ) with a depth of 300 mm, 1.5×12 m (ПНТП) and 3×12 m (ПКЖН) with a depth of 450 mm, that are directly supported on the top chords of the roof trusses. The slabs are secured to the truss chords by welding to the latter short angles embedded in the slab (Fig. 7-3*a*). Large-size slabs are also covered with a heat-insulating material (if the slabs do not

simultaneously perform the functions of such material), a binding course and a layer of roofing felt.

When reinforced concrete slabs are supported on the truss angles, bending of the outstanding leg of the latter should not be feared, seeing that with the development of common deformation of the slab and the angle, the support pressure of the slab will move toward the back side of the angle (Fig. 7-3*b*). It is only essential to prevent shear of the angle material under the support pressure of the slab. For this reason with these truss chord angles ($t<10$ mm for slabs 6 metres long and $t<14$ mm for slabs 12 metres long) bearing plates 10-12 mm thick should be used on the angles (Fig. 7-3*c*).

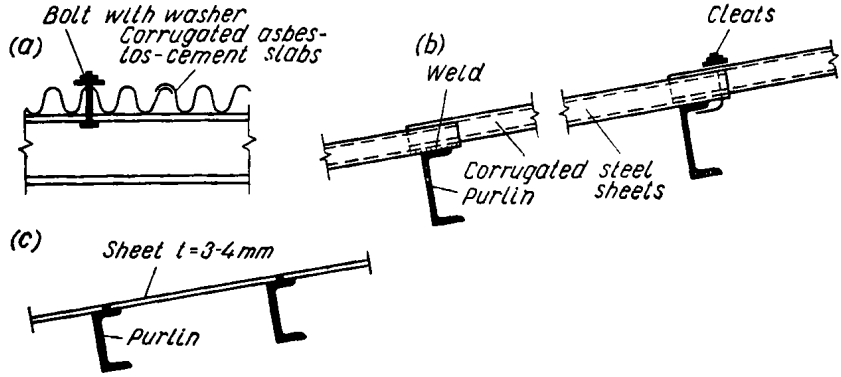


Fig. 7-4. Types of roofing without heat insulation

Cold (without heat insulation) roofing is employed in hot shops and in cold buildings. It is made of corrugated asbestos-cement sheets with a strengthened section (Fig. 7-4*a*) or, in certain cases, of corrugated steel sheets (Fig. 7-4*b*), or a solid sheet (Fig. 7-4*c*) on purlins. Cold roofing can also be designed in the same way as its heat-insulated counterpart, using reinforced concrete slabs, but without heat insulation.

To ensure the shedding of water, roofs are generally given a slope or pitch that depends on the material of the roofing. Ordinarily the following pitches are used for roofs:

for roofing made from roll materials . . . . . $i=\frac{1}{8}$ to $\frac{1}{12}$

for asbestos-cement corrugated slab roofing . . . . . $i=\frac{1}{3}$ to $\frac{1}{4}$

for corrugated steel sheet roofing . . . . . $i=\frac{1}{5}$ to $\frac{1}{7}$

Note. The above fractions denote a rise of 1 metre over the length shown in the denominator.

The pitch of a roof is created, as a rule, by designing the roof trusses with a sloping top chord.

There are distinguished single (also called lean-to, single-pitch or simple) roofs and ridge (also known as saddle or span) roofs.

The purlins are either solid (rolled) or latticed. Rolled purlins made from I sections or channels are heavier than latticed ones, but are considerably simpler and cheaper to fabricate, which explains their greater use. Rolled purlins are installed on the sloping

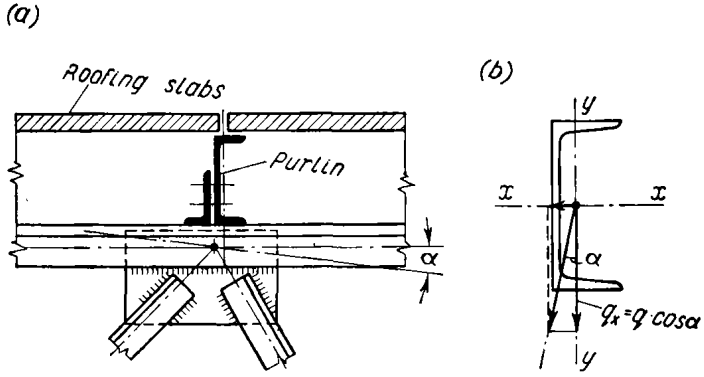


Fig. 7-5. Unsymmetrical bending of purlins

top chords of trusses. Being located at an angle to the plane in which the load acts, they are subjected to unsymmetrical bending (Fig. 7-5*a*).

Purlins are investigated for the combined action of the dead roof and the snow loads. This summary load is resolved into its components along the main axes of the purlin section, viz., q_x normal to the pitch and q_y along the pitch (the pitch component). These components bend the purlin in two planes (Fig. 7-5*b*).

The maximum design moments are respectively equal to

$$M_x = \frac{q_x L_x^2}{8} = \frac{q \cos\alpha L_x^2}{8}$$

$$M_y = \frac{q_y L_y^2}{8} = \frac{q \sin\alpha L_y^2}{8} \qquad (7\text{-}1)$$

It is obvious that at the extreme two points of a purlin section the stresses induced by the simultaneous action of M_x and M_y will be added together. In this case the total stress must not exceed the design strength

$$\sigma = \sigma_x + \sigma_y = \frac{M_x}{W_x} + \frac{M_y}{W_y} \leqslant R \qquad (7\text{-}2)$$

Every effort should be made to ensure the stress σ_y being considerably less than the stress σ_x, since beam sections have a relatively small moment of inertia with respect to the axis y-y.

To reduce the stress σ_y caused by the pitch component of the load, the purlins are generally braced with sag rods or brace rods arranged in the middle of their span, in the plane of the roofing. Thus the magnitude of σ_y is halved (Fig. 7-6). Besides, the brace rods facilitate aligning of the purlins in erection. This improves the conditions

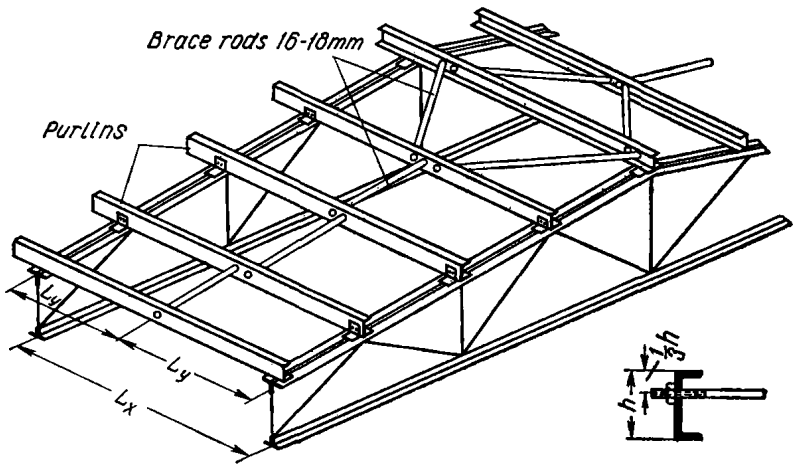


Fig. 7-6. Installation of brace rods between purlins

for the laying of the reinforced concrete slabs. Rolled simple beams secured against the loss of stability and carrying a statical load are analyzed with account taken of plastic deformations.

Hence, the working expression for checking the stress in a rolled purlin is as follows

$$\sigma = \frac{M_x}{1.12W_x} + \frac{M_y}{1.2W_y} \leqslant R \qquad (7\text{-}3)$$

Besides, it is also necessary to check the deflection of a purlin in the plane of its maximum stiffness.

Rolled purlins are connected to trusses, as a rule, by means of unfinished bolts, and with the aid of short angles welded to the chords of the trusses. Channel purlins are connected as shown in Fig. 7-7*a*, and I-section ones as illustrated in Fig. 7-7*b*.

Latticed purlins are employed with a lattice made from round steel bars (bar purlins, Fig. 7-8*a*). Bar purlins can be installed vertically and inclined, i. e., perpendicular to the chords of the trusses (with roof pitches up to 1 in 7). In all cases they must be braced

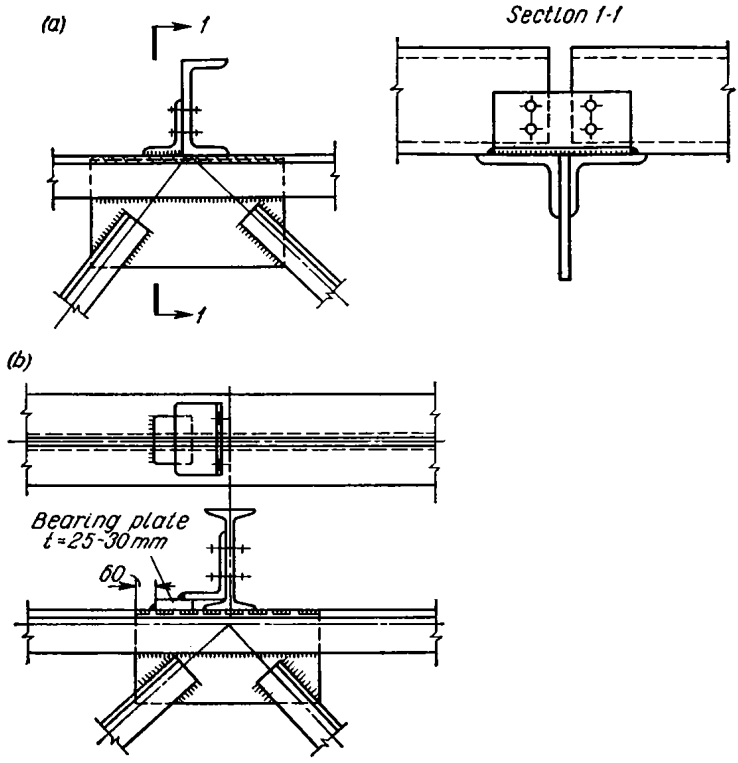


Fig. 7-7. Securing of purlins

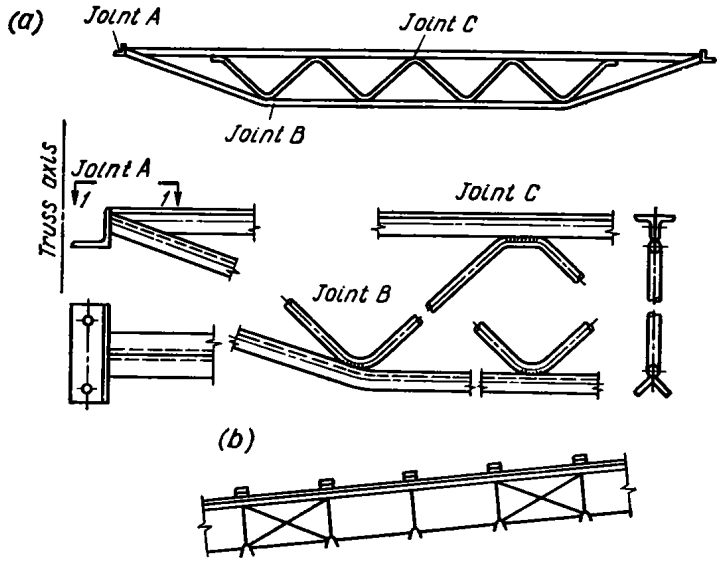


Fig. 7-8. Round steel bar purlin

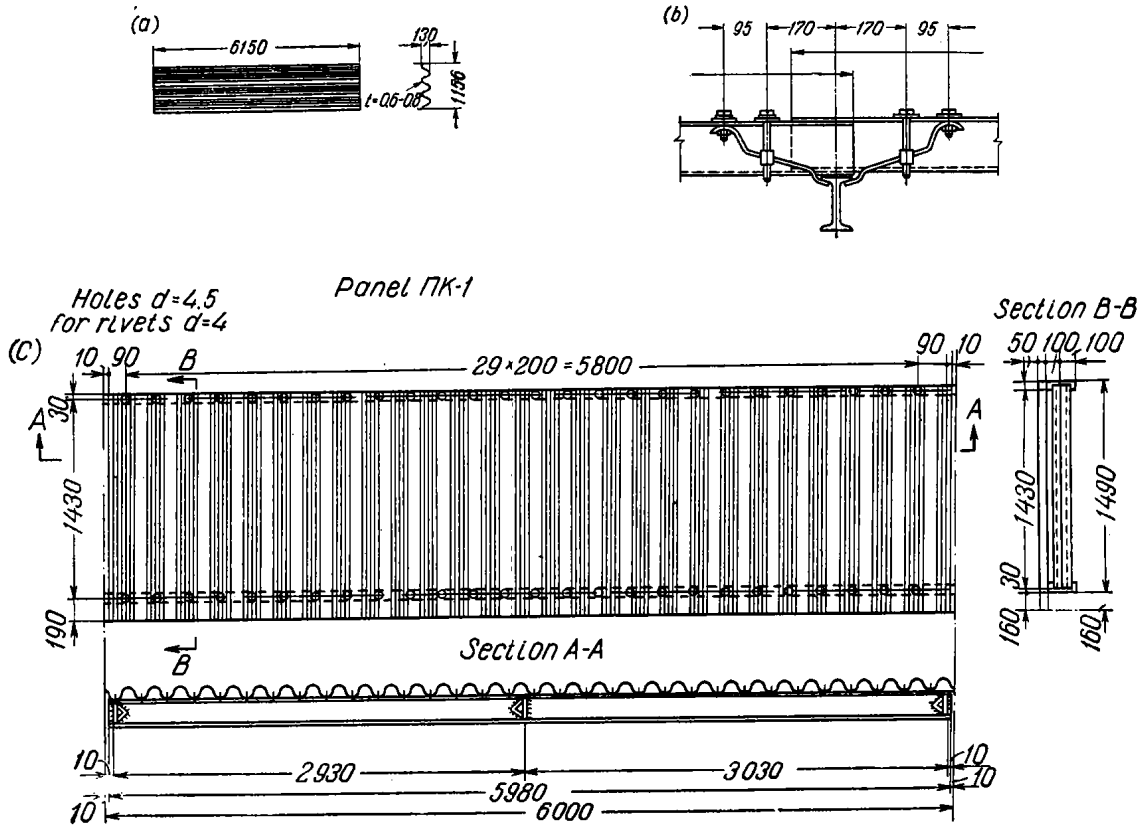


Fig. 7-9. Corrugated aluminium roofing without heat insulation

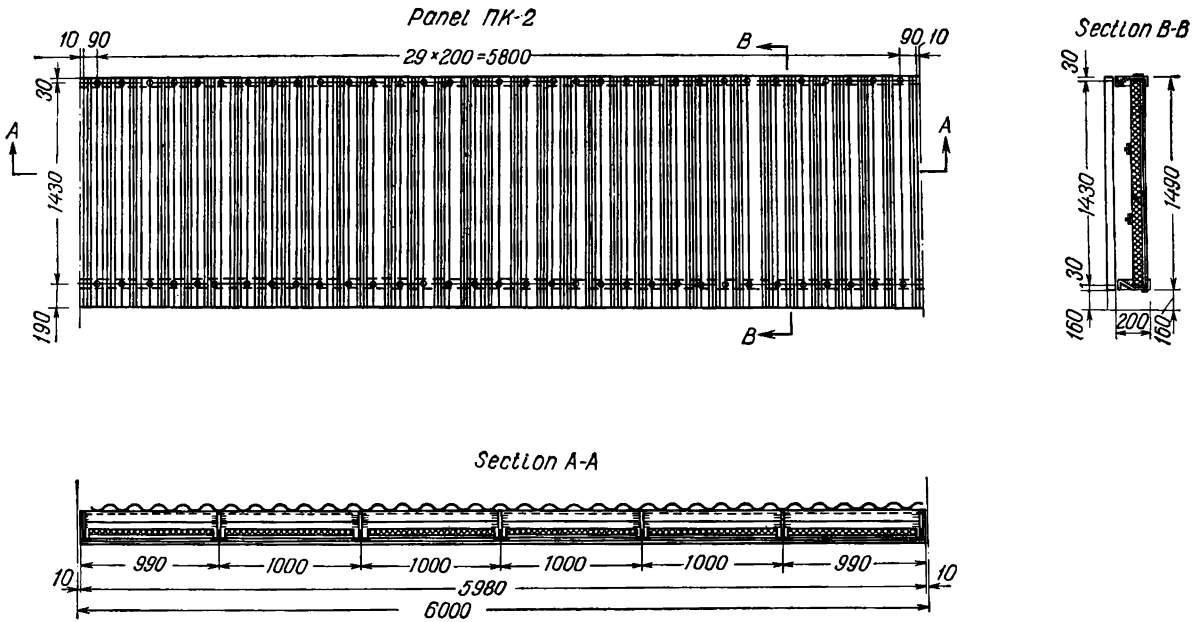


Fig. 7-10. Heat-insulated aluminium roofing

with stays in a lateral direction (both the top and the bottom chords The stays (Fig. 7-8*b*) can be made from 6-mm wire.

It is good policy to use latticed purlins with spans over 6 metr in length.

A shortcoming of large-size reinforced concrete roof slabs is the great weight, which leads to a high consumption of metal for tl load-carrying members. To reduce the weight of the roof, it is goc to employ aluminium roofing plates, with or without heat insul tion. An analysis of the efficiency of employing aluminium roofir

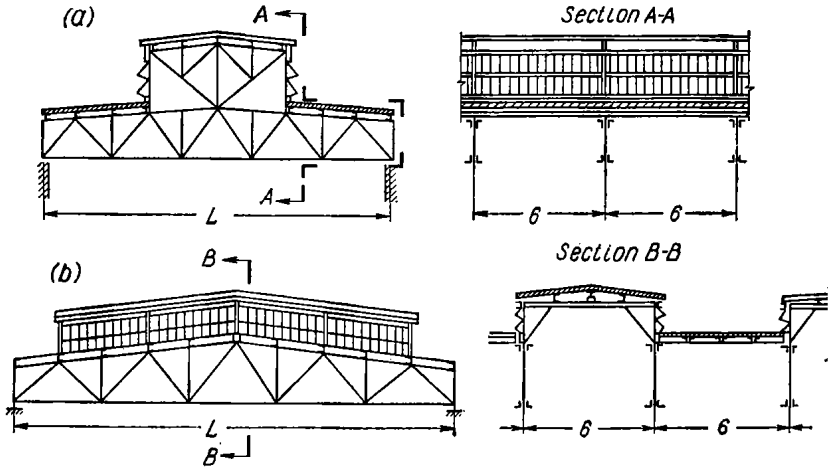


Fig. 7-11. Longitudinal and transverse skylights

in industrial construction has shown that the total saving in the co of the structural part of the buildings due to the reduction in tl weight of the roof is over 10%.

Cold roofing (without heat insulation) can be made from corr gated aluminium sheets 0.6-0.8 mm thick, with a corrugation dep of 130 mm and length of 350 mm, the sheet being 6 metres lor (Fig. 7-9*a*). These sheets, which are 1,150 mm wide (in projection are placed on steel purlins (Fig. 7-9*b*) and are secured to them l means of special cleats, or are designed in the form of special pane (Fig. 7-9*c*).

Warm (heat-insulated) roofing can be made with two plies layers in the form of panels. The upper ply consists of a corrugat aluminium sheet, and the lower one is a sheet of asbestos plywoc carrying sheets of a heat-insulating material such as slag wool foamed plastics (Fig. 7-10). A panel with a length of 12 metr must be strengthened with a strut.

It should be remembered that there must be no contact whats ever, not at a single point, between aluminium sheets and ste

elements. For this reason galvanized or cadmium-plated bolts are used for fastening the panels to the steel trusses.

Roof trusses sometimes carry *skylights*, i. e., members designed for lighting the premises from above and for natural ventilation (aeration). Skylights can be arranged along a building (longitudinal skylights, Fig. 7-11*a*) and across a building (transverse skylights, Fig. 7-11*b*). Metal window sashes or shutters are suspended from the members of the skylight. The entire roofing load is transmitted to the roof trusses through the purlins and skylights, or directly through the large-size reinforced concrete slabs, if the latter are employed.

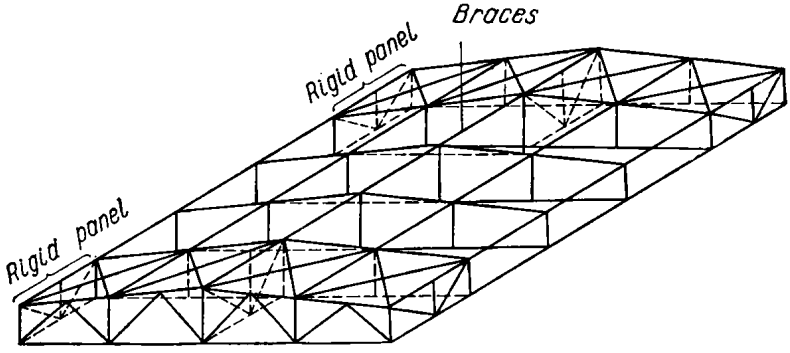


Fig. 7-12. Ties between roof trusses

It is assumed that the loads are applied strictly in the plane of the trusses. Actually this is not the case, and the load is applied with a certain eccentricity caused by requirements of design. This circumstance, as well as the necessity of preventing buckling of the top compression truss chords, requires the installation of stays or ties outside the plane of the truss.

The ties used are horizontal ones in the plane of the top chord and vertical ones between the trusses, locating them at the ends of the building or of the temperature block (Fig. 7-12).

The intermediate trusses are connected to the rigid tie panels by means of purlins or braces. The problems involved in the design of stays and ties in industrial buildings are considered in Sec. 9-1.

7-2. TYPES OF TRUSSES. DETERMINATION OF GENERAL DIMENSIONS. SPACING OF TRUSSES

Trusses are distinguished both with respect to the configuration of the chords and to the arrangement of their web members (lattice). As regards the configuration of their chords, trusses are classified into *parallel-chord* (Fig. 6-13*a*), *single-pitch* (Fig. 7-13*b*), *trapezoidal*

(Fig. 7-13*c*) and *triangular* (Fig. 7-13*d*) trusses. The configuration of trusses is selected depending upon their designation, the material of the roofing, the system of water shedding, as well as with regard to factors of economy. Roof trusses with a trapezoidal configuration are the ones mainly used in industrial structures with roofing made from a roll material (roofing felt, etc.).

A permanent shape of a truss under any load is attained by using web members that form a system of triangles. The web or lattice of a truss is called a *diagonal* one (flat top Pratt or Howe trusses) if it is formed by a continuous zigzag arrangement of diagonals and verticals, all the diagonals of one half of the truss being directed to the same side (Fig. 7-13*b*). The lattice is referred to as a *triangular* one (single Warren trusses) if the zigzag is formed only by diagonals alternately directed to different sides. Most frequently use is made of a triangular lattice with additional verticals (Fig. 7-13*a* and *c*), since the total length of its zigzag and the number of joints are less than in a diagonal lattice, while the additional verticals reduce the panel size in the top truss chord. In this system verticals are not required for creating a constant shape of the truss.

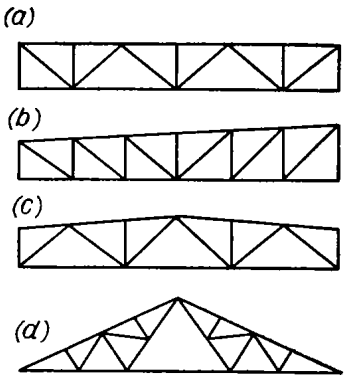


Fig. 7-13. Types of trusses

The general dimensions of a truss are its span and depth. For purposes of standardization, the spans of roof trusses in industrial buildings have been unified and are based on a module of 6 metres, i. e., they have a length of 18, 24, 30, 36 and 42 metres. For simplification of their fabrication and design, the unified standard steel trusses must have a standard geometrical layout for different spans. Examples of unified standard layouts of industrial building roof trusses are pictured in Fig. 7-14. The length of the top chord panel in standard trusses is taken equal to 3 metres.

The optimal depth h at the middle of a trapezoidal truss span is determined by conditions of minimum weight and the required stiffness (deflection). Here attention must also be paid to the possibility of rational transportation, with a view to the maximum concentration of fabrication work at plants. The minimum weight of such trusses is approximately obtained when the weight of the chords equals that of the lattice (including the gusset plates), which will be the case with a comparatively large truss depth-to-span ratio $\left(\frac{h}{L} \cong \frac{1}{5}\right)$. Such a large depth of a roof truss is not convenient

from the standpoint of transportation and erection, since in this instance the truss would have to be transported by separate elements with assembly in the field, which requires a lot of time and great outlay.

To make possible transportation by railway, the maximum overall dimensions of a member should not exceed (for the 1,524-mm gauge railways of the USSR) 3.8 m in a vertical direction and 3.2 m in a horizontal one (Fig. 7-15). The length of a four-wheel flat car is 9 metres and of an eight-wheel one is 13 metres. With a view to inscribing the overall dimensions of roof trusses within the limits of the railway clearance gauge so that only assembly of two halves of a truss would be required in the field, the depth of a truss at the middle of the span, as a rule, is taken not over 3.8 metres (between the extreme points of a member). Using a pitch of the top roof truss chord of 1 in 12, we obtain the depth at the support h_s, and thus all the overall dimensions of the trusses (Fig. 7-14*a*). It will be good practice to use the same depth of a truss at the supports h_s for trusses with different spans. This will permit designers to use a single standard geometrical layout and ensure standardization of the fastening elements. In standard trusses, the depth h_s is taken equal to 2.2 metres (between the backs of the angles).

A solution is possible, however, in which the depth of the truss at the middle exceeds 3.8 metres (with a top chord pitch of 1 in 8), while the truss well lends itself to fabrication and transportation at the expense of an insignificant increase in the number of field joints (three joints instead of two, see Fig. 7-14*b*). For this end the bottom chord in the middle panel is shipped as an independent element, while the main part of the truss is transported upside down. In this instance the clearance depth of the truss is determined by the depth of the third vertical from the support.

The weight of standard trusses g in kg/m^2 of area covered, depending upon the design load q (in kg/m^2), is

For trusses with a span of	$L=18$ m	$g=2.2+\frac{q}{125}$
Ditto,	$L=24$ m	$g=2.78+\frac{q}{54.2}$
Ditto,	$L=30$ m	$g=4.44+\frac{q}{34.7}$
Ditto,	$L=36$ m	$g=5.27+\frac{q}{21}$

The most advantageous angle between the diagonals and the bottom chord is 45-50 deg in a triangular lattice and 35-45 deg in a diagonal one.

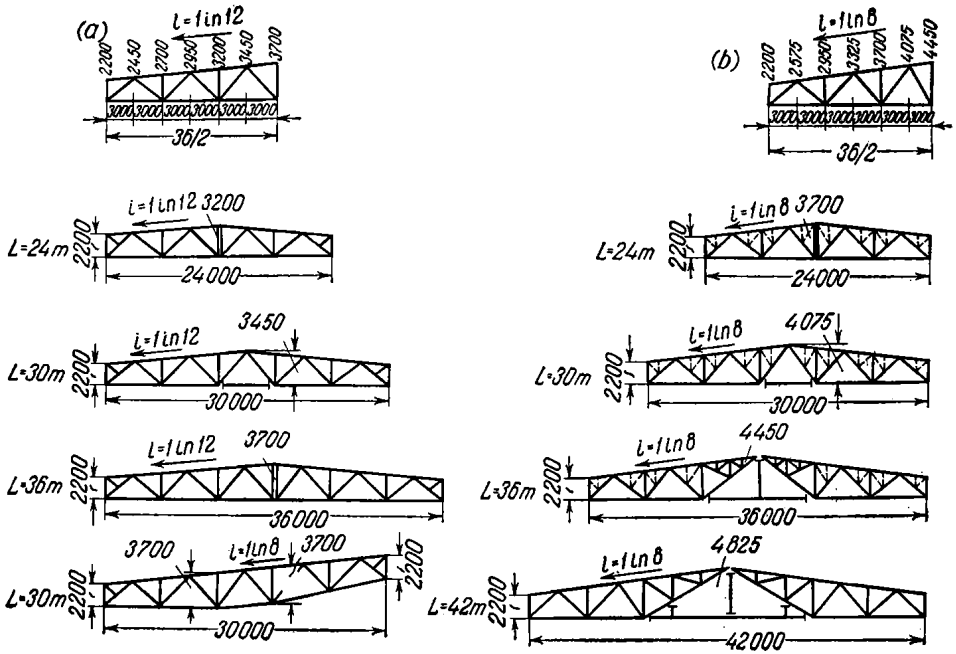


Fig. 7-14. Unified standard layouts of roof trusses

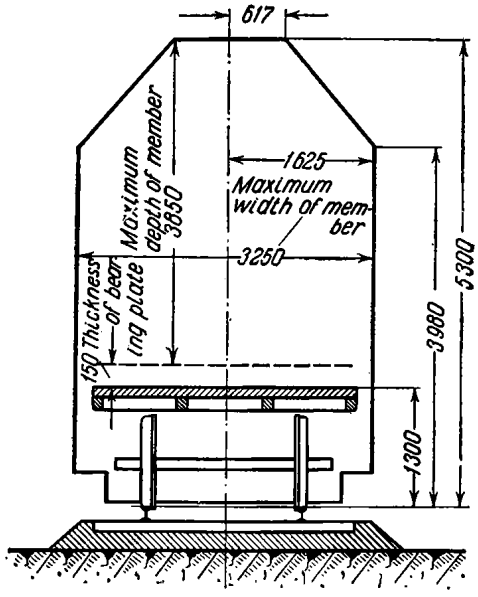


Fig. 7-15. Railway clearance gauge used on USSR 1,524-mm gauge railways

The first, support, diagonal, whose direction determines the entire lattice system, may be directed either upward (Figs. 7-13*c* and 7-14) or downward (Fig. 7-13*a* and *b*). In the practice of designing industrial and residential buildings, the most frequent case is the use of support diagonals directed upward. With such a solution the stiffness of the shop or other premises is ensured more reliably when the truss performs the functions of a frame beam (see Chapter 9), and the design of the support joint and the arrangement of the ties is better from a constructive viewpoint. Trusses with the support diagonal directed downward have an insignificant advantage in erection, consisting in that the point of support is located higher than the centre of gravity of the truss.

It should be noted that standardization of steel structural member elements is an important measure for the industrialization of erection work, inasmuch as it sharply reduces the number of types and sizes of members, and improves the conditions for batch production. This speeds up work and reduces its cost. Being designed with greater attention, standard members, as a rule, are lighter than their individually designed counterparts. The employment of standard members cuts the time required for designing, since standard members are in essence an assortment of members designed for withstanding definite loads.

Standard members differ somewhat from individually designed ones, since the range of members produced should be such that the smallest possible number of types and sizes of members will meet the requirements of the greatest possible variety of conditions in which they can be used. This introduces special features into their design, which should be performed, as a rule, in two stages:

1. Working out and substantiation of standard designs of members and their types and sizes.
2. Compilation of the working drawings and designing of the constructive elements in accordance with the schemes accepted for use.

It is obvious that standardization of members and structures, i. e., the establishment of the standardization modules (relation between the main dimension parameters) must precede any development of standard members, inasmuch as all of the various constructive elements should be easily connected to one another. When standard designs are available, designing mainly consists in selecting the members of a structure, selecting and laying out of the standard elements of the members, and working out of the erection diagrams.

When large-size reinforced concrete slabs 1.5 metres wide are employed in roofs without purlins, the slabs rest with their ribs not only on the panel points, but also on the middle of the panels. This induces an additional bending moment in the top chord of the truss, which adversely affects the entire truss (an increase of weight of

4-5%). To eliminate this bending moment, it will be excellent practice to introduce into the lattice additional struts (shown by dotted lines in Fig. 7-14*b*) for taking the local load and thus precluding flexure of the truss chord.

Triangular trusses are employed only in roofs with steep pitches (Fig. 7-16). Here the tendency should be to use such a configuration of the top chord that the forces in it at the middle of the span will approximately equal those at the supports. This can be achieved

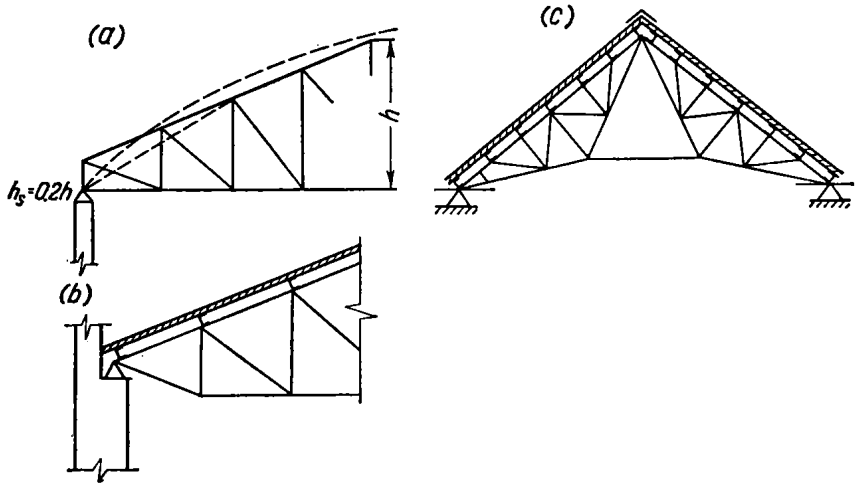


Fig. 7-16. Triangular trusses

by providing a small vertical with a height of $h_s=0.2h$ at the support (Fig. 7-16*a*). With steep roofs, however, this leads to an increase in the depth of the truss, and to a support joint that is inconvenient to design. Consequently, it is better to transfer the support to the upper joint of the truss (Fig. 7-16*b*). The lattices of such trusses are generally of the diagonal type, since in a triangular lattice the ascending diagonals would form a too acute angle with the top chord and would require large gusset plates for connecting them.

In steep roofs that are sometimes encountered (with a pitch of 1 in 1 or 45 deg) complicated strutted triangular trusses are employed, sometimes with a raised bottom chord (Fig. 7-16*c*).

The distance between trusses (the spacing of the trusses) is established when solving the layout of the structure as a whole, with account taken of standardization of the members and parts of the structure.

As the result of investigations aimed at determining the optimal spacing of trusses, standardized spacings of 6 and 12 metres have found the widest use in designing practice.

7-3. ANALYSIS OF TRUSSES

(1) Determination of Loads

Roof trusses are investigated for the following kinds of loads, transmitted to them as concentrated forces at the joints:

1. Dead load formed by the weight of the roofing and that of the members themselves.

2. Live load caused by snow, wind, suspended hoisting and conveying equipment, etc.

The majority of these loads are uniformly distributed ones.

First the uniformly distributed load per square metre is computed, next the tributary area per joint (panel point) is determined, and then the concentrated force acting on each joint of the truss is found

$$P = bd \sum q_i n_i \tag{7-4}$$

where q_i = service (working) uniformly distributed load per square metre of horizontal projection

n_i = load factor for the corresponding load

d = length of truss chord panel to which load is applied

b = distance between trusses (truss spacing).

With steep roof pitches, the load induced by the weight of the roofing g_r should be taken equal to $q_r = \frac{g_r}{\cos \alpha}$, where α is the pitch of the roof in degrees.

The load induced by the weight of the roof trusses, ties, braces, purlins and skylights referred to 1 square metre of building area can be taken from Table 9-3.

The weight of glazing is taken equal to 35 kilograms per square metre of glazed area. The snow load is taken in accordance with the prescriptions of the USSR Building Standards and Regulations. In this instance the snow, as a rule, is considered to be distributed over the whole area of the roof. Sometimes when investigating roof trusses the possibility is considered of the snow being only on one side (on half of the truss span), which results in a greater load on the middle diagonals and may even cause a change in the sign of the force. However, seeing that the section of the middle diagonals, owing to the relatively small forces in them, is most frequently selected from considerations of design (with an adequate slenderness ratio), there will in most instances be no practical necessity of taking such a location of the snow load into account. The wind load is considered in trusses only when the top chord is inclined at an angle exceeding 30 deg.

(2) Determination of Forces in Elements of Trusses

The main assumption made when investigating trusses is that all the bars are pin-connected at the joints.

This assumption is generally possible with small ratios between the depth of the bar section h and its length L $\left(\frac{h}{L} \leqslant \frac{1}{10}\right)$.

The simplest way of determining the forces in the elements of roof trusses induced by the dead load is the graphical one, by plotting Cremona's force plans as for a statically determinate truss. It is also possible to compute the forces analytically.

When a roof is designed without purlins the top chord of a roof truss, besides compressive forces, may also resist local bending due to the supporting of a large-size slab at the middle of the panel (though, as has been indicated above, in this instance, with respect to the consumption of steel, it is more advantageous to install additional struts). By considering the top chord as a continuous beam resting on supports with elastic settling (the joints of the truss) it will be possible to find the relieving influence of the support moments on the moments at the middle of the panel. The results of calculations have proved that this influence is not large. For this reason in the first panel with a length of d the moment induced by the local load P_{loc} is generally determined in the same way as in a simple beam

$$M_1 = \frac{P_{loc} d}{4}$$

In the remaining panels the relieving influence can be taken into account by introducing the factor 0.9, viz.,

$$M_2 = 0.9 \frac{P_{loc} d}{4}$$

7-4. SELECTION OF TRUSS ELEMENT SECTIONS

(1) Determining Area of Element Section

The elements of trusses are mainly subjected to axial forces, namely, tension and compression.

The most convenient section of roof truss elements from the viewpoint of design, and therefore the most widespread one, is a section made up from two angles in the form of a T shape (Fig. 7-17*a-c*). Owing to the presence in the section of two identical elements, a member is obtained that is symmetrical with respect to the vertical plane. The joints of the trusses are formed with the aid of gusset

plates (gussets), to both sides of which the chord and web elements are secured. A section of two angles with a clearance between them necessary for the gusset can be made up of equal leg (Fig. 7-17*a*) and unequal leg angles placed with their wide (Fig. 7-17*b*) or narrow (Fig. 7-17*c*) legs outstanding. Other sections (Fig. 7-17*d* and *e*) are also possible.

The sections of all the elements of roof trusses can also be formed of single angles (Fig. 7-17*f*). In such trusses the consumption of steel is the same as in ordinary trusses. However, less labour is

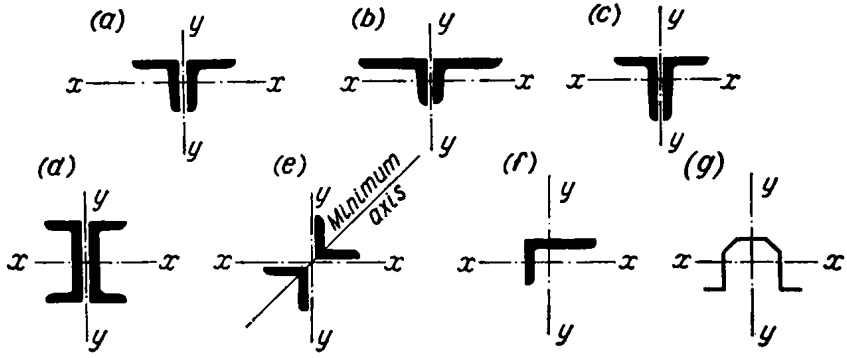


Fig. 7-17. Sections of roof truss elements

required for their manufacture owing to the smaller number of parts, and to there being no necessity of turning over the members when performing the welding jobs. Because of certain difficulties in the design of the joints and unsymmetrical application of the loads, however, good practice dictates the use of such trusses mainly in premises with an aggressive medium, since they have no slots between the section elements and are therefore better to paint.

It is also possible to use thin-wall (lightweight) bent sections (Fig. 7-17*g*) in the compression elements of trusses.

The required sectional area of a tension element is determined from expression (1-6)

$$A_{req\cdot n} \geqslant \frac{F}{R} \tag{7-5}$$

In tension elements, the sections should be of a rigid shape to prevent deformation of the truss in transportation and erection, and also bending of the element under its own weight. For this reason the Building Standards and Regulations do not permit the slenderness ratio of tension elements to exceed 400 (see Table 7-1).

The required sectional area of a compression element is found from expression (2-14)

$$A_{req.gr} \geqslant \frac{F}{\varphi R} \tag{7-6}$$

For truss web (lattice) elements in which the compressive force is relatively low, and small sections with a low stiffness are obtained, and which, therefore, may easily be deformed during fabrication and erection, a factor of service conditions $k_s=0.8$ should be introduced for reducing the design strength (see Table 2 of Appendix I). At the same time, an increase in the eccentricity of force application in compression elements is dangerous.

For the given force F and design strength R the minimum sectional area is obtained using the maximum value of the buckling factor φ, which is found from Table 1 of Appendix II depending on the slenderness ratio λ of the element. Hence, the problem of determining the required sectional area of a compression element must be solved by the method of successive approximations, preliminarily assuming various values of the factor φ. For the first approximation the following values of this factor can be taken: $\varphi=0.7$-0.8 for the chords and $\varphi=0.5$-0.6 for the web elements.

Thus the main parameter influencing the selection of the type of section to be used for a compression element of a truss is the factor φ.

(2) Effective Length of Compression Roof Truss Elements

In the critical state the loss of stability (buckling) of a compression element is possible in any direction. Let us consider two main directions—in the plane of the truss and perpendicular to this plane.

Deformation of the top chord of a truss upon the loss of stability in the plane of the truss may take place as shown in Fig. 7-18*a*, i. e., between the panel points of the truss. Such deformation corresponds to the main case of buckling with a length coefficient of $k=1$ [see Sec. 2-4, equation (2-16)]. The effective length of a compression top chord in the plane of the truss is therefore taken equal to its geometrical length (between panel point centres)

$$L_e = L$$

For the diagonals (except for the support one, which is considered as a continuation of the chord) and the verticals the effective length in the plane of the truss is taken equal to

$$L_e = 0.8L$$

in view of a certain constraining of their ends due to the presence of tension elements connected to the gusset. Indeed, upon the loss

of stability, a compression diagonal (or vertical) secured to the gusset tends to turn it, but the tension elements also connected to the gusset resist this turning, thus acting as a sort of constraint, which makes it possible to employ a length coefficient of $k=0.8$.

The effective length of compression elements in buckling perpendicular to the plane of the truss is determined by the distance between their fixed points. Thus, in the roof truss with a skylight and

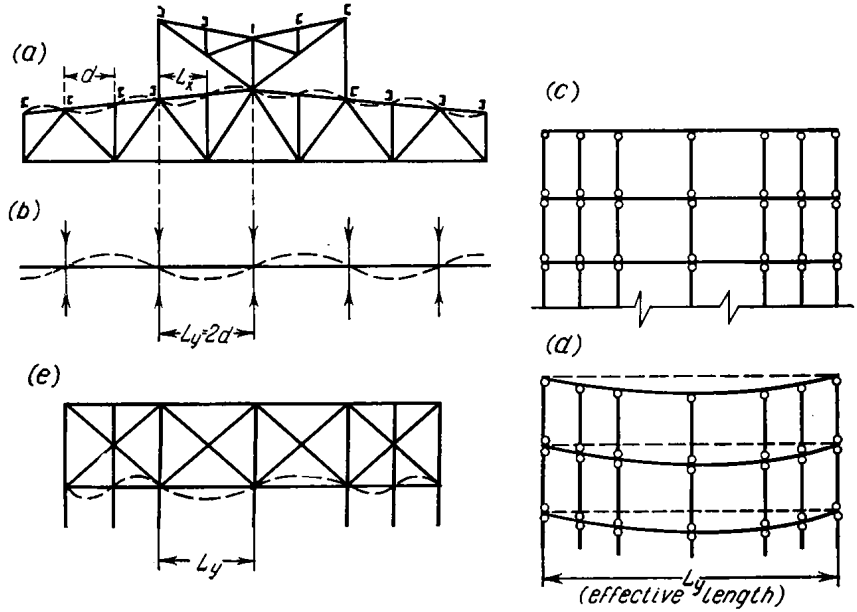


Fig. 7-18. To determination of the effective length of truss top chord elements

purlins illustrated in Fig. 7-18*a* the section of the top compression chord under the skylight may be investigated for buckling in the plane of the truss with the effective length taken equal to the length of the panel, i. e., $L_x=d$, and for buckling at right angles to the plane of the truss (in a top view) with the effective length taken equal to $L_y=2d$ (Fig. 7-18*b*). For the remaining sections of the compression top chord, the effective lengths in buckling in and perpendicular to the plane of the truss, owing to the presence of purlins at the joints, will be identical, i. e., $L_x=L_y=d$. It should be remembered that purlins are supporting points for the top chord only if they are restrained to prevent free displacement in a horizontal plane. Thus, for example, Fig. 7-18*c* pictures a top view of trusses with purlins pin-connected to the trusses and permitting their free horizontal displacement. As a result, buckling of the top chord may occur as

depicted in Fig. 7-18*d*, i. e., with an effective length of the chord equal to the entire span of the truss. Only after ties or stays have been installed in the plane of the top chord (Fig. 7-18*e*) will there be ensured (within the limits of elastic behaviour of the bracing horizontal truss) fixation of the purlins restraining the points of the top compression chord in a lateral direction, and thus determining its effective length in buckling perpendicular to the plane of the truss. Generally the ties are located in such a way that there will be a joint fixed in place by these ties in every second panel (it is difficult to secure a tie in the middle of a purlin from the viewpoint of erection conditions). For this reason it is general practice, in determining the slenderness ratio for the compression chord of a roof truss in buckling perpendicular to the truss plane, to take the effective length of the chord equal to the double length of a panel ($L_y=2d$), even when there are purlins at every joint (if only the purlin is not fastened to a tie).

The effective length of compression diagonals and verticals in buckling at right angles to the plane of the truss is taken equal to their geometrical length.

In a roof without purlins, the large-size slabs are welded to the top chords of the trusses. Thus they ensure lateral stability of the top chords. During erection, however (or when there is a skylight), ties may nevertheless be required (see Chapter 9).

(3) Selecting Type of Sections

The type of angles to be used for the top compression chords of roof trusses is selected with a view to the minimum consumption of metal that will ensure equal stability of the chord in all directions, as well as to creating the stiffness perpendicular to the plane of the truss necessary to ensure convenient transportation and erection.

Since the effective lengths of the chord in buckling in the plane and normal to the plane of the truss often differ considerably from each other ($L_y=2L_x$), then to ensure equality of the slenderness ratios it is essential that the radii of gyration also differ from each other ($r_y \cong 2r_x$). This condition is met by installing unequal leg angles with their long legs outstanding from the plane of the truss (Fig. 7-17*b*).

If each joint of the top chord is fixed in some way by ties or roof slabs (i. e., $L_y=L_x$), equal stability of the chord is ensured by a section formed of unequal leg angles installed with their short legs outstanding from the plane of the truss (Fig. 7-17*c*), since in this case $r_x \cong r_y$. However, trusses with such a section of the top chord are not convenient for transportation and erection. They easily bend out of their plane, which leads to additional expenditure for

straightening the members and as a result of delays in construction. For this reason it will not be good to use such a section for the top chord. Sections made from equal leg angles, from the viewpoint of the most advantageous ratios of the radii of gyration, are only slightly inferior to sections made from unequal leg angles. This is offset, however, by the variety of standard equal leg angles being considerably greater than that of their unequal leg counterparts.

Thus it can be considered that the most rational shapes for the top chords of roof trusses are either sections made from two unequal leg angles installed with their long legs facing outwards, or from equal leg angles.

For compression support diagonals having identical effective lengths in buckling in both planes ($L_x = L_y$), the unequal leg angles may be arranged as illustrated in Fig. 7-17*c*, since such a section has approximately equal radii of gyration.

It will be excellent practice to employ unequal leg angles placed with their long legs facing outwards for the support diagonals, halving the effective length of the diagonal in the plane of the truss with the aid of a strut (Fig. 7-14*a* and *b*). Such a design is advantageous not only from the viewpoint of economy in metal, but also because the strut strengthens the overhanging element of the top chord and prevents its being bent during transportation of the trusses.

The remaining compression diagonals and verticals, between whose effective lengths there is an insignificant difference ($L_x = 0.8L$, $L_y = L$), are most frequently designed of equal leg angles (Fig. 7-17*a*), inasmuch as the ratio between the radii of gyration r_x and r_y in such a section approximately corresponds to the indicated ratio between the effective lengths. In all the sections considered above common stressing of the two shapes is ensured by the installation of small filler plates in the spaces between the gussets (see Fig. 7-29).

For tension elements, the type and arrangement of the angles is not so important, since here the determining factor is the net sectional area. This is why they are made both from equal leg angles and from unequal leg ones installed with their long legs facing outwards (if it is necessary to increase the rigidity in a lateral direction to ensure better erection conditions). In shops with heavy service conditions, the lateral slenderness ratio of the bottom chord should not exceed 250 (see Table 7-1), which with a distance between ties along the bottom chord of 12 or 18 metres also often requires the use of unequal leg angles.

Other types of sections are seldom employed, and only if there are specific requirements for design. Thus, for example, chords made from two channels (Fig. 7-17*d*) are employed when they are subjected not only to an axial force, but also to a considerable local moment originated by a load applied between the panel points of

the trusses (the suspension of conveyors from the bottom chord, the top chord of trussed crane girders, etc.).

The verticals of trusses are designed with a T section formed of two equal leg angles. A cross-shaped section of two angles (Fig. 7-17*e*) is generally employed for the verticals of trusses when it is necessary to secure other elements of the member to them that are connected to the trusses at right angles in the top view. The minimum radius of gyration of a cross-shaped section is greater than that of a T section, and therefore the use of the former is also advantageous where stiffness is the main requirement. A section formed of one angle is not used for the main elements of ordinary trusses owing to the asymmetry of its connection. The use of such a section is possible only for unimportant auxiliary elements that are considerably understressed. In accordance with the standards, when investigating compressed single angles secured by one leg a service condition factor of $k_s=0.75$ is introduced (see Table 2 of Appendix I).

(4) Selection of Sections

When selecting the sections of truss elements, the tendency should be to use the smallest possible quantity of different numbers and sizes of angle shapes in order to simplify rolling and reduce transportation costs (since rolling at the mills is specialized by sections). It is usually found possible to select rational sections of roof truss elements by employing angles of 6-8 different standard sizes.

Selection of the sections is commenced with the compression element, which resists the largest design force. Next the element is selected, which withstands the minimum force (most frequently with a view to the limiting slenderness ratio), and thus the range of angle sections is established. When selecting angle sections for the compression elements, the tendency should be to use angles of the smallest possible thickness, since their radii of gyration have the relatively greatest value.

To avoid the possibility of bending during transportation, a minimum angle section of 50×5 is taken. Of considerable importance for the selection of the sections is the restriction of the limiting slenderness ratio of the elements, necessitated by the tendency to prevent sagging, vibrations under a dynamic load, bending in transportation of the truss elements, etc.

The limiting slenderness ratios established by the standards for the elements of steel members are contained in Table 7-1.

In structures not taking dynamic loads, the slenderness ratio of tension elements is checked only in a vertical plane.

With small forces in compression elements, the section is selected with regard to the given limiting slenderness ratio λ_l. First the

TABLE 7-1 **Limiting Slenderness Ratios λ for Compression and Tension Elements**

Name of elements	Compression elements	Tension elements		
		under direct action of dynamic load	under static load	in buildings with heavy service conditions
Chords, support diagonals and verticals of trusses transmitting support reactions . .	120	250	400	250
Other truss elements	150	350	400	300
Roofing ties (except brace rods)	200	400	400	300

required radius of gyration is found

$$r_{req} = \frac{L_e}{\lambda_l}$$

and in accordance with this radius the corresponding angles are selected. To facilitate determining the required dimensions of the sections, which will permit designers to select the necessary angle sizes, the approximate values of the radii of gyration for different sections of elements made from angles are presented in Table 7-2.

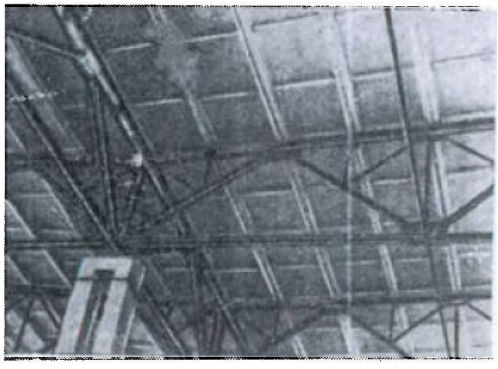


Fig. 7-19. Roof trusses with one end angle

When selecting the sections of the truss chords the most rational section could be taken for each panel (in accordance with the change in the force), but this would lead to the necessity of having a large number of joints and a great diversity of sections. Such a design will considerably increase the labour requirements for manufacture, introduce complications in the ordering of the metal, and almost entirely preclude the possibility of obtaining economy of it (owing to the necessity of making joints). For this reason it is usual practice to select chords of one section in trusses with a span up to

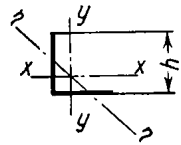
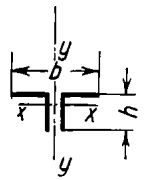
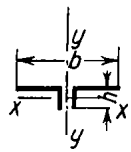
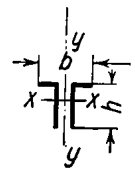
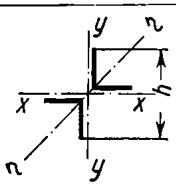
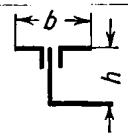
24 metres, this section being selected with a view to the maximum force. In roof trusses with spans of 24 metres and more, it will be good policy to change the section of the chords along the length of the truss, the changes being made, as a rule, only in the width of the chords. The retaining of a single angle thickness will facilitate splicing with the aid of splice angle straps (see Figs. 7-24 and 7-25).

In trusses that are pin-supported on columns, it is possible, for purposes of weight reduction, to do away with the second angle of the top chord section in the first (zero) panel, where the force induced by the vertical loads is equal to zero. In such cases generally only one angle of the section selected for the adjacent panel is continued in this panel (Fig. 7-19). This measure results in an economy of steel of about five per cent per truss.

When the section is changed over the length of the chord, it should be seen that the section centres of gravity of two elements being connected are displaced by not more than five per cent of the truss chord depth (to avoid the appearance of considerable additional moments). In this case such an additional moment may be left out of consideration. With a greater eccentricity, however, it should be taken into account when selecting the section of the chord elements.

Having selected the section of an element, the design stresses in

TABLE 7-2 Approximate Values of Radii of Gyration for Sections of Elements Consisting of Angles

Sections of elements	Radii of gyration	
	r_x and r_{η}	r_y
	$r_{\eta}=0.195h$	$r_x=r_y=0.3h$
	$r_x=0.3h$	$r_y=0.2b$
	$0.28h$	$0.24b$
	$0.32h$	$0.2b$
	$r_{\eta}=0.185h$	$0.21h$
	$0.3h$	$0.17b$

it should be checked, compression and flexural elements being investigated for stability not only in the plane of the truss, but also perpendicular to its plane by means of expression (8-40), which takes into account the influence of local bending.

The thickness of the gussets used to form the joints of trusses is selected depending on the magnitude of the maximum force in the support diagonal, the same thickness of the gussets being used, as a rule, throughout the truss. In trusses with large spans, the support gussets can be two millimetres thicker than those of the intermediate joints.

The thicknesses of truss gussets recommended for use are contained in Table 7-3.

TABLE 7-3 **Recommended Thicknesses of Truss Gussets**

Maximum design force in support diagonal, tons	Up to 20	20-45	45-75	75-115	115-165	165-225	225-300
Thickness of gussets, mm	8	10	12	14	16	18	20

The results of the investigations and of selecting the sections of the elements are generally drawn up in a tabular form (see example 7-1).

Example 7-1. Investigate and select the sections of the elements of an industrial building roof truss (Fig. 7-20*a*). The span of a truss is 30 metres, the spacing of the trusses in a longitudinal direction is 12 metres. The trusses carry a skylight 12 m wide and 4 m high. Over the trusses are laid large-size reinforced concrete slabs 3×12 m in size, a heat insulating course of foam concrete slabs 80 mm thick, a levelling asphalt binder course 20 mm thick and a damp course of roll material. Snowfall should be taken as for Moscow.

The truss is to be made from grade BCт.3nc steel to GOST 380-60.

Solution. 1. *Determination of Design Loads.* The design loads acting on the truss are calculated in Table 7-4.

The total uniformly distributed load on the skylight is $340+112\cong450$ kg/m^2, on the remaining part of the truss it is $340+159\cong500$ kg/m^2.

2. *Determination of Panel Point Loads on Truss.* The panel point loads are computed in Table 7-5. The forces P_1, P_2 and P_4 applied at the panel points have been obtained as the product of the total uniformly distributed load and the corresponding tributary area. The force P_3 includes the weight of the edge slab and the skylight glazing.

3. *Determination of Forces in Truss Elements.* We determine the forces in the truss elements graphically by plotting Cremona's force plan (Fig. 7-20*b*). The forces found in this way are entered in Table 7-6.

4. *Selection of Sections.* We begin selection of the sections with the most loaded element of the compression top chord (panel 6-13) having a design force of

TABLE 7-4 **Table of Design Loads**

Nature of load	Load	Unit	Service load, kg/m²	Load factor	Design load, kg/m²
Dead	Damp course	kg/m²	10	1.1	11
	Asphalt binder course 20 mm thick ($\gamma = 1.8$ t/m³)	Ditto	36	1.1	40
	Heat insulation of foam concrete slabs 80 mm thick ($\gamma = 0.6$ t/m³)	Ditto	48	1.2	58
	Large-size reinforced concrete slabs	Ditto	173	1.1	190
	Weight of steel members (roof trusses, skylights and ties) [1]	Ditto	37	1.1	41
	Total, uniformly distributed dead load	—	—	—	340
Dead concentrated	Concentrated load on extreme verticals of skylight: weight of edge slab 1×12 m in size	kg	1,550	1.1	1,700
	Weight of skylight glazing 3 m high and 12 m long (weighing 35 kg/m²)	Ditto	1,260	1.1	about 1,400
	Total dead concentrated load on extreme verticals of skylight	kg	—	—	3,100
Live [2]	Snow on skylight ($c_{sn} = 0.8$)	kg/m²	100× ×0.8= =80	1.4	112
	Snow on remaining part of truss ($c_{sn} = 1 + 0.2 \frac{12}{30-12} = 1.133$)	Ditto	100× ×1.133= =113.3	1.4	159

[1] The weight of the metal members can be approximately taken from Table 9-3 on page 391.
2) The snow loads are taken in accordance with the Building Standards and Regulations. The snow load for the region of Moscow is 100 kg/m². The factor c_{sn} allows for the distribution of the snow over the roof.

$F = -204$ tons, and effective lengths of $L_x = 3$ m in the plane of the truss and $L_y = 6$ m perpendicular to the plane of the truss. With such a ratio of the effective lengths a section consisting of two unequal leg angles with their long legs facing outwards will be the most rational one.

TABLE 7-5 **Table of Panel Point Loads**

Designation of panel point load	Calculation of panel point loads	Magnitude of force in tons
P_1	$500\times1.5\times12$	9
P_2	$500\times3\times12$	18
P_3	$500\times1.5\times12+450\times1.5\times12+3{,}100$	20.2
P_4	$450\times3\times12$	16.2
$F_{r\cdot A}$	$9+2\times18+20.4+16.2+\frac{16.2}{2}$	89.5

We take a section made from two angles ⊓ 250×160×16 with an area $A=2\times63.6=127.2$ cm² and radii of gyration $r_x=4.58$ cm and $r_y=11.93$ cm.

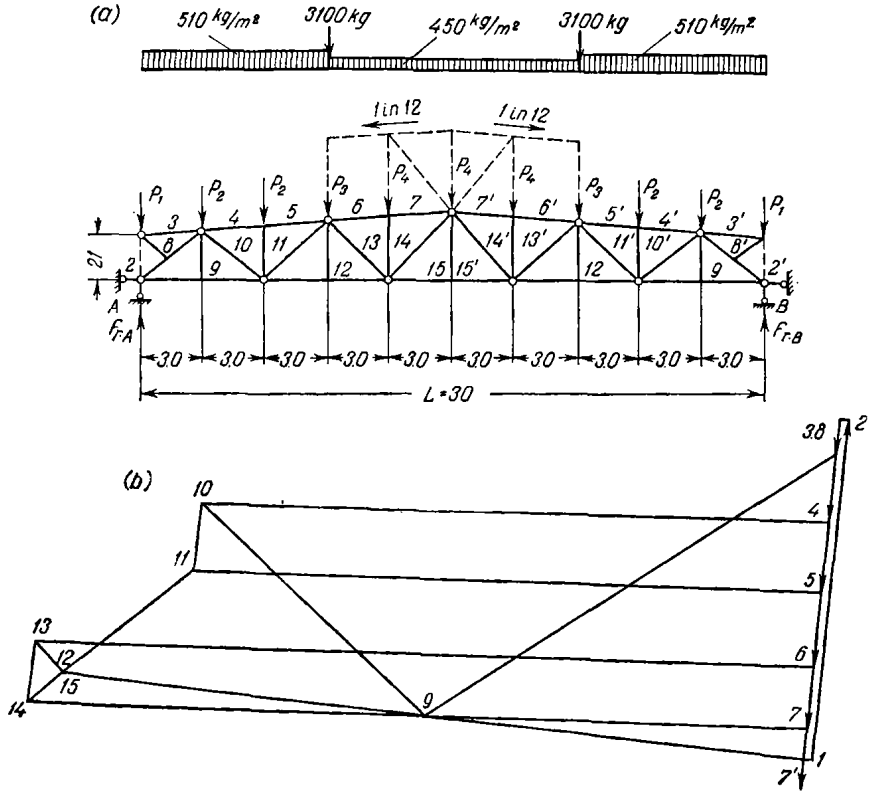


Fig. 7-20. To analysis of roof truss

TABLE 7-6 **Selection of Sections of Roof Truss Elements**

Name of element	Designation of element	Design forces, tons	Section	Area A, cm²	Effective lengths, cm			Radii of gyration, cm	Slenderness ratios		φ_{min}	k_s	Stress, kg/cm²
					L_x	L_y	r_x	r_y	λ_x	λ_y			
Top chord	4-10	−165	⊓200×12	94.2	300	300	6.22	8.69	48	35	0.896	1	1,960
	6-13	−204	⊓250×160×16	127.2	300	600	4.58	11.93	66	50	0.830	1	1,930
Bottom chord	1-9	+103	⊥⊥100×14	52.6	600	600	3.00	4.75	200	127		1	1,960
	1-12	+197	⊥⊥200×125×16	99.6	600	1,800	3.52	9.77	171	185		1	1,980
	1-15	+197	⊥⊥200×125×16	99.6	600	1,800	3.52	9.77	171	185		1	1,980
Diagonals	8-9	−131	⊓160×12	74.8	191	362	4.94	7.09	39	54	0.878	1	2,000
	9-10	+79	⊓125×8	39.4	306	382	3.87	5.60	79	68		1	2 000
	11-12	−44	⊓125×8	39.4	332	414	3.87	5.60	86	74	0.714	0.8	1,950
	12-13	+11	⊓ 75×5	14.78	332	414	2.31	3.56	144	116		1	744
	14-15	+13.5	⊓ 75×5	14.78	360	449	2.31	3.56	156	126		1	912
Verticals	10-11	−19.3	⊓ 80×6	18.76	208	260	2.47	3.80	85	69	0.720	0.8	1,780
	13-14	−21.5	⊓ 90×6	21.2	248	310	2.78	4.18	89	74	0.696	0.8	1,820
Strut	—	0	⌊ 75×5	7.39	157		r_{min}=1.49		λ_{max}=105				

The thickness of the gussets has been taken from Table 7-3, namely, t=16 mm for the support gussets, and t=14 mm for all the remaining ones.

The weight of one truss is 6.2 tons.

The slenderness ratios of the element in and perpendicular to the plane of the truss will be

$$\lambda_x = \frac{L_x}{r_x} = \frac{300}{4.58} = 66 \quad \text{and} \quad \lambda_y = \frac{L_y}{r_y} = \frac{600}{11.93} = 50$$

Using the maximum value of the slenderness ratio found from Table 1 of Appendix II, we find the buckling factor $\varphi = 0.830$. The stress is checked according to expression (2-14)

$$\sigma = \frac{F}{\varphi A_{gr}} = \frac{204{,}000}{0.830 \times 127.2} = 1{,}930 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

We check the stresses for the remaining elements of the truss in a similar manner, introducing for the compression diagonals (except the support ones) a service condition factor of $k_s = 0.8$ (see page 281). The results of all the operations involved in the selection of the sections are entered directly in the table (Table 7-6) without any intermediate calculations.

7-5. DESIGN OF TRUSSES. DETAILS OF JOINTS

(1) Centering of Elements. Configuration and Fastening of Gussets

The design of a truss is commenced with drawing of the centre lines forming the geometrical layout of the member. When doing this, strict attention must be paid to seeing that the centre lines

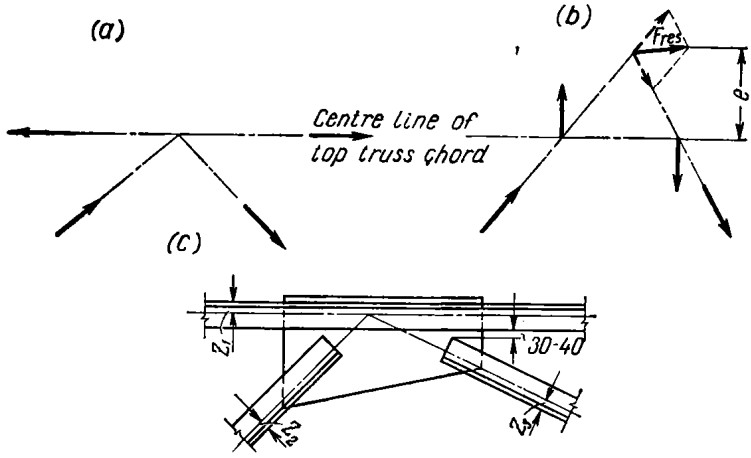


Fig. 7-21. Centering of elements at truss joints

of elements converging at a joint intersect at its centre (Fig. 7-21*a*). Only in this instance is it possible to ensure balancing of the forces converging at the joint. The centering of the elements illustrated in Fig. 7-21*b* is not to be tolerated, since in this instance the forces

converging at the joint will not be balanced, and an additional moment $M = F_{res}\, e$ will appear that will bend the chord. This additional moment cannot be neglected, since it affects the conditions of equilibrium of the joint, and the stresses induced by it relate to the category of principal ones (see Sec. 1-6).

Next the contour lines of the elements are so added to the drawing as to ensure the centre lines coinciding with the centre of gravity of the section or being as close to it as possible. When doing this in welded trusses, the distance z from the centre of gravity to the back end (Fig. 7-21*c*) is rounded off to the nearest greater integer that is

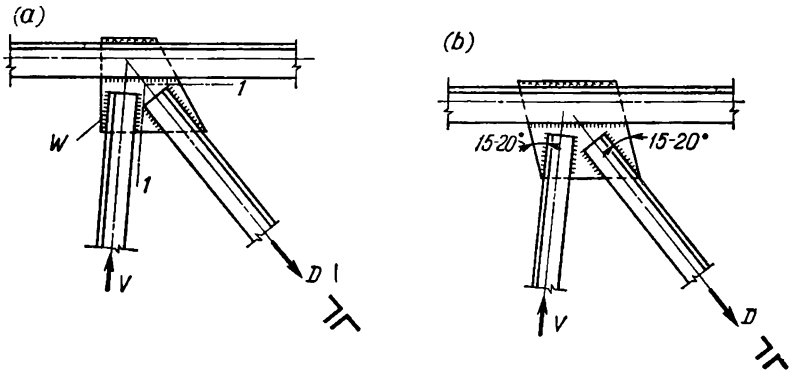


Fig. 7-22. Design of gussets

a multiple of five; in riveted trusses the angles are centred according to the rivet gauge lines. The angles of the web members, as a rule, should be cut off at right angles to the centre line, leaving a space of 30-50 mm between the ends of these elements and the chord. The configuration of the gussets at the joints, which is determined by the conditions of arranging the welds or rivets used for connecting the web elements, should be as simple as possible. Seeing that the gusset transmits the force from one element to another, each of its sections must be strong and capable of withstanding the corresponding force field.

Figure 7-22*a* depicts an improper design of a gusset. The latter has a smaller area along section *I-I* than the sectional area of the diagonal element consisting of two angles being connected at the joint, and may therefore fail. Besides, the weld *W* located at the back end of the vertical angles and taking the greater part of the vertical force cannot transmit it to the gusset, owing to the absence of place for a normal force field. This part of the gusset will be considerably overstressed. Figure 7-22*b* shows a properly designed

gusset having an angle of about 15-20 deg between the edge of the gusset and the respective element.

It is better to secure gussets to chord angles on two sides of the latter—the back and the edge (Fig. 7-23*a*), since otherwise the chord angles may be easily bent back owing to chance causes (for example,

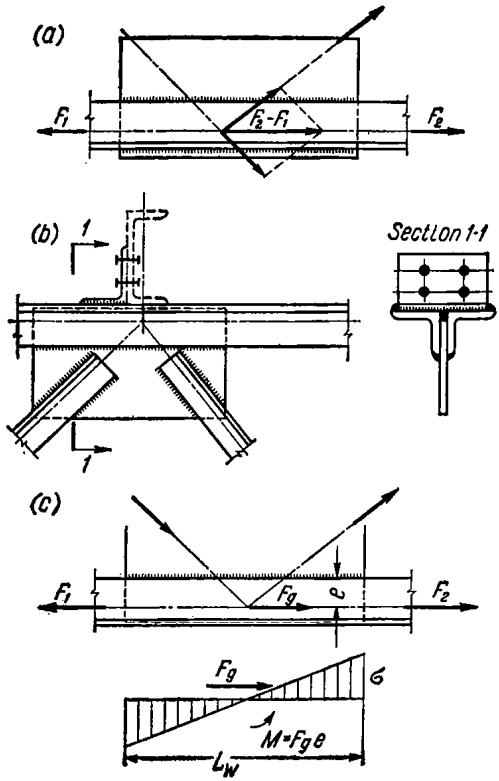


Fig. 7-23. Connection of gusset to truss chord

during transportation). It is not always convenient from the viewpoint of design, however, to let the gusset protrude beyond the surface of the chord, for instance, when installing purlins along the top chord that are connected to short angles (Fig. 7-23*b*). In this case an interval of 5 mm is left between the edge of the gusset and the back of the chord angle, and the gusset is secured only by means of the weld at its leg edge. It is desirable in this instance to weld up the space formed between the backs of the short angles and the gusset, but this weld cannot be considered as one subject to investigation, since it is difficult to ensure its good penetration (the weld is a caulking,

and not a penetration, one). Thus the main load-carrying welds to be investigated in this instance are the welds applied at the chord angle edge.

The force for which the connection of the gusset is investigated, and which tends to displace it with respect to the chord, is the resultant of the forces in the web elements converging at the given joint. In a particular case, when there is no external load applied to the panel point of a straight chord, this force F_g is equal to the difference between the forces in the adjacent panels of the chord (Fig. 7-23*a* and *c*)

$$F_g = F_2 - F_1$$

The force F_g is applied at the centre of the joint and is directed along the chord centre line. If the gusset does not protrude beyond the face of the chord, this force will induce in the welds located at the edge of the chord angles not only shear along their length, but also bending caused by the moment $M = F_g e$ (Fig. 7-23*c*).

Generally the normal stresses originated by bending are not large, and for this reason the weld is investigated only for shear with a reduced design strength of the weld (by about 15-20%).

The principles of designing riveted joints are in essence the same, only differing in that rivets are used instead of welds.

The dimensions determining the gusset size to be used are dictated, the same as in welded trusses, by the conditions of connecting the diagonals. In this instance use can be made of an additional short angle to connect especially heavy diagonals. Here the number of rivets on one of the legs of the short angle is increased by 50% over the design number. The minimum number of rivets that may be used to connect an element at a joint is two. For this reason in the connections of elements with small stresses the rivets are arranged with the maximum possible pitch (see Sec. 5-2). First the rivets are located along the ends of the connection, and only then are the intermediate dimensions established, if possible, as multiples of five millimetres.

(2) Chord Splices

Owing to the limited length of rolled shapes, as well as with a view to conditions of transportation, it becomes necessary to break up large-span trusses (over 18 metres long) into separate shipping elements, designing the field splices, as a rule, at the middle of the span. Here the main rule of splicing must be observed that the sectional area of the splice plates or straps should never be less than that of the elements being spliced. Truss chord splices may be located both at a joint or in a panel. It will be more convenient to

locate the chord splice at a joint, since part of the gusset is used as a splice plate. The simplest design of a splice is the covering of the chord angles with splice ones having the same section. Figure 7-24*a* pictures a welded splice, and Fig. 7-24*b* a riveted splice of a bottom truss chord. In a welded splice the vertical legs of the splice angle

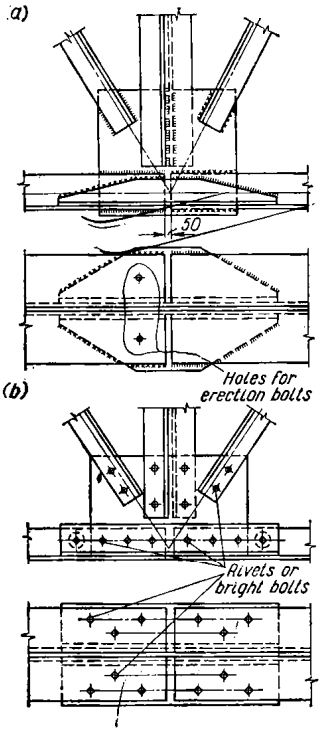


Fig. 7-24. Splices of bottom chord

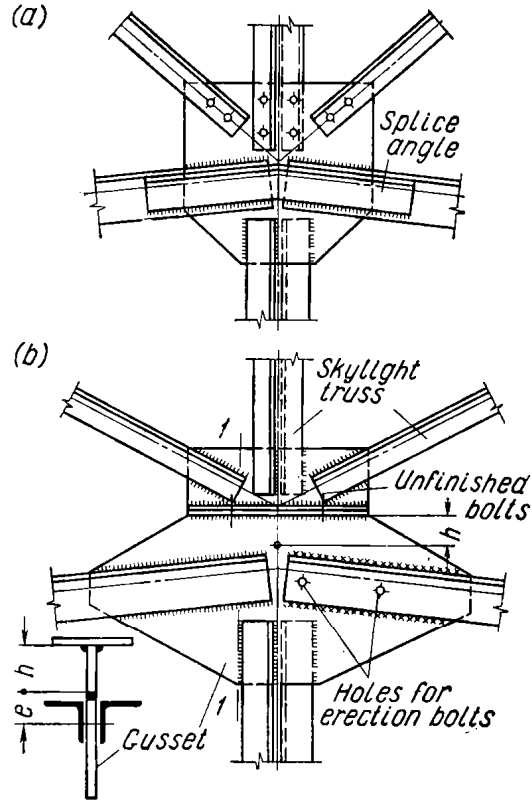


Fig. 7-25. Splices of top chord

are partly cut away in order to avoid concentration of the welds at the edges. To ensure more uniform transmission of the force, it is good to cut long angle legs obliquely.

The splice of a top chord, usually located at the ridge of the truss, may be designed similar to the splice of a bottom chord, covering it with bent splice angles. Such a splice is illustrated in Fig. 7-25*a*, the gusset protruding upward for connection of the skylight member. This splice, which in essence repeats the idea of riveted ones, can also be designed in a different way, as shown in Fig. 7-25*b*. Here the T section of the gusset completely compensates the section of

the two angles. It is only desirable to so appoint the dimension h that the centre of gravity of the T gusset coincides with the axis of the chord angles. If they do not coincide, the gusset must be analyzed not only for compression, but also for bending induced by a moment equal to the axial force in the chord, multiplied by the eccentricity e of the force with respect to the centre of gravity of

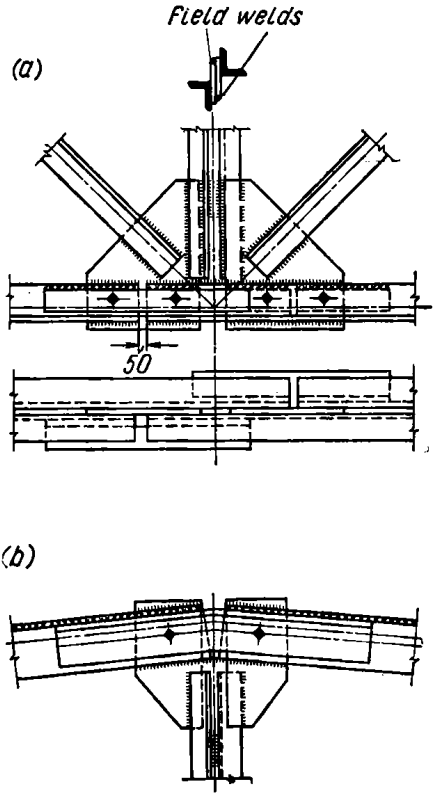


Fig. 7-26. Splices of chords when dividing truss into two identical shipping elements

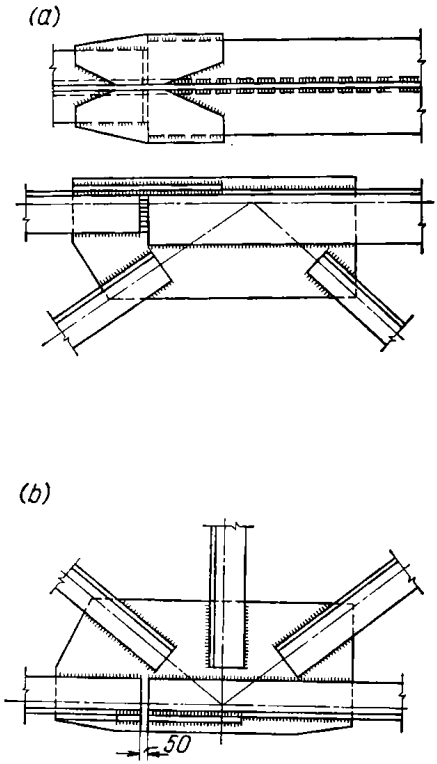


Fig. 7-27. Splices of chords having different sections

the gusset (Fig. 7-25*b*, Section *1-1*). To facilitate application of the welds at the backs of the chord angles, the width of the horizontal plate should not exceed $2h$. The design of the splice pictured in Fig. 7-25*b* is convenient for erection owing to the presence of a horizontal seat on which the skylight member is installed in the form of a separate truss (Fig. 9-26*b*).

To increase the number of identical member elements used and, therefore, to reduce the labour requirements and costs in fabricating

members, the tendency should be to divide trusses into two absolutely identical shipping elements. A design of truss splices meeting this requirement is shown in Fig. 7-26. An excellent solution here is a splice of the bottom chord for trusses with a span of 24 and 36 metres, in which the diagonals converge toward the middle of the chord (Fig. 7-26*a*). Here the gusset is split in the middle, while the splices of the chord angles are staggered. In this design two whole

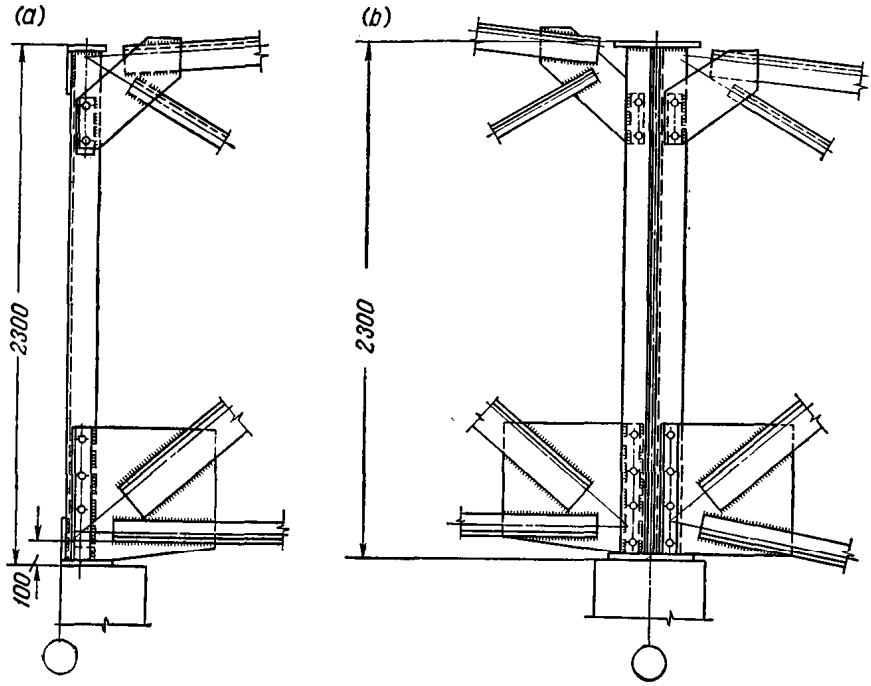


Fig. 7-28. Supporting of roof trusses on reinforced concrete columns

angle sections are obtained along the axis of the chord, while only one angle terminates at the splices, which is covered by the splice angle and the gusset. Figure 7-26*b* shows the corresponding design of a top chord, also with a split gusset. For purposes of resisting the shear force that may appear owing to an unsymmetrical load, twin fillers made from plates (Fig. 7-26*a*) or short angles are provided in the cross-shaped verticals between the vertical angles. Thus the design of the truss splice is absolutely symmetrical.

When angles with a different leg thickness are employed in the chord of a truss, the chords can be spliced with the aid of gussets and plate straps (Fig. 7-27*a* and *b*).

(3) Support Joints

Roof trusses can be carried on reinforced concrete columns, brick walls or steel frame (skeleton) elements of industrial and civic buildings, viz., steel columns or secondary trusses. The design of the connections of trusses to steel columns and secondary trusses is considered in detail in Chapter 9.

An example showing how a roof truss is supported on a reinforced concrete column is illustrated in Fig. 7-28*a*. The bearing plate of the truss, generally 16-20 mm thick, is secured to the column by means of anchor bolts with a diameter of 22-24 mm. The dimensions of the plate are determined on the basis of the design compressive strength of the support material.

To standardize the design of the support joints of trusses resting on reinforced concrete and steel columns, as well as on secondary trusses, the support joint depicted in Fig. 7-28*b* can be used. Here the support load is transmitted through the milled edge of the support gusset, strengthened with side straps, onto special column caps. The bearing plate of the column cap after installation is welded to the plate of the column head-piece. The trusses are connected to the column caps by means of unfinished bolts.

For trusses with a span up to 42 metres there is generally no requirement that the support connections be movable.

(4) Details

As has already been indicated, compression elements of trusses consisting of two angles should be connected to each other in the intervals between the gussets by means of small connecting plates (Fig. 7-29*a*). If this is not done, the longitudinal compressive force F may cause each angle, which resists a force of $F/2$, to buckle

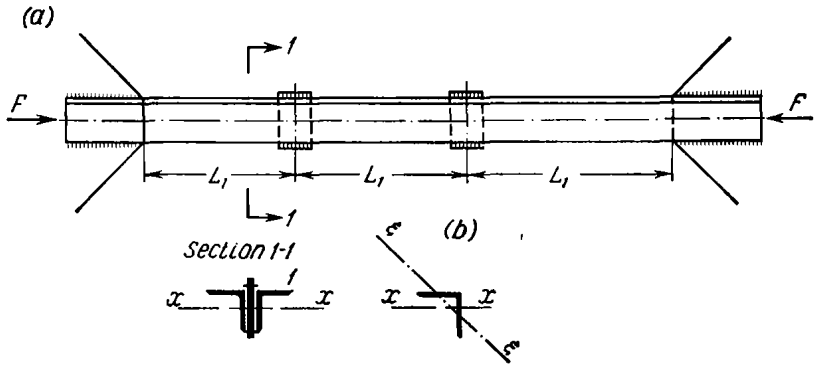


Fig. 7-29. Arrangement of connecting plates in truss elements

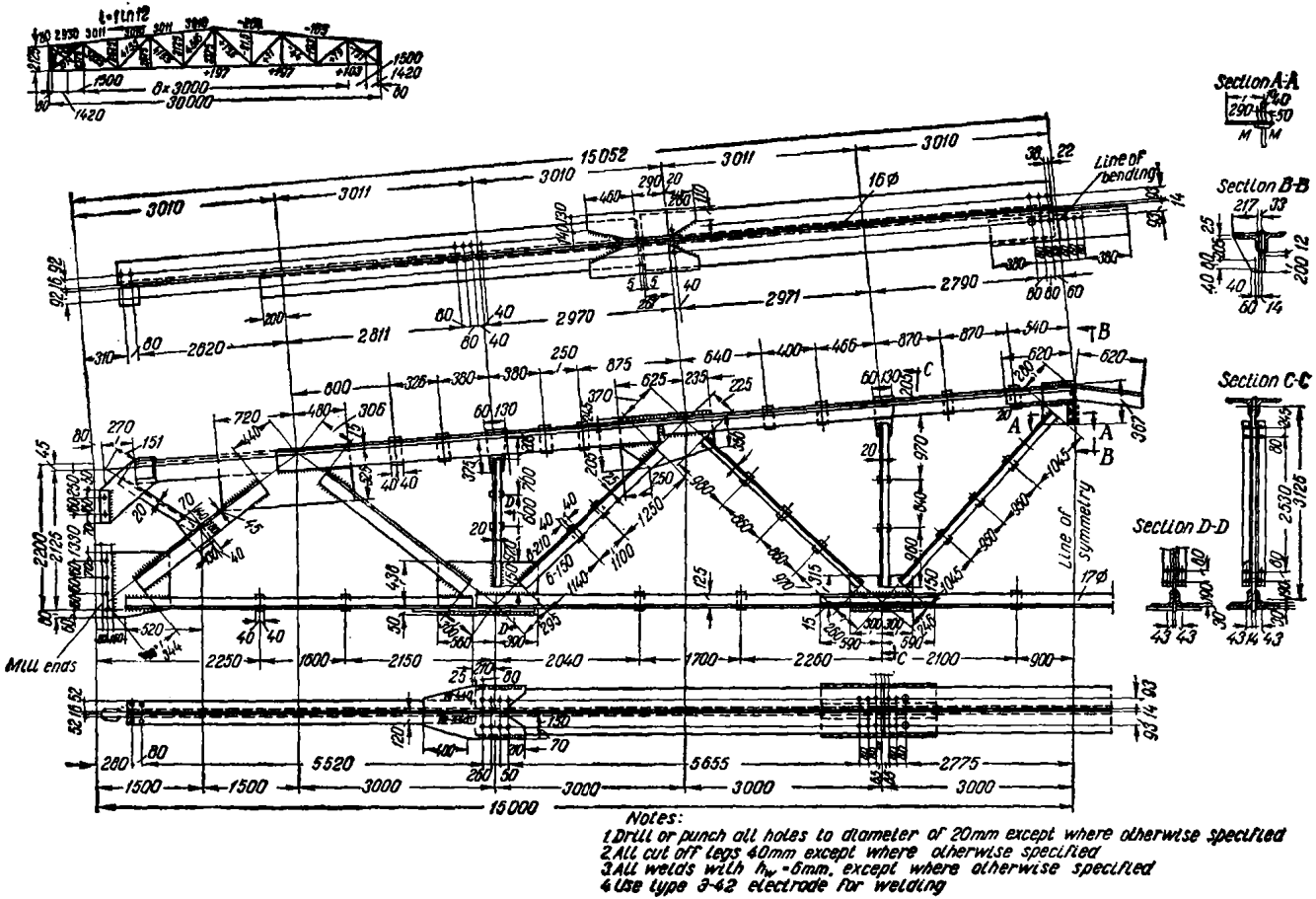

Section A-A
Section B-B
Section C-C
Section D-D
Line of bending
Line of symmetry
Mill ends
15052
15000
Notes:
1 Drill or punch all holes to diameter of 20mm except where otherwise specified
2 All cut off legs 40mm except where otherwise specified
3 All welds with h_w -6mm, except where otherwise specified
4 Use type 3-42 electrode for welding

Fig. 7-30. Working drawing of truss

independently of the other one, since in a single angle the minimum radius of gyration with respect to the axis ξ-ξ (Fig. 7-29*b*) is considerably smaller than that of the section made from two angles with respect to the axis *x*-*x*. These plates are arranged along the length of the compression elements at distances from each other of $L_1 = 40r$ (where r is the radius of gyration of an angle with respect to the axis parallel to the plane which the plates are arranged in). To secure better common stressing of both angles of tension elements in trusses, connecting plates are also installed, but spaced at distances of $L_1 \leqslant 80r$.

The plates are usually made 60-100 mm wide.

Figure 7-30 is an example of a working drawing of a roof truss that illustrates the material contained in the present Chapter.

CHAPTER EIGHT

Columns

8-1. TYPES OF COLUMNS

Columns serve to transmit the load from members located above them through the footings to the soil. Depending on how the load is transmitted, axially loaded and eccentrically loaded columns are distinguished.

Axially loaded columns resist a longitudinal force applied along the axis of the column and inducing uniform compression of its cross section.

Eccentrically loaded columns, besides axial compression induced by the longitudinal force, also withstand bending originated by a moment.

Every column consists of a body or shaft, which is the main load-carrying element, a head-piece (capital) serving as a support for the member above the column and distributing the load over the section of the column, and a base (footing) that distributes the concentrated load from the column over the surface of the foundation and secures the column in the foundation.

Columns are divided with respect to type into columns with a constant and with a changing section in height, as regards the design of the shaft section into solid and open-web columns, and with respect to the method of fabrication into welded and riveted ones.

8-2. AXIALLY LOADED COLUMNS

In axially loaded columns, the load is applied either directly to the centre of the column section (Fig. 8-1*a*), or symmetrically with respect to the axis of the shaft (Fig. 8-1*b*). When designing axially loaded columns, attention should be devoted to ensuring equal stability of the column, i. e., equal slenderness ratios of the column with respect to the principal axes of the section.

The required sectional area of a column shaft is found from the basic expression for analyzing compression members [see expression (2-14)]

$$\sigma = \frac{F}{\varphi A} \leqslant R \qquad (8\text{-}1)$$

namely

$$A_{req} \geqslant \frac{F}{\varphi R} \tag{8-2}$$

With a given design load F acting on the column and a design strength R, the minimum area A_{req} will be obtained in such a column

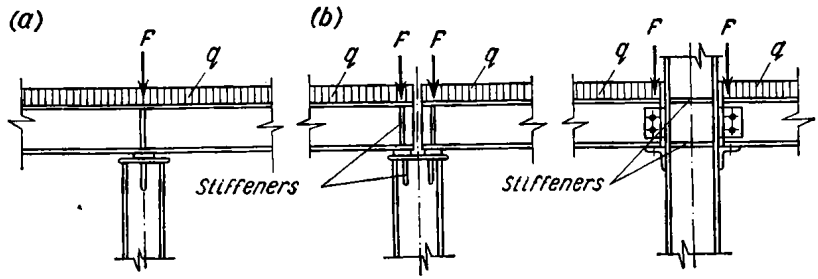


Fig. 8-1. Axially loaded columns

whose buckling factor φ is the highest. The effective length of a column (see Sec. 2-4, and also Table 2-4)

$$L_e = kL$$

depends upon the character of the restraint of the column ends.

Let us consider axially loaded solid and open-web columns.

(1) Solid Columns

Types of Sections. A solid column shaft is formed of one or several rolled shapes or plates connected together by welding or riveting. The types of sections of solid columns are shown in Fig. 8-2. The most rational section from the standpoint of behaviour of the material is a tubular one (Fig. 8-2*a*), which, however, is seldom used in practice. The principal section of solid axially loaded columns is a welded I section built up of three plates (Fig. 8-2*b*), although in such a section it has not been found possible to completely adhere to the condition of equal stability. A single rolled I section is rarely employed as a compression element owing to the considerable difference between the moments of inertia I_x and I_y. It may be used as an independent section only in columns that are braced in height perpendicular to the y-axis. Otherwise it requires strengthening with plates (Fig. 8-2*c*).

Welded I sections built up of three elements (Fig. 8-2*b* and *c*) can be manufactured with the wide use of automatic welding; the

accessibility of all the surfaces of the column simplifies the design of connections with adjacent elements.

Use is sometimes made of sections consisting of three rolled shapes (Fig. 8-2*d*). Such sections, however, are heavier than the ordinary ones.

Solid riveted columns consist of plates and angles (Fig. 8-2*e*).

Analysis and Design of Solid Column Shaft. The analysis of a column is commenced with the determination of the loads acting

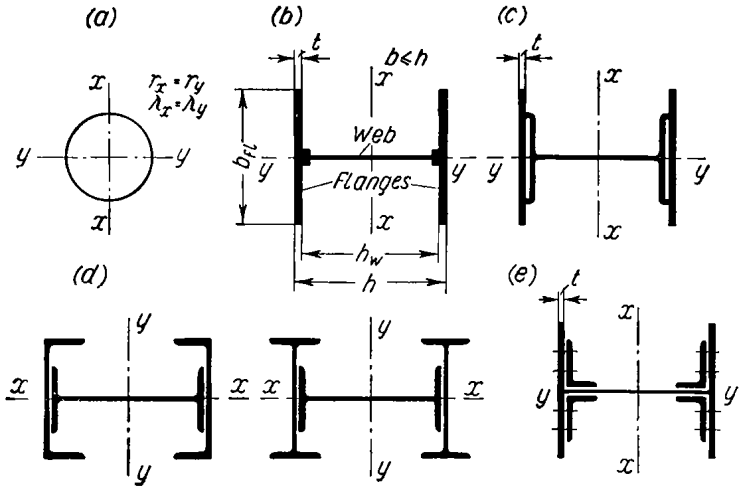


Fig. 8-2. Sections of axially loaded solid columns

on it. Next, for purposes of selecting the section, the required sectional area of the column shaft is computed from expression (8-2). For this end an approximate value of 0.75 to 0.85 is preliminarily assigned to the buckling factor φ.

The dimensions of the section are established with a view to the following considerations. Plates with a thickness of $t=8$ to 40 mm are used for the flanges, and with a thickness of $t=6$ to 16 mm for the web, depending on the size of the column. The depth of the column section h in structures of an ordinary type, i. e., with a height of the column $H=10$ to 20 metres, is taken equal to at least $h=\left(\frac{1}{15}\right.$ to $\left.\frac{1}{20}\right)H$.

The width of the flange plates should be so selected that the plate will not lose its stability under the action of the normal compressive stresses (see Fig. 6-26*c*). This requirement is in essence similar to that indicated for selecting the section of beam compression flanges (see page 174). In columns, however, it is obviously desirable to

have the loss of local stability of the flanges occur with critical stresses somewhat higher than those of the column as a whole, and these stresses, as is known, are a function of the column slenderness ratio. For this reason the building standards give the maximum design width of overhang of the plate (flange) depending on the slenderness ratio of the column (see Table 8-1).

TABLE 8-1 **Maximum Ratios Between Overhang of Plate $\frac{b_{fl}}{2}$ and Its Thickness t_{fl}**

Grade of steel	Values of overhang-to-thickness ratio with slenderness ratio λ equal to				
	25	50	75	100	125
Steel 3 and steel 4	14	15	16.5	18	20
14Г2, 15ГС, 10Г2С, 10Г2СД and 15ХСНД	12	13	14.5	16.5	18.5
10ХСНД	11	12.5	14	16	17.5

In a similar way the slenderness ratio of the column web, i. e., the maximum ratio between the design depth of a solid column web and its thickness (see page 194) depends on the degree of its restraint in the flanges. The latter, in turn, depends on the slenderness ratio of the column as a whole. For this reason the maximum slenderness ratio of a solid column web, as stipulated by the building standards, is determined from the equation

$$\lambda_w = \frac{h_w}{t} = 40\sqrt{\frac{2,100}{R}} + 0.2\lambda \quad (8\text{-}3)$$

but should not be taken over 75.

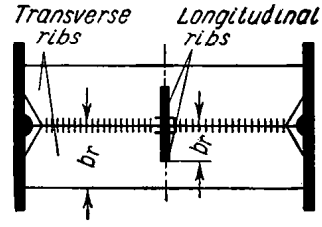


Fig. 8-3. Stiffening ribs of column

With understressing of a column, the maximum values of the ratios $\frac{b}{2t_{fl}}$ and $\frac{h_w}{t}$ can be increased $\sqrt{\frac{R\varphi}{\sigma}}$ times (here $\sigma = \frac{F}{A}$ and φ is the buckling factor).

With ratios exceeding the values obtained from equation (8-3), stability of the web is not ensured and it must be braced with twin longitudinal stiffening ribs (Fig. 8-3). It is recommended to include the ribs in the design area of the section. Besides, with $\frac{h_w}{t} \geqslant 70$

transverse ribs are installed that are spaced along the height of the column at distances of at most $(2.5\text{-}3)h_w$.

The dimensions of transverse and longitudinal ribs in welded columns are established from considerations of design, namely, the width of a transverse rib $b_r \geqslant \frac{h_w}{30}+40$ mm, the thickness $t_r \geqslant \frac{b_r}{15}$, the width of a longitudinal rib $b_r \geqslant 10t_w$ and the thickness $t_r \geqslant 3/4t_w$. The longitudinal ribs prevent wavy buckling of the web, while the transverse ones increase the stiffness of the section by bracing the flanges.

The section of a riveted column is formed of a plate (web) and four unequal leg angles whose short legs are riveted to the web. Such a section is sometimes reinforced with flange plates (Fig. 8-2*e*). The ratio between the width of the unframed overhang of the plate (counting from the nearest row of rivets) and its thickness should not exceed the values contained in Table 8-1.

The webs of riveted columns are braced with stiffening ribs (made from angles) in the same way as welded columns.

Having selected the dimensions of the section that comply with the design requirements, the actual slenderness ratio λ and the buckling factor φ corresponding to it are determined for the column, after which the stresses are checked by means of expression (8-1).

To ensure normal service, the building standards limit the maximum slenderness ratio for columns and their elements (Table 8-2).

TABLE 8-2 **Maximum Slenderness Ratio of Columns**

Name of member	Elements of member	Maximum slenderness ratio
Columns and verticals	Principal	120
Ditto	Secondary (verticals of wall framework, verticals of skylights, elements of column lattices)	150

In welded columns the elements of the section are connected to each other by means of continuous welds, taking the size of the weld $h_{weld} \cong 0.5t_{web}$ (6 to 10 mm). When automatic welding is used, the flange welds should have a uniform size over the entire length of the column, whereas with manual welding it will be good policy to increase the size of these welds at the connections of beams, girders and collar beams, as well as at the base (over lengths of about one metre).

In riveted columns the flange rivets should be arranged with the maximum pitch, but not exceeding $18t_{min}$, where t_{min} is the lowest value of the thicknesses of the elements being connected.

Example 8-1. Select the section of a column shaft resisting a design load of $F=292$ tons applied vertically at the centre of the head-piece. The height of the column $H=7.2$ m. The column has a base with fixed restraints in both directions, while the top end is pin-connected. The shaft is a welded I section (Fig. 8-4) made from grade ВСт. 3кп steel.

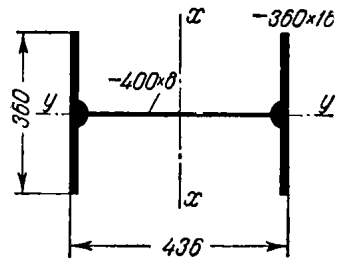


Fig. 8-4. To example 8-1

Solution. In accordance with the end conditions of the column, we find the effective length of the shaft

$$L_e = kH = 0.7 \times 7.2 = 5.04 \text{ m}$$

We preliminarily assume the value of the buckling factor $\varphi=0.75$.

From expression (8-2) we find the required sectional area

$$A_{req} = \frac{F}{R} = \frac{292{,}000}{0.75 \times 2{,}100} = 185 \text{ cm}^2$$

We distribute this area A_{req} between the elements of the section:

flanges	2 plates	$36 \times 2 = 144$ cm²
web	plate	$40 \times 1 = 40$ cm²
Total		184 cm²

To determine the slenderness ratio of the column shaft, we compute the moments of inertia and radii of gyration of the section

$$I_x = \frac{1 \times 40^3}{12} + 2 \times 36 \times 2\left(\frac{42}{2}\right)^2 = 68{,}830 \text{ cm}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{68{,}830}{184}} = 19.3 \text{ cm}$$

$$I_y = 2 \times \frac{2 \times 36^3}{12} = 15{,}500 \text{ cm}^4 \quad r_y = \sqrt{\frac{15{,}500}{184}} = 9.2 \text{ cm}$$

The maximum slenderness ratio of the column

$$\lambda_y = \frac{L_e}{r_y} = \frac{504}{9.2} = 55$$

hence the buckling factor $\varphi_y = 0.875$.

The greatest stress corresponding to the maximum slenderness ratio is found from expression (8-1)

$$\sigma = \frac{F}{\varphi A} = \frac{292{,}000}{184 \times 0.875} = 1{,}810 \text{ kg/cm}^2 < R = 2{,}100 \text{ kg/cm}^2$$

Incomplete utilization of the design strength points to an excess of material in the section. For this reason, the thickness of both the flanges and the web is

reduced by 2 mm without changing, for purposes of retaining the stiffness, the width and the depth of the section (Fig. 8-4). We calculate anew the sectional area and the design stress

$$A' = 2 \times 36 \times 1.8 + 40 \times 0.8 = 161.6 \text{ cm}^2$$

$$I_y' = 2 \times \frac{1.8 \times 36^3}{12} = 14{,}000 \text{ cm}^4$$

$$r_y' = \sqrt{\frac{14{,}000}{161.6}} = 9.3 \text{ cm}$$

$$\lambda_y' = \frac{504}{9.3} = 54 \quad \varphi_y' = 0.878$$

$$\sigma = \frac{292{,}000}{161.6 \times 0.878} = 2{,}060 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

(2) Open-Web Columns

Types of Sections and Lattices. An open-web column shaft consists of two or more rolled sections connected to each other in the chord planes with batten plates (Fig. 8-5) or lattices.

The principal advantage of open-web columns is the possibility of ensuring equal stability in them.

Open-web columns are sufficiently economical as concerns the consumption of metal. At the same time more labour must be spent in their fabrication, since the multitude of short welds makes it difficult to employ automatic welding.

The section of an open-web column shaft is generally formed of two channels located with their flanges facing inside the section (Fig. 8-6*a*). The arrangement of the channels with their flanges facing outward (Fig. 8-6*b*), with the same overall dimensions of the section, is less advantageous from the viewpoint of the consumption of metal, and is employed only in riveted columns to make riveting more convenient.

A section made from I shapes (Fig. 8-6*c*) is used only with considerable loads that preclude the employment of channels.

A section built up of four angles (Fig. 8-6*d*) is employed for compression elements having a large length (masts, crane booms, etc.), in which a definite stiffness is required in both directions. Such a section is sufficiently economical, and a comparatively light design is obtained, but the presence of lattices in four planes results in high labour requirements for fabrication.

The lattice of open-web columns is generally designed of single angles with a maximum slenderness ratio of an element of $\lambda = 150$. The lattice used is either of the simple triangular type (Fig. 8-7*a*) or with braces or diagonals (Fig. 8-7*b*). The lattice is secured to the branches of the column either by welding or riveting, it being per-

mitted to centre the angles on the outer edges of the branches. Columns with batten plates (Fig. 8-7*c*) are simpler to fabricate, have no outstanding lattice angles, and their appearance is better; columns with lattices are considerably stiffer, especially against torsion.

Behaviour of Open-Web Column Shaft Under Load. The two branches of an open-web column shaft are connected by means of batten plates or lattices into a single member. If there were no such connection, each branch would be subjected under load to buckling with respect to its own axis (Fig. 8-8*a*, axis *1-1*). When batten plates or lattices are installed, the stiffness of the shaft as a whole grows considerably, seeing that both branches resist the load together, as a single section, withstanding buckling with respect to axis *y-y* (Fig. 8-8*a*). This axis, as distinguished from the *material axis x-x*, which intersects the body of the column, is known as the *free axis*. The slenderness ratio λ_x of an open-web column shaft with respect to the material axis *x-x* is equal to the slenderness ratio of one branch with respect to the same axis, since $r_x = \sqrt{\frac{2I_x}{2A}} = \sqrt{\frac{I_x}{A}}$. On the other hand, the slenderness ratio with respect to the free axis *y-y* depends on the distance between the branches (dimension $2a$ in Fig. 8-8*a*).

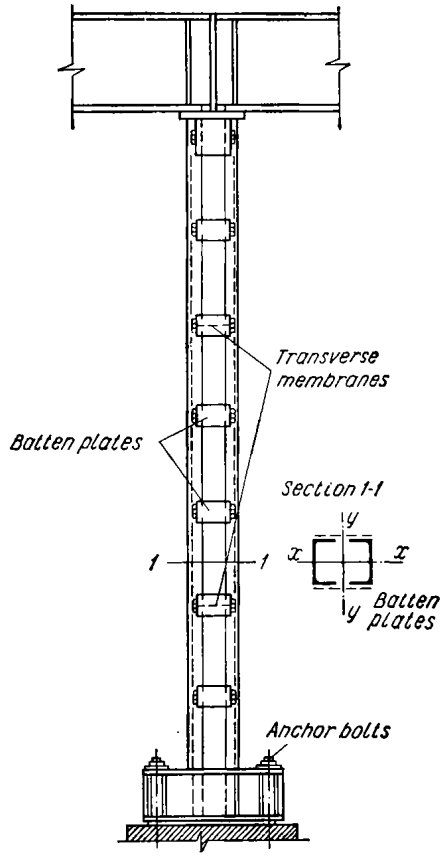


Fig. 8-5. Open-web column with batten plates

The moment of inertia I_y of a section made up of two branches is expressed by the equation

$$I_y = 2\,(I_{br} + a^2 A_{br}) \qquad (8\text{-}4)$$

where I_{br} = moment of inertia of one branch with respect to its own axis *1-1*

A_{br} = sectional area of one branch

a = distance from axis of branch *1-1* to free axis of column *y-y* (Fig. 8-8*a*).

It would seem that the slenderness ratio of a column shaft with respect to the free axis should be found from the equation

$$\lambda_y = \frac{L_y}{r_y} \tag{8-5}$$

where L_y is the effective length of the shaft in respect to the axis *y-y*.

In actual conditions, however, the slenderness ratio of the column with respect to the free axis is found to be greater owing to elastic

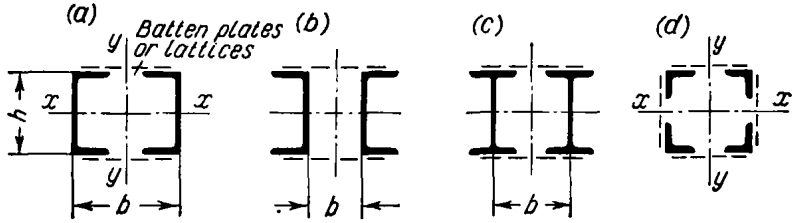


Fig. 8-6. Section of open-web columns

yielding of the batten plates or lattices. This so-called *equivalent* or *reduced* slenderness ratio is equal to

$$\lambda_{red} = k\lambda_y$$

where $k>1$ is a reduction factor for a built-up column shaft depending upon the deformability (yielding) of the batten plates or lattices.

For columns with batten plates

$$k = \sqrt{1 + \left(\frac{\lambda_{br}}{\lambda_y}\right)^2}$$

and for columns with lattices

$$k = \sqrt{1 + \frac{C_a A}{\lambda_y^2 A_d}}$$

Thus the reduced slenderness ratio will be:

for columns with batten plates

$$\lambda_{red} = \sqrt{\lambda_y^2 + \lambda_{br}^2} \tag{8-6}$$

for columns with lattices

$$\lambda_{red} = \sqrt{\lambda_y^2 + C_a \frac{A}{A_d}} \tag{8-7}$$

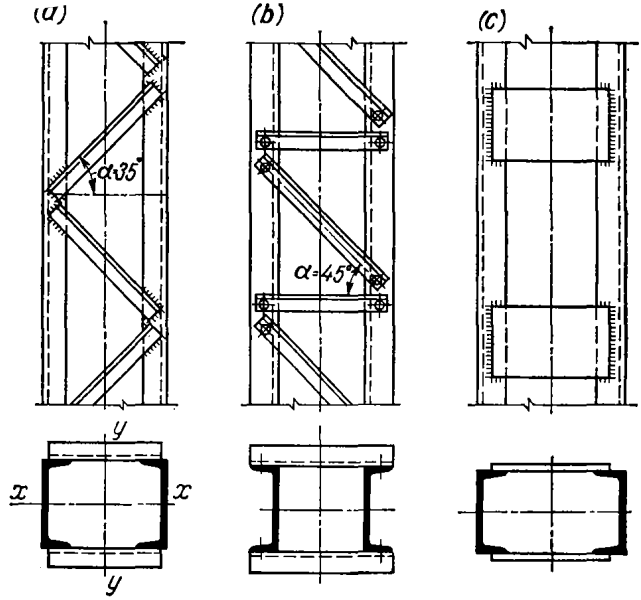


Fig. 8-7. Types of open-web column lattices

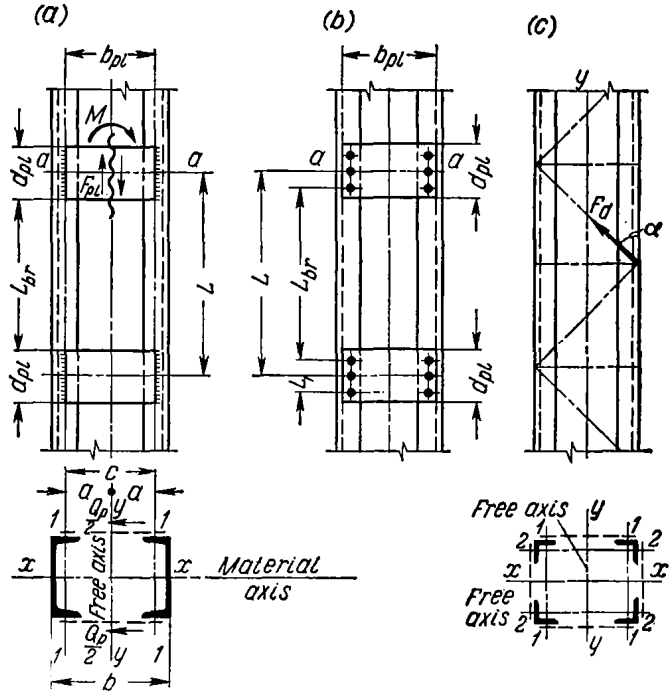


Fig. 8-8. To analysis of open-web axially loaded columns

Here $\lambda_y = \frac{L_y}{r_y}$ = slenderness ratio of entire column shaft with respect to free axis determined from equation (8-5)

$\lambda_{br} = \frac{L_{br}}{r_{br}}$ = slenderness ratio of a part of the branch between batten plates with respect to own axis (Fig. 8-8*a* and *b*)

A = sectional area of entire shaft

A_d = sectional area of two diagonals of a lattice (in two planes)

C_a = coefficient whose value depends upon the angle α between the diagonals of the lattice and the column branch:

with $\alpha =$	30°	40°	45-60°
$C_a =$	45	31	27

The second term in the radicand of equations (8-6) and (8-7) takes into consideration the slenderness ratio of the branches and the yielding of the batten plates or lattice, and thus determines the required location of the latter, since with a change in these quantities the reduced slenderness ratio also changes.

The design slenderness ratio used to determine the buckling factor φ will be the greater of the two ratios λ_x or λ_{red}. By moving apart the branches (i. e., by increasing the distance a in Fig. 8-8*a*) it is easily possible to lower the value of λ_{red} without any noticeable increase in the quantity of metal required, and thus meet the stipulation that $\lambda_{red} \leqslant \lambda_x$; accordingly, when selecting the section of a built-up column, the required slenderness ratio is generally determined with respect to the material axis.

For columns consisting of four branches (Fig. 8-8*c*), the reduced slenderness ratio will be

$$\lambda_{red} = \sqrt{\lambda^2 + A\left(\frac{C_{a\cdot 1}}{A_{d\cdot 1}} + \frac{C_{a\cdot 2}}{A_{d\cdot 2}}\right)} \qquad (8\text{-}8)$$

in which λ = maximum slenderness ratio of the entire column with respect to the free axis

$A_{d\cdot 1}$ and $A_{d\cdot 2}$ = sectional area of two diagonals of lattices located in planes that are respectively perpendicular to axes *1-1* and *2-2*

$C_{a\cdot 1}$ and $C_{a\cdot 2}$ = the same as C_a for the respective planes.

In the branches of open-web columns, the slenderness ratios of the separate branches should not exceed 40 in columns with batten plates and should not be more than the reduced slenderness ratio of the entire column in columns with lattices.

In axially loaded columns the connecting elements—the plates or lattices—are investigated for the lateral (shear) force that may

appear upon bending induced by the critical force (Fig. 8-9*a*), which, as is known, depends for a given material only on the geometrical dimensions of the column shaft. In accordance with the building standards, the magnitude of this fictitious shear force induced by buckling is determined, depending on the section of the shaft, from the equations

$$\left.\begin{array}{l}\text{for steel 3 } Q = 20A_{gr} \text{ kg} \\ \text{for steel 5 and low-alloy steel } Q = 40A_{gr} \text{ kg}\end{array}\right\} \quad (8\text{-}9)$$

where A_{gr} is the gross section of the column shaft, cm^2.

The shear force Q is taken constant over the length of the shaft (Fig. 8-9*b*) and is equally distributed between the planes of the batten plates (lattices).

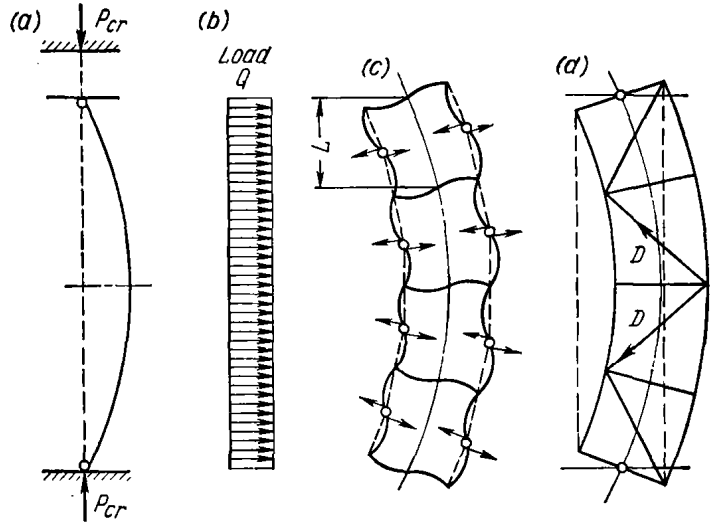


Fig. 8-9. To analysis of columns with batten plates and lattices

Under the action of the lateral (shear) force the column bends. The batten plates resist bending and shear in their plane as elements of a truss without diagonals (Fig. 8-9*c*), while the elements of lattices withstand axial forces, like the diagonals and verticals of a truss (Fig. 8-9*d*). Columns with lattices are less subject to deformation than their counterparts with plates, and for this reason they are the main ones used for heavy loads.

Analysis and Design of Open-Web Column Shafts. As in solid columns, selection of the section of an open-web column shaft is

commenced with calculation of the required sectional area on the basis of the design load and design strength of the material. For this purpose a value of 0.7 to 0.9 is preliminarily assigned to the buckling factor φ. After this, the required sectional area of one branch is found from the equation

$$A_{br \cdot req} = \frac{F}{2\varphi R} \tag{8-10}$$

With a view to this area, the nearest number of channel or I shape is selected from the catalogue of standard shapes, and its slenderness ratio with respect to the material axis x-x is determined. Next expression (8-1) is used to check the design stress in the column for the selected section on the basis of the slenderness ratio with respect to the material axis x-x. The next step is to plan the layout of the section and check it with respect to the free axis. It is essential to so arrange the branches of the section and design the lattice as to comply with the condition

$$\lambda_{red} \leqslant \lambda_x$$

If for columns with batten plates, we assume as the first approximation that its branches are spaced at such a distance $2a$ from each other, whereupon

$$\lambda_{red} = \sqrt{\lambda_y^2 + \lambda_{br}^2} = \lambda_x$$

then in this instance the required slenderness ratio of the column with respect to the free axis will be

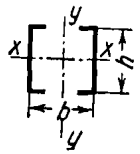
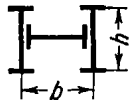
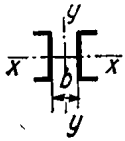
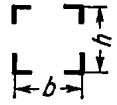
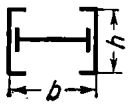
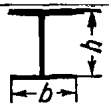
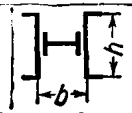
$$\lambda_{y \cdot red} = \sqrt{\lambda_x^2 - \lambda_{br}^2} \tag{8-11}$$

Ordinarily the slenderness ratio of one branch is taken within the limits of $\lambda_{br}=30$ to 40. Having found the value of $\lambda_{y \cdot req}$, the required radius of gyration can be computed from equation (8-5), namely, $r_y = \frac{L_y}{\lambda_{y \cdot req}}$, and the latter can be used to find the required moment of inertia I_y and the corresponding dimension a from equation (8-4).

To facilitate the selection of column sections, Table 8-3 gives approximate values of the radius of gyration for various sections of solid and open-web columns.

The batten plates are so located in columns as to ensure compliance with the previously assumed slenderness ratio of a branch, i. e., $L_{br}=\lambda_{br} r_{br}$. Here the effective length of a branch is taken equal to the clear distance between plates in welded columns (Fig. 8-8*a*), and to the distance between the extreme rivets of adjacent plates in riveted columns (Fig. 8-8*b*).

TABLE 8-3 **Approximate Values of Radius of Gyration for Column Sections**

Section	r_x	r_y	Section	r_x	r_y
	0.38h	0.44b		0.32h	0.49b
	0.38h	0.6b		0.43h	0.43b
	0.32h	0.4b		0.42h	0.24b
	0.32h	0.58b			

The length of a batten plate b_{pl} depends on the distance between the branches. In welded columns, overlapping of the plates onto the branches is about 40-50 mm. The width of the plate d_{pl} is established from the viewpoint of properly locating the welds or rivets used to secure the plate to the branch of the column. The thickness of a plate is taken from 6 to 12 mm, and in heavy columns must be checked by analysis. Besides, it should be seen that the ratio $\frac{b_{pl}}{t_{pl}}$ does not exceed 50.

Batten plates and their connections to the branches of a column are investigated for the shear force $F_{sh.pl}$ and the moment M_{pl} acting in the plane of the plate and appearing in it as the result of the action of the fictitious shear force Q bending the column (Fig. 8-10a). Investigation is conducted with the aid of the equations

$$F_{sh.pl} = \frac{Q_p L}{c} \tag{8-12}$$

$$M_{pl} = \frac{Q_p L}{2} \tag{8-13}$$

in which L=distance between batten plate centres
c=distance between axes of branches

$Q_p = \frac{Q}{2}$ = shear force acting on system of plates located in one plane (Fig. 8-8*a*)

Q = shear force computed from equations (8-9).

The adequate strength of a plate is checked by means of the expression

$$\sigma = \frac{M_{pl}}{W_{pl}} \leqslant R \tag{8-14}$$

The welds securing each end of the plate to a branch of the column are investigated for the action of the resultant stress originated by the action of the moment and the shear force (Fig. 8-10*b*). The following expression is used for this purpose

$$\sigma_{res} = \sqrt{\sigma_w^2 + \tau_w^2} \leqslant R_{w \cdot f} \tag{8-15}$$

where

$$\sigma_w = \frac{M_{pl}}{W_w} = \frac{6M_{pl}}{0.7h_w d_{pl}^2}$$

$$\tau_w = \frac{F_{sh \cdot pl}}{A_w} = \frac{F_{sh \cdot pl}}{0.7h_w d_{pl}}$$

Should the plates be riveted to the branches of the column, they are investigated for the same stresses as their welded counterparts (Fig. 8-10*c*). Here the extreme most heavily loaded rivets are the ones to be analyzed.

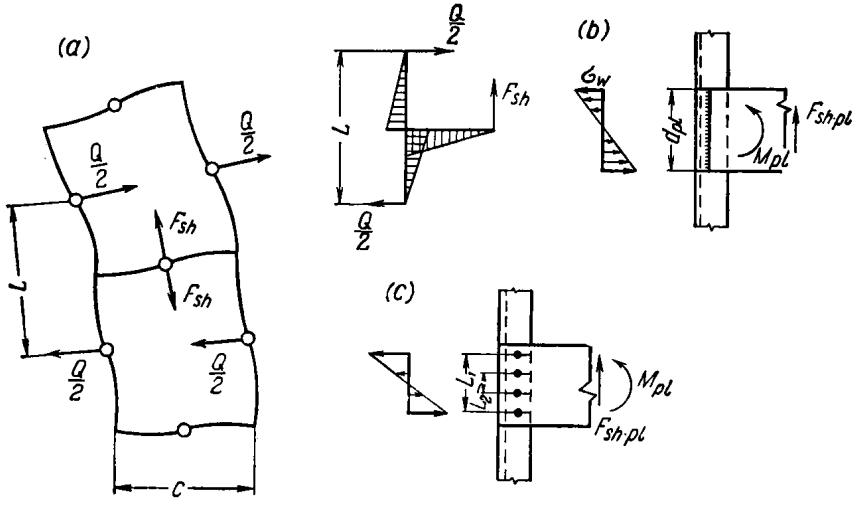


Fig. 8-10. To analysis of batten plate connections

The horizontal component of the force (induced by the moment) shearing the extreme rivet will be

$$F_1 = M_{pl} \frac{L_1}{\sum L^2} \tag{8-16}$$

where $\sum L^2 = L_1^2 + L_2^2 + \ldots$, depending on the number of pairs of rivets symmetrically arranged with respect to the axis of the plate (see page 206).

The vertical component of the force (induced by the shear force) in the extreme rivet is equal to

$$F_{sh \cdot 1} = \frac{F_{sh \cdot pl}}{N} \tag{8-17}$$

where N is the number of rivets on one side of the batten plate.

The design (resultant) force F_r must not exceed the force that can be resisted by one rivet, i. e.,

$$F_r = \sqrt{F_1^2 + F_{sh \cdot 1}^2} \leqslant F'_{r \cdot sh} \tag{8-18}$$

The elements of a lattice are investigated for the axial forces that appear in them owing to the action of the shear force Q induced by buckling.

The compressive force in a diagonal (when the lattices are located in two parallel planes) is determined in the same way as in the elements of a truss lattice

$$F_d = \frac{Q}{2 \sin \alpha} \tag{8-19}$$

where α is the angle between the diagonal and the branch.

As already indicated, for members made from steel 3 the fictitious shear force Q induced by buckling is taken equal to $20A_{gr}$ kilograms (where A_{gr} is the sectional area of the entire column in cm^2). The stress in a diagonal should not exceed the design strength

$$\sigma = \frac{F_d}{\varphi A_d} = \frac{20A_{gr}}{2 \sin \alpha \varphi A_d} \leqslant k_s R \tag{8-20}$$

Here A_d is the sectional area of one lattice diagonal.

When selecting the section of diagonals made from single equal leg angles connected along one leg, the design strength is reduced by application of a service condition factor $k_s = 0.75$. With unequal leg angles whose long leg is connected to the column, the factor k_s is not applied.

With regard to the service condition factor indicated above, the connection of the diagonals to the branches of the column is computed for the force

$$F_{j \cdot d} = \frac{F_d}{k_s} \tag{8-21}$$

in which $F_{j.d}$ =design force of joint
F_d = force in diagonal.

The minimum section of lattice elements employed in welded columns is an angle 45×4, while the minimum section of the lattice elements of a riveted column is dictated by the rivet diameter to be used. The maximum slenderness ratio of the lattice elements is taken equal to $\lambda = 150$.

The lattice diagonals are generally centred on the outer edge of the branch (Fig. 8-8*c*), the diagonals in welded columns being centred on the angle backs (Fig. 8-7*a*), and in riveted ones on the rivet gauge lines (Fig. 8-7*b*).

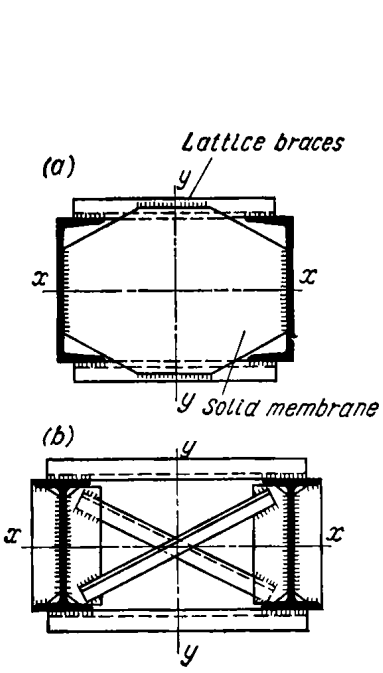


Fig. 8-11. Membranes of open-web columns

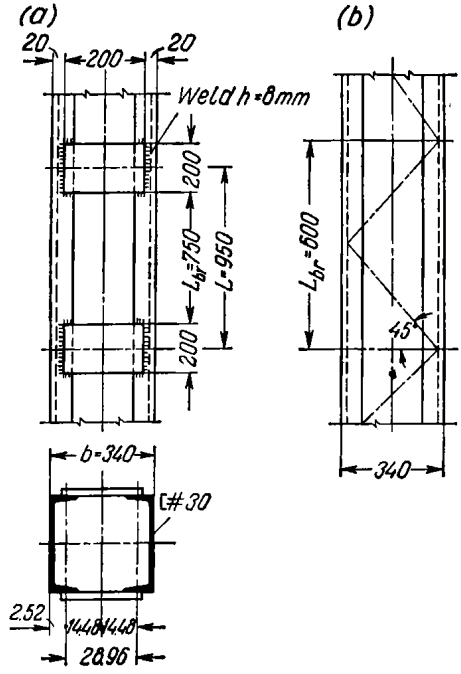


Fig. 8-12. To example 8-2

To prevent twisting of open-web column shafts, transverse membranes are installed at about every four metres along the height of the shaft, regardless of the size of the lattice. They can be solid (Fig. 8-11*a*) or built up of angles (Fig. 8-11*b*).

Example 8-2. Select the section of a welded, axially loaded column consisting of two channels (Fig. 8-12), and also investigate and design two alternatives of the connecting elements , viz., batten plates and a lattice made from angles.

The design load on the column is 145 tons, the height of the column is 6.8 metres. The column is pin-supported at the bottom, and its head-piece is pinned by means of ties to prevent horizontal displacements. The material is grade BCт.3 steel, and type Э42 electrodes are to be used for welding.

Solution. 1. *Selection of Column Section.* We commence selection of the section by determining the required sectional area of one branch from equation (8-10), for which purpose we assume a buckling factor of $\varphi=0.9$

$$A_{br\cdot req}=\frac{F}{2\varphi R}=\frac{145{,}000}{2\times 0.9\times 2{,}100}=38.4 \text{ cm}^2$$

The nearest section in the catalogue of standard shapes is channel No. 30 with a sectional area of $A_{br}=40.5$ cm^2 and radii of gyration of $r_x=12$ cm and $r_y=2.84$ cm.

We check the column for stability with respect to the material axis.

The effective length of the column shaft in both directions is the same and equal to $L_e=kL=1\times 6.8=6.8$ m. The slenderness ratio with respect to axis x-x will be

$$\lambda_x=\frac{L_e}{r_x}=\frac{680}{12}=57 \quad \text{and} \quad \varphi=0.869$$

The stress in the column

$$\sigma=\frac{F}{\varphi A}=\frac{145{,}000}{0.869\times 2\times 40.5}=2{,}060 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

Now let us determine the layout of the section, for which purpose we appoint the distance between the plates. Let us assume a slenderness ratio of a branch of $\lambda_{br}=30$. The distance between the plates, or the length of a branch, will then be $L_{br}=\lambda_{br} r_y=30\times 2.84=85$ cm. To observe the condition of equal spacing along the height of the column (Fig. 8-5), we shall use a length of $L_{br}=75$ cm. To find the minimum distance between branches essential for ensuring equal stability of the column ($\lambda_{red}\cong\lambda_x$), we determine the required slenderness ratio with respect to the free axis of the section from equation (8-11)

$$\lambda_{y\cdot red}=\sqrt{\lambda_x^2-\lambda_{br}^2}=\sqrt{57^2-30^2}=48$$

The required radius of gyration with respect to the free axis will be

$$r_y=\frac{L_y}{\lambda_{y\cdot req}}=\frac{680}{48}=14.2 \text{ cm}$$

From Table 8-3, for a section consisting of two channels, we find $r_y=0.44b$, whence the required distance

$$b=\frac{r_y}{0.44}=\frac{14.2}{0.44}=32.3 \text{ cm}$$

We take $b=34$ cm (Fig. 8-12).

We check the stability of the column with respect to the free axis, for which we must preliminarily find from equation (8-6) the reduced slenderness ratio of the column with the section selected above. First we determine λ_y, after finding I_y and r_y

$$I_y=2\,(I_{br}+a^2A_{br})=2\,(327+14.48^2\times 40.5)=17{,}500 \text{ cm}^4$$

$$r_y=\sqrt{\frac{17{,}500}{2\times 40.5}}=14.7 \text{ cm} \quad \lambda_y=\frac{680}{14.7}=46.2$$

The slenderness ratio of a branch

$$\lambda_{br}=\frac{75}{2.84}=26.4$$

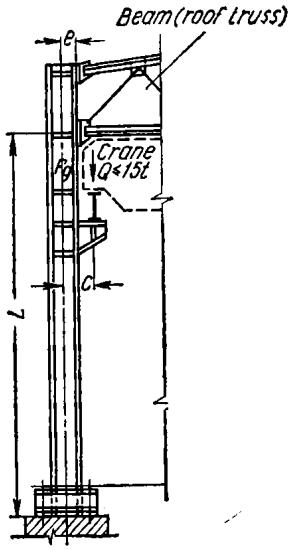


Fig. 8-13. Constant dimension column

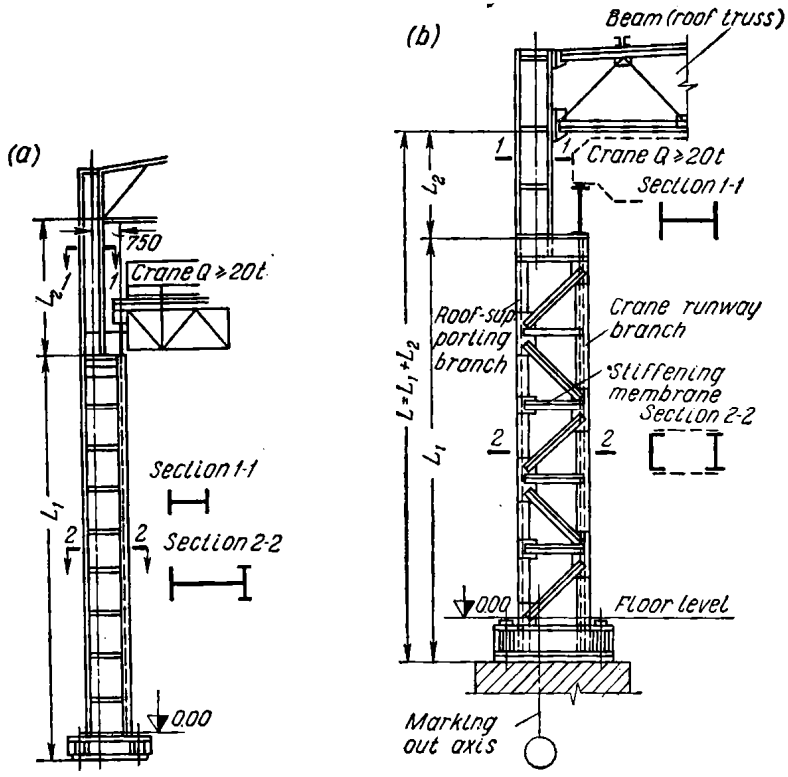


Fig. 8-14. Variable dimension (stepped) columns:
a—solid; *b*—open-web

part, while the upper part with a length of L_2 is termed the *roof-supporting part*. In outside columns, i.e., with the cranes located only on one side, the section consists of the inside crane runway branch directly resisting the crane load, and the outside, roof-supporting branch (Fig. 8-14). In solid columns, both branches are connected by a solid plate, while in open-web ones use is made of lattices consisting of angles arranged in two planes.

3. *Divided Columns* (Fig. 8-15). It will be good policy to use such columns in shops with a heavy crane load (with cranes having a lifting capacity of over 150 tons) and a relatively small height (up to 15-20 metres). The crane runway upright of a divided column is connected to the roof-supporting column by means of a number of horizontal batten plates. Owing to the low stiffness of these plates in a vertical plane, the crane runway upright withstands only axial compression induced by the crane load, without transmitting it to the roof-supporting branch.

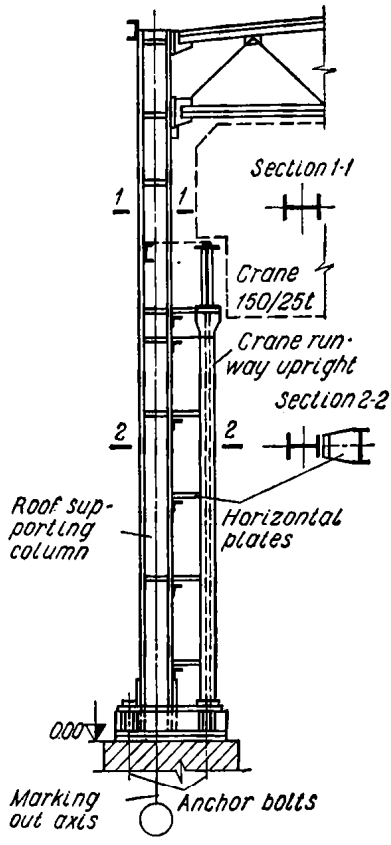


Fig. 8-15. Divided column

(2) Types and Dimensions of Eccentrically Loaded Column Sections

If constant dimension columns are employed, the depth of the section h is usually taken equal to $h \cong \frac{1}{15} L$ for columns with a height of 10-12 metres, $h \cong \frac{1}{18} L$ for columns with a height of 14-16 metres and $h \cong \frac{1}{20} L$ for columns with a height of $L \geqslant 20$ metres. As a rule welded I sections are used in these columns.

In variable dimension columns, the depth of the section of the roof-supporting part is selected within the limits of $\frac{1}{8}$ to $\frac{1}{12}$ of its

height L_2. This dimension with cranes having a medium lifting capacity is generally taken equal to 500 mm, it being increased only with a large value of L_2 and with heavy cranes (Q exceeding 100 tons), as well as when it is necessary to design a passageway through the web of a column (page 372). In such cases the depth of the column

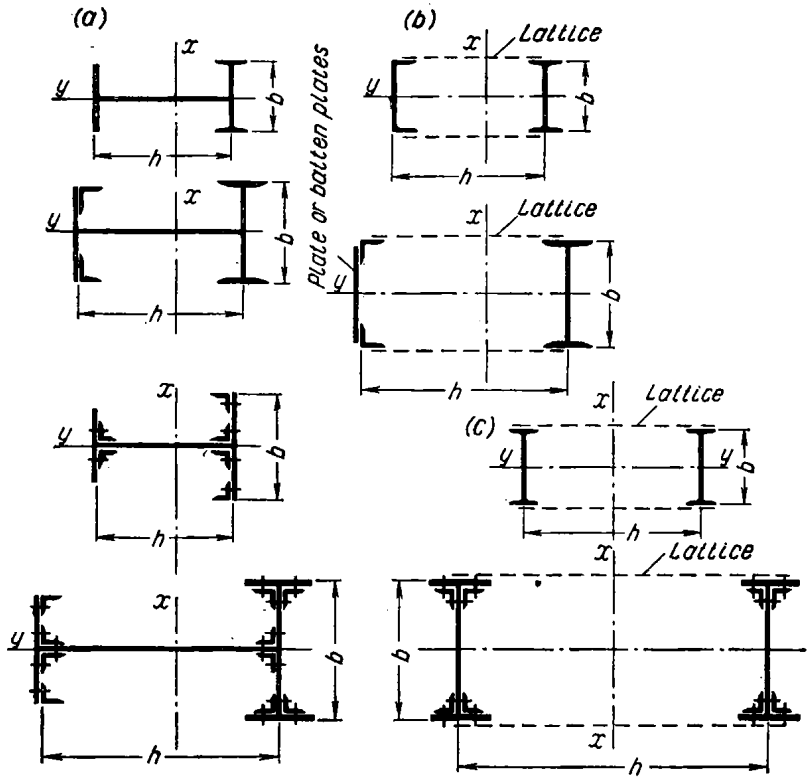


Fig. 8-16. Types of sections of stepped column crane runway parts

section is generally taken equal to 1,000 mm. The axis of the roof-supporting part above the crane runway, as a rule running along the middle of the section, is made to coincide with the marked out centre line of the side structure. The distance from this line to the outer surface of a column is taken equal to 250 or 500 mm.

The section of the roof-supporting part of stepped columns is ordinarily taken in the form of a welded symmetrical I section (Fig. 8-14).

The sections of the crane runway part of stepped columns can be solid (Fig. 8-16*a*) and latticed (Fig. 8-16*b*). The sections of outside

columns having one crane runway branch are unsymmetrical (Fig. 8-16*a* and *b*), while the sections of the middle columns in multispan buildings with cranes having an identical lifting capacity are symmetrical (Fig. 8-16*c*).

The crane runway branches of stepped columns, as a rule, are designed with an I section, whereas the outside (roof-supporting) branch generally has a channel section or is made from a plate with a smooth external surface. This is necessary for convenient connection of the wall panels. Both branches, if possible, should be designed of rolled shapes.

The depth of the section of a stepped column crane runway part is determined by the standard spans of crane bridges, which are multiples of 0.5 metre, and the spans of the shop, which are multiples of 3 metres. The distance D_a between the axis of the crane track and the marking out axis (Fig. 8-17) is in the majority of cases taken equal to 0.75 or 1 metre, in view of the clearances that must be provided between the crane and the column.

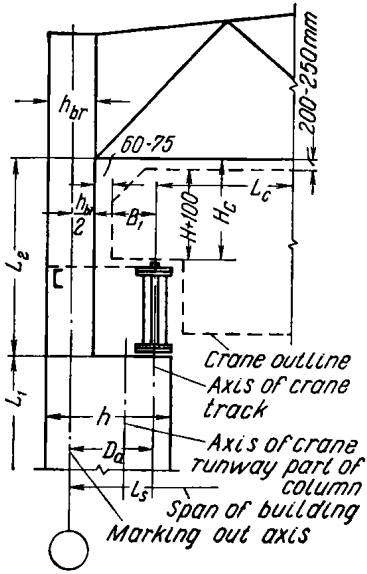


Fig. 8-17. Location of axes and crane clearances in stepped column

The depth of the column section h also depends on the height of the column H, since the latter determines the stiffness of the structure characterized by horizontal deflections. Table 8-4 contains the ratios

TABLE 8-4 **Minimum Ratios Between Depth of Section h and Height of Crane Runway Part of Column L_1**

Height of crane runway part of column L_1, m	Solid columns	Open-web columns
Up to 10-12	$\frac{1}{10}$ - $\frac{1}{14}$	$\frac{1}{9}$ - $\frac{1}{12}$
15-20	$\frac{1}{12}$ - $\frac{1}{16}$	$\frac{1}{11}$ - $\frac{1}{14}$
25-30	$\frac{1}{15}$ - $\frac{1}{20}$	$\frac{1}{13}$ - $\frac{1}{17}$

between the depth of the section h and the height of the crane runway part of the column L_1 generally used in practice, and ensuring the required stiffness of the member.

Solid columns have a somewhat greater stiffness than their open-web counterparts, and are simpler to manufacture; however, when having a width of about 1.2-1.5 metres and above they are less economical.

(3) Analysis and Design of Eccentrically Loaded Column Shafts

Solid Columns. When analyzing columns in which the compressive load is applied eccentrically with respect to the axis of the column, it is always possible to transfer the compressive load to the axis, adding at the same time a bending moment. By the *axis of a column* is meant the line connecting the centres of gravity of the column sections. In variable-dimension or stepped columns the axis is also stepped (Figs. 8-17 and 8-22*b*), which is taken into account when determining the moments acting on a column. Thus the section of an eccentrically loaded solid column is selected and investigated for the action of the longitudinal force F applied along the axis and the moment M, whose values have been obtained by statical analysis of a frame or of a separately designed column.

The type of column to be used, as well as the type and depth of its section, is generally selected when designing the layout of a structure as a whole.

When selecting a section, its area is planned in accordance with the selected depth.

As has been indicated in Sec. 2-6, the stability of an eccentrically compressed bar in the plane of action of a moment can be checked by means of the expression [see expression (2-36)]

$$\sigma = \frac{F}{\varphi_{ec} A_{gr}} \leqslant R \tag{8-22}$$

where φ_{ec} is a factor applied for reducing the design strength to the value of the critical stress of the given element. This factor is determined for columns with a solid section according to Table 7 of Appendix II, depending on the slenderness ratio of the column λ and the equivalent eccentricity m_1.

The equivalent eccentricity m_1 is determined from the equation

$$m_1 = \eta m = \eta \frac{e}{\rho} = \eta \frac{M}{F} \frac{A}{W} \tag{8-23}$$

where $e=\frac{M}{F}$ = eccentricity of the longitudinal force in the plane of bending, the moment M in stepped columns being taken as the maximum one over the length of the part of the column having a constant section

$\rho=\frac{W_{gr}}{A_{gr}}$ = core radius, cm

η = coefficient allowing for the influence of the shape of the section.

When estimating the required section area A_{req}, the core radius can be preliminarily determined by taking the radius of gyration as a function of the depth of the section (from Table 8-3) and assuming a value of the distance z_i between the most compressed branch and the centre of gravity of the section

$$\rho=\frac{W}{A}=\frac{r^2}{z_i} \tag{8-24}$$

The value of z_i for symmetrical sections (columns of middle rows) is equal to $z_i=0.5\,h$, where h is the depth of the section. For unsymmetrical sections (external columns) the value of z_i ranges from $0.45h$ to $0.5h$. The coefficient η for columns varies approximately from 1.2 to 1.3.

Having taken the mean value of $\eta=1.25$, and also $r_x=0.45h$ (see Table 8-3), one may assume for a first approximation when selecting the section of a column that

$$m_1=\eta\frac{e}{\rho}=1.25\,\frac{e\,0.45h}{(0.45h)^2}\cong 2.8\,\frac{e}{h} \tag{8-25}$$

The slenderness ratio of the column can be taken within the limits of $\lambda=50$-90 (on the average $\lambda=70$).

With great eccentricities (m_1 exceeding 4) the influence of the normal force and the value of the column slenderness ratio are reduced, and therefore in this instance it is possible to use the two-term expression

$$\sigma=\frac{E}{A\varphi}+\frac{M}{W}\leqslant R \tag{8-26}$$

This expression can also be used to approximately plan the required sectional area A_{req}.

Assuming that $\varphi=0.8$ and $\rho=0.45h$, we obtain

$$A_{req}=\frac{F}{R}\left(\frac{1}{\varphi}+\frac{e}{\rho}\right)=\frac{F}{R}\left(1.25+2.2\,\frac{e}{h}\right) \tag{8-27}$$

The slenderness ratio of the column λ, as has already been indicated, is determined from equation (2-16)

$$\lambda = \frac{L_e}{r} = \frac{kL}{r}$$

The effective length, determined by the length coefficient k, is the basis of investigations of column shafts for stability and may be considered as the equivalent length of a pinned-end shaft of the same stiffness.

For constant dimension columns with clearly defined end conditions (free upper end, pinned ends, or fixed ends) the length coefficient k is taken from Table 2-4.

The effective lengths of columns that serve as the uprights or posts of lateral buiding frames are determined by solving the equations of stability drawn up for column shafts. Here account is taken of the deformation of their elastic axis as a function of a dimensionless argument $u = L\sqrt{\frac{F}{EI}}$.

The minimum positive root of the equation of stability characterizes the critical value of the force and determines the critical value of the argument u_{cr} and, correspondingly, the length coefficient k.

The coefficients u_{cr} and k are related by the Euler equation for column shafts with different end constraints

$$F_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{(kL)^2} = \frac{u_{cr}^2 EI}{L^2} \qquad (8\text{-}28)$$

in which $k = \frac{\pi}{u_{cr}}$.

As a rule, the coefficients of the stability equations are transcendental functions of the argument u, and these equations have no general solution. Their numerical solutions can be expressed through the parameters of the frame and be represented in the form of tables.

When determining the values of the critical forces for the posts of a frame, a number of assumptions are made that simplify investigations.

1. It is assumed that the frame is loaded only by forces applied at its joints.

2. A single-span frame is considered to be under the simultaneous action of the critical load on both stanchions or columns. This permits designers to disregard the influence of the stiffness and of the load applied to the adjacent column, and to consider the column as a separate shaft either with the bottom end fixed and the top one free (Fig. 8-18*a* and *b*), or pinned at the bottom and elastically constrained at the top (Fig. 8-18*c*).

3. In multiple-bay frames (with two or more spans), in which there is employed a rigid roof, or a system of longitudinal ties or stays for bracing the tops of the columns, it is assumed to be impossible that the loads on all the columns simultaneously reach the critical value. In accordance with this assumption the top support of the frame column is considered as undisplaceable (with a pinned or fixed column-to-beam connection). It is thus also possible in this instance to consider the column as a separate one having different end conditions.

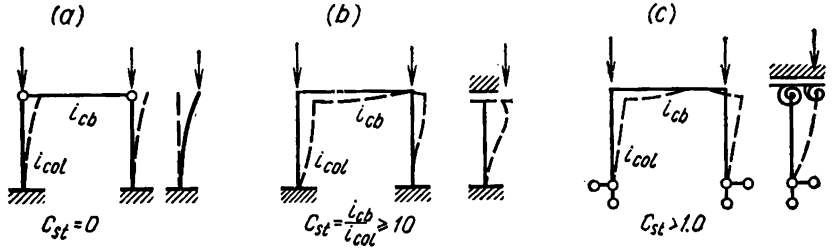


Fig. 8-18. To consideration of single-span frame stability

For columns with constant dimensions that are rigidly (elastically) connected to the collar beam of the frame and have different constraints in the foundation, the coefficient k is taken from Table 8-5 depending on the ratio between the linear stiffness of the collar

TABLE 8-5 **Values of Coefficient k for Constant-Dimension Columns**

End conditions in foundation	Values of k with c_{st} equal to							
	0	0.2	0.3	0.5	1.0	2.0	3.0	≥10
Fixed	2.0	1.5	1.4	1.28	1.16	1.08	1.06	1.0
Pinned	—	3.42	3.0	2.63	2.33	2.17	2.11	2.0

beam and that of the column

$$c_{st}=\frac{i_{cb}}{i_{col}} \tag{8-29}$$

where $i_{cb}=\frac{I_{cb}}{L_{cb}}$

$i_{col}=\frac{I_{col}}{L}$

I_{cb} = moment of inertia of the collar beam at the middle of the span
L_{cb} = span of collar beam
I_{col} = moment of inertia of column
L = height of column up to collar beam.

If a roof truss serves as the collar beam of a frame, then

$$I_{cb} = 0.75\,(A_{t\cdot ch} z_t^2 + A_{b\cdot ch} z_b^2) \qquad (8\text{-}30)$$

in which $A_{t\cdot ch}$ and $A_{b\cdot ch}$ = sectional areas of top and bottom chords

z_t and z_b = distances from centre of gravity of each chord to common centre of gravity of both chords at the middle of the span.

With a pinned connection of a collar beam to a column, the coefficient (ratio) c_{st} is taken equal to zero.

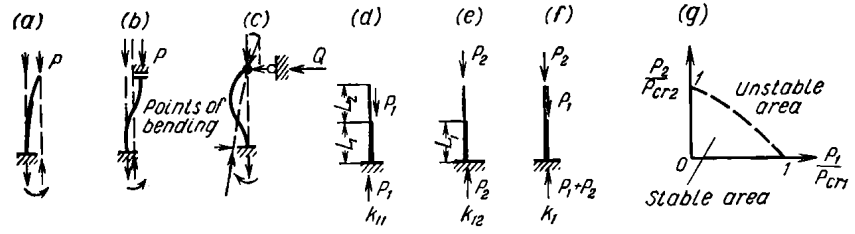


Fig. 8-19. To determination of coefficients k for single-stepped columns

For the stepped columns of single-storey frames in industrial buildings that are fixed in their foundations by means of anchor bolts, the values of the length coefficient are determined separately for the lower part of the column (k_1) and for its upper part (k_2). Here two cases are possible.

In the first case, with single-span frames, the column of the frame is considered as a separate post having free horizontal displacement at the top (similar to Fig. 8-18 *a* and *b*), i. e., it is assumed that the column has no horizontal support reaction and that the shear force $Q = 0$. Here a definite line of the compressive forces is obtained, which is parallel to the undeformed column axis (Fig. 8-19*a* and *b*).

The condition of stability is determined by means of one equation depending on two parameters

$$c_1 = \frac{i_2}{i_1} = \frac{I_2 L_1}{I_1 L_2} \quad \text{and} \quad c_2 = \frac{u_2}{u_1} = \frac{L_2}{L_1}\sqrt{\frac{F_2 I_1}{F_1 L_2}} \qquad (8\text{-}31)$$

where L_1, I_1, F_1 = height, moment of inertia and longitudinal force of lower part of column

L_2, I_2, F_2 = ditto of upper part of column.

The values of the coefficient k_1 are given for different values of the parameters c_1 and c_2 in Tables 10 and 11 of Appendix II, the former being for pinned and the latter for fixed connection of the collar beam to the column.

When considering parts of columns with different values of L, I and F, it is assumed that the ratio between the critical forces in separate parts of the column is the same as between the maximum longitudinal forces acting in the column, i. e., that the column reaches its critical state upon a simultaneous and proportional increase of the loads in both of its parts (simple loading)

$$\frac{F_{cr_2}}{F_{cr_1}} = \frac{F_2}{F_1} = \text{const}$$

It can thus be assumed that

$$c_2 = \frac{u_2}{u_1} = \frac{u_{cr_2}}{u_{cr_1}} = \frac{k_1}{k_2}$$

whence the value of k is determined, which is limited by the actual conditions of service of the column

$$k_2 = \frac{k_1}{c_2} \leqslant 3 \tag{8-32}$$

Since in the analysis of frames different load combinations are possible and thus various values of the forces F are obtained, then when determining the effective lengths, the maximum value of the force F on separate parts of the columns is taken, and the resulting value of k is used for other load combinations.

In frames with ratios of $\frac{L_2}{L_1} \leqslant 0.6$ and $\frac{F_1}{F_2} \geqslant 3$, the values of the coefficient k differ slightly from the mean figures. For this reason, for purposes of simplifying analysis, the building standards permit the value of the length coefficient k to be taken from Table 8-6.

TABLE 8-6 **Values of Coefficient k for Single-Stepped Columns of Single-Storey Industrial Building Frames**

Shear force at top end	Constraint of upper end	For lower part, k_1, with		For upper part, k_2
		$0.3 \geqslant \frac{I_2}{I_1} \geqslant 0.1$	$0.1 \geqslant \frac{I_2}{I_1} \geqslant 0.05$	
$Q=0$	Free end	2.5	3	3
	End fixed only against rotation	2	2	3
$Q \neq 0$	Immovable pin-supported end	1.6	2	2.5
	Immovable end fixed against rotation	1.2	1.5	2

In the second case, when the frames cover two or more bays, the upper end of a column when determining the effective length can be considered as fixed, i. e., the shear force in the upper end of the column $Q \neq 0$ and the line of compressive forces may deviate from the vertical (Fig. 8-19*c*). When there is a step, and two forces P_1 and P_2 (Fig. 8-19*f*) are applied, the condition of adequate stability is determined by means of two equations with transcendental coefficients and two parameters. To avoid the compilation of a multitude of tables for different ratios of P_1 to P_2, the building standards propose an approximate method of investigation consisting in the following.

The column is considered twice, first under the action of the force P_1 alone, the critical force P_{cr_1} and the coefficient k_{11} being determined for the lower part (Fig. 8-19*d*), and then under the action of the force P_2 alone, the critical force P_{cr_2} and the coefficient k_{12} also being determined for the lower part of the column (Fig. 8-19*e*). When both forces are applied simultaneously (Fig. 8-19*f*) the sum of the two ratios between the acting forces and the critical ones will characterize (in a system of coordinates representing these ratios) an area that can be divided into stable and unstable parts.

Indeed, each value of the ratio $\frac{P_1}{P_{cr_1}}$ is less than or equal to unity, and the same is true for the ratio $\frac{P_2}{P_{cr_2}}$. Therefore, upon laying off unity along each of the coordinate axes and connecting the end points by a straight line (to ensure a margin of stability) we obtain a boundary straight line (actually—a convex curve). The stable area will be below this line and the unstable one above it (Fig. 8-19*g*). The equation of this boundary straight line can be written as follows

$$\frac{P_1}{P_{cr_1}} + \frac{P_2}{P_{cr_2}} = 1 \tag{8-33}$$

Assuming that

$$P_{cr_1} = \frac{\pi^2 E I_1}{(k_{11} L_1)^2} \quad P_{cr_2} = \frac{\pi^2 E I_1}{(k_{12} l_1)^2} \quad \frac{P_1 + P_2}{P_2} = m_p$$

$$\text{or } P_2 = \frac{P_1}{m_p - 1}$$

and substituting in equation (8-33), we get

$$P_1 \left[\frac{k_{11}^2}{\frac{\pi^2 E I_1}{L_1^2}} + \frac{k_{12}^2}{\frac{\pi^2 E I_1}{L_1^2} (m_p - 1)} \right] = 1 \tag{8-34}$$

Taking into account that

$$P_1 + P_2 = \frac{P_1 m_p}{m_p - 1} \text{ or } P_1 = \frac{(P_1 + P_2)(m_p - 1)}{m_p}$$

then upon substituting this expression for P_1 in equation (8-34) and assuming that when both forces act simultaneously

$$P_1+P_2=\frac{\pi^2 EI_1}{k_1^2 L_1^2}$$

we get

$$k_1=\sqrt{\frac{k_{11}^2(m_p-1)+k_{12}^2}{m_p}} \qquad (8\text{-}35)$$

For the upper part of columns $k_2=\frac{k_1}{c_2}\leqslant 3$

Thus the coefficient k_1 is determined as a function of k_{12}—the length coefficient of the lower part of the column with $P_1=0$, and of k_{11}—the length coefficient of the lower part with $P_2=0$.

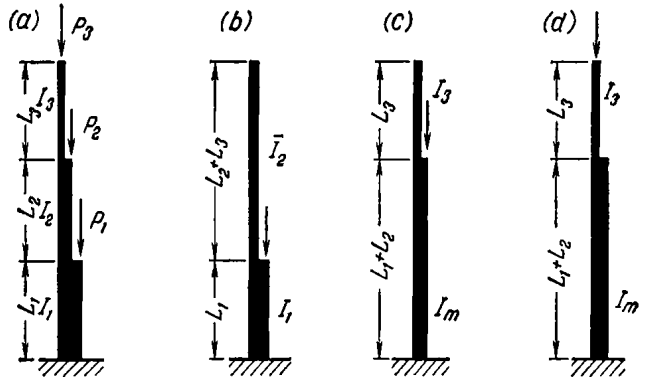


Fig. 8-20. To determination of coefficients k for double-stepped column

The values of k_{12} and k_{11} as a function of the ratios $\frac{L_2}{L_1}=n$ and $\frac{I_2}{I_1}=\beta$ are given in Tables 12, 13, 14 and 15 of Appendix II.

A comparison of the above solutions with the precise one gives a slight deviation towards increasing the margin of stability (within two to eight per cent).

A similar approximate solution is proposed by the USSR Building Standards and Regulations for double-stepped columns (Fig. 8-20*a*). Here the tables drawn up for solving single-stepped columns are used introducing moments of inertia averaged over the length of the parts of the columns.

If we designate (Fig. 8-20*a*)

$$\frac{P_1}{P_3}=s_1 \quad \frac{P_2}{P_3}=s_2 \quad \frac{I_2}{I_1}=\beta_2 \quad \frac{I_3}{I_1}=\beta_3 \quad \frac{L_2}{L_1}=n_2 \text{ and } \frac{L_3}{L_1}=n_3$$

then the coefficient k_1 for the lower part will be

$$k_1 = \sqrt{\frac{s_1\bar{k}_1^2 + (s_2\bar{k}_2^2 + \bar{k}_3^2)(1+n_2)^2\frac{I_1}{I_m}}{1+s_1+s_2}} \qquad (8\text{-}36)$$

where $I_m = \frac{I_1L_1+I_2L_2}{L_1+L_2}$ is the mean value of the moment of inertia for the part of the column L_1+L_2. The coefficients $\bar{k}_1$, $\bar{k}_2$ and $\bar{k}_3$ are determined in the same way as for single-stepped columns according to the diagrams in Fig. 8-20*b*, *c* and *d*, and following the prescriptions contained in Table 8-7.

TABLE 8-7 **Table for Determination of Coefficients $\bar{k}_1$, $\bar{k}_2$ and $\bar{k}_3$**

Shear force at upper end	Upper end conditions	$\bar{k}_1$ (Fig. 8-20*b*)	$\bar{k}_2$ (Fig. 8-20*c*)	$\bar{k}_3$ (Fig. 8-20*d*)
$Q=0$	Free end	$\bar{k}_1=2$	$\bar{k}_2=2$	$\bar{k}_3=k_1$, according to Table 10 of Appendix II with $c_2=\frac{L_3}{L_1+L_2}\sqrt{\frac{I_m}{I_3}}$
	End fixed only against rotation	$\bar{k}_1=k_1$, from Table 11 of Appendix II with $c_2=0$	$\bar{k}_2=k_1$, from Table 11 of Appendix II with $c_2=0$	$\bar{k}_3=k_1$, from Table 11 of Appendix II with $c_2=\frac{L_3}{L_1+L_2}\sqrt{\frac{I_m}{I_3}}$
$Q\neq0$	Immovable pinned end	$\bar{k}_1=k_{11}$, from Table 13 of Appendix II	$\bar{k}_2=k_{11}$, from Table 13 of Appendix II	$\bar{k}_3=k_{12}$, from Table 12 of Appendix II
	Immovable end fixed against rotation	$\bar{k}_1=k_{11}$, from Table 15 of Appendix II	$\bar{k}_2=k_{11}$, from Table 15 of Appendix II	$\bar{k}_3=k_{12}$, from Table 14 of Appendix II

In this case the value of I_2 (Fig. 8-20*b*) is found from the equation

$$\bar{I}_2 = \frac{I_2L_2+I_3L_3}{L_2+L_3}$$

The length coefficient for the middle part of the column length is determined from the equation

$$k_2 = \frac{k_1}{c_3} \tag{8-37}$$

where

$$c_3 = n_2 \sqrt{\frac{P_2 + P_3}{(P_1 + P_2 + P_3)\beta_2}}$$

The length coefficient for the upper part of the column is determined from the expression

$$k_3 = \frac{k_1}{c_4} \leqslant 3 \tag{8-38}$$

where

$$c_4 = n_3 \sqrt{\frac{P_3}{(P_1 + P_2 + P_3)\beta_3}}$$

When designing the section of a solid column, the computed required area A_{req} should be distributed most advantageously, not forgetting to ensure local stability of the separate elements of the section.

The width of a branch (or of the column section) should be sufficient for ensuring general stability of the column in the direction perpendicular to the plane of the frame. This width is generally taken equal to $b = \frac{1}{20}$ to $\frac{1}{30} L_1$ (the height of the crane runway part of the column). The other, less loaded branch is usually made of the same or nearly the same width, with a view to the convenience of securing the column base plate to the column shaft.

To ensure local stability of the section of the column branches it is essential, the same as in axially loaded columns, to take account of the maximum ratios between the overhangs of the plates and their thickness $\left(\frac{b}{t_{ch}}\right)$, which are established by the building standards depending on the slenderness ratio of the column (see Table 8-1).

If an element is understressed the value of the $\frac{b}{t_{ch}}$ ratio obtained from Table 8-1 can be increased $\sqrt{\frac{R\varphi'}{\sigma}}$ times, but by not more than 25% [here φ' is the smaller of the quantities φ_{ec} or $c\varphi_y$, where the factor c is found from equation (8-41)].

The maximum allowable slenderness ratio of the column web $\lambda_w = h_w/t$ is determined depending on the degree of non-uniformity of the stress distribution in the stress diagram, characterized by the factor $k_n = \frac{\sigma - \sigma'}{\sigma}$ (see page 195), and on the ratio $\frac{\tau}{\sigma}$.

Here σ = design stress in the most compressed fibre of the web, determined without applying the factor φ_{ec}

σ' = stress at the opposite edge of the web

$\tau = \frac{Q}{h_w t}$ = mean shearing stress in the section under consideration.

With $k_n \leqslant 0.4$, the maximum value of the $\frac{h_w}{t}$ ratio is taken, as for axially loaded columns, according to equation (8-3), viz.,

$$\frac{h_w}{t} = 40 \sqrt{\frac{2,100}{R} + 0.2\lambda}$$

but not exceeding 75.

With $k_n \geqslant 0.8$ the maximum slenderness ratio of the web is taken equal to

$$\frac{h_w}{t} = 100 \sqrt{\frac{k_{str}}{\sigma}} \qquad (8\text{-}39)$$

where the coefficient k_{str} is taken from Table 8-8.

TABLE 8-8 **Values of Coefficient k_{str}**

$\frac{\tau}{\sigma}$	Values of k_{str} with factor k_n equal to						
	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0	1.88	2.22	2.67	3.26	4.20	5.25	6.30
0.2	1.88	2.18	2.51	2.90	3.40	3.82	4.11
0.4	1.59	1.76	1.93	2.07	2.25	2.43	2.56
0.6	1.31	1.38	1.48	1.60	1.71	1.80	1.86

Within the interval of $0.4 < k_n < 0.8$ the maximum slenderness ratio of the column web is determined by interpolation.

It is not good to design a web thinner than 8 mm. If adequate stability of the web is not ensured, it can be reinforced with a longitudinal twin rib over the whole height of the column, as indicated above for axially loaded columns. Since such a solution is connected with a considerable increase in the labour requirements for fabricating the column, it may be considered, when the stability of the web is not ensured, that the web is not subjected to any loads, except for its extreme parts adjoining the branches with dimensions of $15t$ on each side (Fig. 8-21). Here the column becomes a sort of an open-web one with the web playing the part of the lattice or lacing.

When the slenderness ratio of the web $\frac{h_w}{t} \geqslant 70$ transverse ribs must be installed at distances of (2.5 to 3) h_w from each other. These

will bind the section into a whole, ensuring its high stiffness against twisting of the column.

In heavy columns, membranes are installed over the width of the entire section at intervals of about 4 metres, and are also intended to ensure adequate stiffness of the column.

Besides checking an eccentrically loaded column in the plane of bending, it must always be investigated for sufficient stability in a plane perpendicular to that which the moment acts in. This is performed by means of the expression

$$\sigma = \frac{F}{c\varphi_y A} \leqslant R \tag{8-40}$$

where φ_y = buckling factor taken in accordance with Table 1 of Appendix II depending on the slenderness ratio λ_y in a direction perpendicular to the plane which the moment acts in

c = factor accounting for the influence of the moment on the stability of an eccentrically loaded element, with a view to the bending and twisting form of loss of stability, equal to

$$c = \frac{\beta}{1 + \alpha m_x} \tag{8-41}$$

in which $m_x = \frac{e_x}{\rho_x}$ and $e_x = \frac{M_x}{F}$. Here M_x is taken equal to the maximum moment within the limits of the middle third of the length (but not less than half of the maximum moment over the length of the column) for columns with ends fixed against a displacement perpendicular to the plane which the moment acts in, and to the moment at the constraint for columns with free ends.

The coefficients α and β are taken in accordance with Table 8-9.

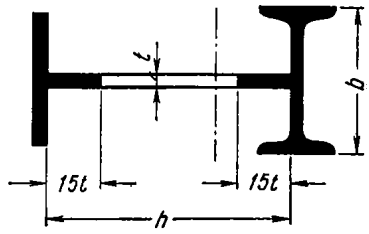


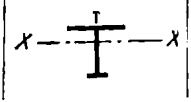
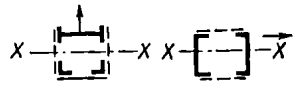
Fig. 8-21. Design section of column with unstable web

In Table 8-9 the quantities I_1 and I_2 are respectively the moments of inertia of the larger or smaller flange relative to the axis of symmetry of the section y-y.

When $\lambda_y \leqslant \lambda_{st}$, and also when $\frac{I_2}{I_1} < 0.5$ and the eccentricity is directed toward the smaller flange, the coefficient $\beta = 1$.

The values of λ_{st} are taken equal to 100 for members made from steel 3 and 4, and to 85 for the group of low-alloy steels with $R = 2{,}900$ kg/cm^2.

TABLE 8-9 **Values of Coefficients α and β**

Type of section and direction of eccentricity	Open sections with eccentricity directed toward		Closed sections with lattices (batten plates) or solid ones
	larger flange or with no eccentricity	smaller flange	
↑			
α	0.7	$1-0.3\frac{I_2}{I_1}$	0.6
β with $\lambda_y > \alpha_{st}$	$\frac{0.6}{\varphi_y}$	$1-\left(1-\frac{0.6}{\varphi_y}\right)\times \left(2\frac{I_2}{I_1}-1\right)$	$\frac{0.6}{\varphi_y}$

When $\lambda_y > \lambda_{st}$ the factor c should not exceed the values given in Table 8-10, while for columns with a closed or box section it should not exceed unity.

TABLE 8-10 **Maximum Values of Factor c with $\lambda_y > \lambda_{st}$**

$\frac{Lt_1}{bh}$	Values of factor c with M/Fh =															
	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	10
0.1	1	0.88	0.69	0.56	0.46	0.39	0.34	0.30	0.27	0.24	0.22	0.18	0.15	0.12	0.10	0.049
0.8	1	0.91	0.77	0.64	0.54	0.47	0.41	0.36	0.33	0.30	0.27	0.23	0.19	0.15	0.12	0.062
1.5	1	0.95	0.85	0.74	0.66	0.58	0.52	0.47	0.43	0.39	0.37	0.30	0.26	0.20	0.18	0.086
≥2.5	1	0.99	0.92	0.85	0.78	0.72	0.66	0.61	0.56	0.52	0.49	0.41	0.36	0.28	0.24	0.126

Notation used in Table 8-10:
h=depth of section;
b and t_1=width and thickness of the more compressed flange.

The effective lengths of columns (when checking stability) in a direction along a building are taken, as a rule, equal to the distances between fixed points (supports or bearings) of the columns, crane girders, joints of tie and collar beam connections, etc.

Open-Web Columns. The lower (crane runway) part of a column is in the majority of cases (when the width exceeds 1.2-1.5 metres)

designed of the open-web type, consisting of two branches connected by lattices (Fig. 8-14*b*).

The analysis of open-web columns is based on the assumption that they behave in the same way as a truss with parallel chords. For this purpose the longitudinal force and the moment acting on the column are distributed among the branches, the forces in which are found from the equation

$$F_{br} = \frac{Fz}{h} \pm \frac{M}{h} \tag{8-42}$$

where z is the distance from the centre of gravity of the column section to the axis of the branch opposite the one being investigated.

In a symmetrical section $z=0.5$, whereas in an unsymmetrical one the distance from the centre of gravity to the branch resisting the greater load varies from $0.4h$ to $0.5h$. Similar to a solid column, the section of the crane runway branch is designed of a rolled or (in very large columns) welded I shape. To ensure adequate general stability of the column in a direction perpendicular to the plane of the frame, the depth of the I shape is selected to be within the limits of $\left(\frac{1}{20} \text{ to } \frac{1}{30}\right)L_1$, which corresponds to a slenderness ratio of $\lambda \cong 60$ to 100.

The roof-supporting or hip branch is generally designed of the channel type, with the same width as the crane runway branch. When no channels of the corresponding number are available, the channel section is made up of angles with batten plates or more frequently with a solid plate (Fig. 8-16*b*).

Both branches are laced together with lattices, usually of the triangular system, arranged in two planes.

The section of the branches is checked by means of expression (8-1) used for an axially loaded column

$$\sigma = \frac{F_{br}}{\varphi A_{br}} \leqslant R$$

Here the buckling factor φ is taken equal to the smaller of two values, the first of which is determined from the slenderness ratio of the branch relative to the axis of the I shape (or channel) having the greater moment of inertia. Here the effective length is taken equal to the distance from the foundation to the crane girder. When finding the second value of φ, the effective length of the branch is taken equal to the distance between joints of the lattice, while the minimum radius of gyration of the branch section is used in the calculations.

The tendency should be to so design the connection of the crane girders to the branch of the column as to avoid eccentrical application of the bearing pressure of the girders with respect to the axis of the

column, i. e., in a direction perpendicular to the plane of the frame. This is easily achieved with continuous girders by aligning the bearing (support) rib strictly along the axis of the column, as well as when the bearing member in light simple girders or beams is designed in the form of bearing plates (see Fig. 6-51*a*). In heavy girders for cranes with a lifting capacity of 100 tons or more, when employing the bearing member illustrated in Fig. 6-51*b*, it is desirable to locate the bearing ribs of the girders closer to their ends to ensure the support reactions being within the limits of the column cross section core. If this is not the case, the branch of the column must also be investigated for buckling in a direction perpendicular to the plane of the frame with a one-sided crane load.

In addition, open-web columns, especially narrow and high ones (whose section depth is less than 1/15th of the length L_1) should be checked for stability on the assumption that the column branches are behaving as a single column with a built-up section. Such a column with lattices or batten plates located in the plane of bending is investigated with the aid of the same expression (8-22), but the buckling factor φ_{ec} is found from Table 8 of Appendix II depending on the reduced slenderness ratio λ_{red} computed from equations (8-6)-(8-8) and the relative eccentricity

$$m_x = \frac{e_x}{\rho} = \frac{M_x}{F} \frac{Ay_1}{I_x} \tag{8-43}$$

where A = sectional area of the entire element

y_1 = distance from the centre of gravity of the section to the axis of the most compressed branch (but not less than the distance to the axis of the branch web)

I_x = moment of inertia of the section.

Each lattice of a column is designed of single angles, the diagonals being arranged at an angle of 45-50 deg to the horizontal. The force in the diagonal of a lattice is determined from equation (8-19) with a view to the maximum actual shear force in the column obtained when investigating the frame, or according to the fictitious shear force (if it is greater) computed from equations (8-9).

The procedure for investigating the diagonal connections and the design requirements are similar to those set forth above for constant dimension columns. The installation of braces reduces the effective length of the column branch. The force in a brace is determined on the basis of the fictitious shear force Q in the corresponding branch.

Example 8-3. Analyze and select the section of a stepped column for a single-bay shop (Fig. 8-22), given the following data:

1. Dimensions: total height of column $L=20.5$ m, height of crane runway part $L_1=15.3$ m, height of part above crane runway (roof-supporting part) $L_2=5.2$ (Fig. 8-22*a*).

2. Design force combinations: in the roof-supporting (upper) part $M_2=+85.6$ t-m, $F_2=70.9$ t; in the crane runway (lower) part $M_1=+346$ t-m, $F_1=189.1$ t, $M_1'=-185.4$ t-m, $F_1'=109.6$ t, $Q=18.92$ t (Fig. 8-22*b*).

The maximum values of the normal forces $F_2=96.1$ t, $F_1=302.1$ t.

In the plane of action of the moment the column is fixed in the foundation and is pin-connected to the collar beam. The members are made from grade ВСт.3кп steel.

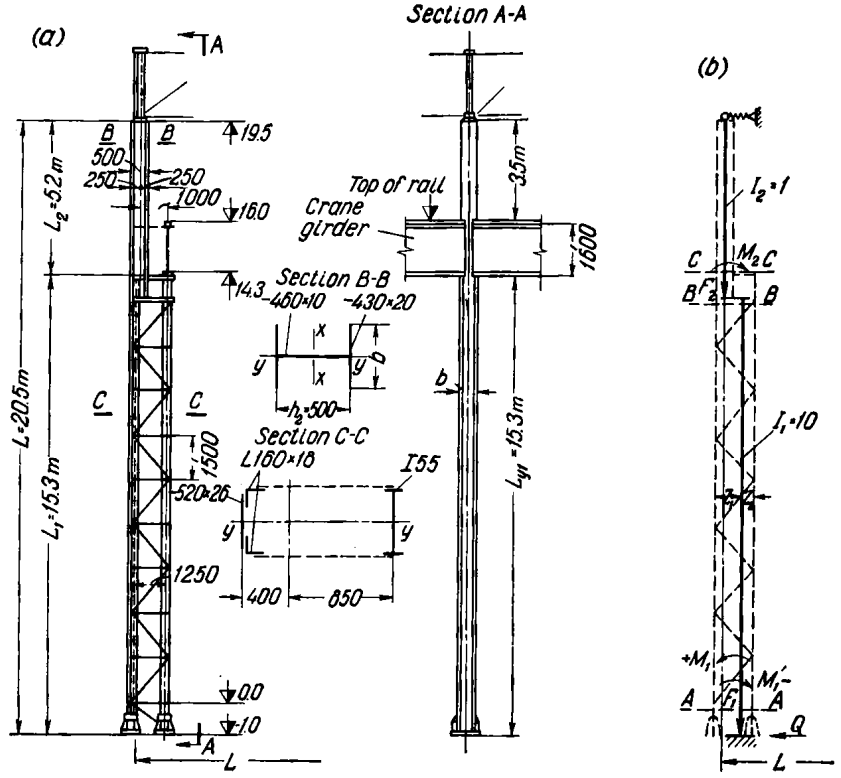


Fig. 8-22. To example 8-3

Solution. 1. *Appointment of Column Section Dimensions.* The type of column and the depth of its sections are selected when designing the layout of the building and its cross section. The following dimensions of the sections have been selected, namely, $h_2=500$ mm for the upper part (which gives $\frac{h_2}{L_2}=\frac{0.5}{5.2}=\frac{1}{10.4}$) and $h_1=1{,}250$ mm for the lower part (which yields $\frac{h_1}{L_1}=\frac{1.25}{15.3}=\frac{1}{12.2}$).

2. *Determination of Effective Lengths.* We determine the possibility of using Table 8-6 for finding the length coefficients. We have $\frac{L_2}{L_1}=\frac{5.2}{15.3}=0.34<0.6$ and $\frac{F_1}{F_2}=\frac{302.1}{96.1}=3.17>3$.

Hence Table 8-6 can be used for finding the length coefficients. This table gives $k_1=2.5$ and $k_2=3$. Thus the effective lengths essential for computing the slenderness ratio of the column in the plane of bending will be

$$L_{x_1}=k_1L_1=2.5\times15.3=38.2\text{ m} \qquad L_{x_2}=k_2L_2=3\times5.2=15.6\text{ m}$$

In a plane perpendicular to the action of the moment (Fig. 8-22*a*) the effective length of the upper part is equal to the distance from the bracing beam to the bottom of the truss: $L_{y_2}=5.2-1.6=3.6$ m, and of the lower part—to the distance from the foundation to the crane girder $L_{y_1}=15.3$ m.

3. *Selection of the Section of the Upper Column Part.* We plan the section of the upper part of the column in the form of a welded I section (Fig. 8-22*a*). For finding the design relative eccentricity m_1 from equation (8-23), we first determine the quantities e, r_x, λ_x, ρ and η, namely, $e=\dfrac{M_2}{F_2}=\dfrac{85.6}{70.9}=1.205$ m or 120.5 cm; from Table 8-3 we find the approximate value of $r_x=0.42h=0.42\times\times50=21$ cm; $\lambda_x=\dfrac{L_{x_2}}{r_x}=\dfrac{1.560}{21}=74.5$.

From the approximate formula (8-25) $m_1=2.8\dfrac{e}{h}=2.8\times\dfrac{1.205}{0.5}=6.64$, and with $\lambda_x=74.5$, from Table 7 of Appendix II we find $\varphi_{ec}=0.165$. Hence the approximate required sectional area will be

$$A_{req}=\frac{F}{\varphi_{ec}R}=\frac{70,900}{0.165\times2,100}=204\text{ cm}^2$$

Another way can be used for approximately determining the required sectional area—see equation (8-24)

$$\rho=\frac{W}{A}\frac{r_x^2}{z}=\frac{21^2}{25}=17.6\text{ cm, where }z=\frac{h}{2}=\frac{50}{2}=25\text{ cm}$$

From Table 9 of Appendix II we find $\eta=1.45-0.003\ \lambda=1.45-0.003\times\times74.5=1.23$;

$k_1=\eta k=\eta\dfrac{e}{\rho}=\dfrac{1.23\times1.205}{17.6}=8.39$. From Table 7 of Appendix II we find φ_{ec} for $\lambda_x=74.5$ and $k_1=8.39$; as a result of interpolation $\varphi_{ec}=0.14$.

Thus $A_{req}=\dfrac{F}{\varphi_{ec}R}=\dfrac{70,900}{0.14\times2,100}=240\text{ cm}^2$.

In view of the necessity of also ensuring stiffness of the column with respect to axis y (with $\lambda_y=50$-60 less than λ_x, seeing that the effective length $L_{y_2}=3.6$ m is less than L_{x_2}), we select the minimum width of the column section b, for which end we find $r_y=\dfrac{L_{y_2}}{\lambda_y}=\dfrac{360}{50}=7.2$ cm. From Table 8-3 we find that $r_y=0.24b$, whence we have $b\geqslant\dfrac{r_y}{0.24}=\dfrac{7.2}{0.24}=30$ cm.

On the basis of the required area we select a section of the web made from a plate 460×10 mm in size, and flanges consisting of plates 430×20 mm in size (Fig. 8-22*a*).

We check the selected section of the upper part of the column, for which purpose we first compute the characteristics of the section, namely, A, I_x, I_y W_x, r_x, r_y, ρ, λ_x, m and m_1:

the sectional area $A=46\times1+2\times43\times2=218$ cm^2

the moments of inertia

$$I_x = \frac{t_w h_w^3}{12} + 2c^2 A_{fl} = \frac{1 \times 46^3}{12} + 2 \times 24^2 \times 86 = 107{,}300 \text{ cm}^4$$

$$I_y = \frac{2t_{fl} b^3}{12} = \frac{2 \times 2 \times 43^3}{12} = 26{,}400 \text{ cm}^4$$

the section modulus $W_x = \frac{I_x}{h/2} = \frac{107{,}300}{25} = 4{,}300 \text{ cm}^3$

the radii of gyration $r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{107{,}300}{218}} = 22.2$ cm $r_y = \sqrt{\frac{I_y}{A}} =$

$= \sqrt{\frac{26{,}400}{218}} = 11.1$ cm

For checking the section in the plane which the moment acts in we determine

$$\rho = \frac{W}{A} = \frac{4{,}300}{218} = 19.7 \text{ cm} \quad m_x = \frac{e}{\rho} = \frac{120.5}{19.7} = 6.13$$

$$\lambda_x = \frac{L_{x_2}}{r_x} = \frac{1{,}560}{22.2} = 70.5 \quad \eta = 1.45 - 0.003\lambda_x = 1.45 - 0.003 \times 70.5 =$$

$$= 1.21 \quad m_1 = \eta\, m = 1.21 \times 6.13 = 7.4$$

From Table 7 of Appendix II with $\lambda = 70.5$ and $m_1 = 7.4$ we find $\varphi_{ec} = 0.155$
We check the design stress

$$\sigma = \frac{F_2}{\varphi_{ec} A_{gr}} = \frac{70{,}900}{0.155 \times 218} = 2{,}090 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

No checking of the local stability of the web and the flanges is required, since for the web $\frac{h}{t} = \frac{48}{1} = 48 < 40 + 0.2 \times 70.5 = 54$ [see equation (8-3) with $R =$ $= 2{,}100$ kg/cm^2), while for the flanges $\frac{b}{2t_{fl}} = \frac{43}{2 \times 2} = 10.75 < 16$ (see Table 8-1), and they are therefore stable under the action of the normal stresses.

We investigate the section of the upper part of the column in a direction perpendicular to the plane of action of the moment by means of equation (8-40). For this end the first step is to find the values of λ_y, φ_y and the factor c from equation (8-41): $\lambda_y = \frac{L_{y_2}}{r_y} = \frac{360}{11.1} = 32.6$; $\varphi_y = 0.941$ (from Table 1 of Appendix II); $c = \frac{\beta}{1 + \alpha m_x} = \frac{1}{1 + 0.7 \times 6.13} = 0.189$ (seeing that with $\lambda_y < 100$ the coefficient $\beta = 1$), after which we compute

$$\sigma = \frac{F_2}{c \varphi_y A} = \frac{70{,}900}{0.189 \times 0.941 \times 218} = 1{,}820 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

4. *Selection of the Section of the Lower Column Part.* The design combinations of loads in section *A-A* (Fig. 8-22*b*) are:

for the crane runway branch $F_1' = 109.6$ tons, $M_1' = -185.4$ t-m;

for the outer branch $F_1 = 189.1$ tons, $M_1 = +346$ t-m.

The effective lengths of the lower part of the column are $L_{x_1} = 38.2$ m and $L_{y_1} = 15.3$ m.

We design the lower part of the column of the open-web type, the crane runway branch being made of a rolled I shape, while the outer, roof-supporting

branch—of angles with a plate (Fig. 8-22*a*). The lattice is designed with panels of 1.5 m.

The depth of the section $h_1=1.25$ m is established upon selecting the layout of the building (see example 9-1). The distance from the centre of gravity of the section to the crane runway branch is taken equal to $z_2=0.6$ m $(0.48h_1)$, and to the outer branch $z_1=0.65$ m (Fig. 8-22*b*).

Seeing that the positive moment acting in the direction of the outer branch was found to be greater than the negative one, we assume a displacement of the axis from the middle of the column nearer to the outer branch, namely, $z_1=0.5$ m $(0.4h_1)$ and $z_2=0.75$ m $(0.6h_1)$. The approximate maximum loads on the branches [from equation (8-42)] will be

in the crane runway branch

$$F_{c\cdot br}=\frac{F_1'z_1}{h_1}+\frac{M_1'}{h_1}=\frac{109.6\times0.5}{1.25}+\frac{185.4}{1.25}=192.5 \text{ t}$$

in the outer branch

$$f_{o\cdot br}=\frac{F_1z_2}{h_1}+\frac{M_1}{h_1}=\frac{189.1\times0.75}{1.25}+\frac{346}{1.25}=391 \text{ t}$$

The width of the column (the depth of the crane runway branch section) is generally taken within the limits of $\frac{1}{25}L_1$ to $\frac{1}{30}L_1$, i.e., $\frac{1}{25}\times1\,530$ to $\frac{1}{30}\times$ $\times1{,}530$ or 50 to 60 cm.

Having assumed the slenderness ratio of the crane runway branch in buckling perpendicular to the plane of action of the moment $\lambda=70$, we find $\varphi=0.81$ and the approximate required sectional area of the branch

$$A_{req}=\frac{F_{c\cdot br}}{\varphi R}=\frac{192{,}500}{0.81\times2{,}100}=113 \text{ cm}^2$$

I shape No. 55 ($A=114$ cm^2) corresponds to this area and the dimension found above.

In a similar way we find the approximate sectional area of the outer branch, assuming $\varphi=0.75$

$$A_{req}=\frac{F_{o\cdot br}}{\varphi R}=\frac{391{,}000}{0.75\times2{,}100}=249 \text{ cm}^2$$

We select a section made from angles 160×18 and a plate 520×26 (Fig. 8-22*a*) so arranged that the depth of the section will equal 55 cm.

We determine the geometrical characteristics of the section.

Crane runway branch—I shape No. 55, data from list of standard shapes

$$A_{c\cdot br}=114 \text{ cm}^2,\quad I_y=55{,}150 \text{ cm}^4,\quad I_{x0}=1{,}350 \text{ cm}^4,$$
$$r_y=22 \text{ cm},\quad r_{x0}=3.44 \text{ cm}$$

Outer branch—plate 520×26 and 2 angles 160×18 ($A_{ang}=54.8$ cm^2)

$$A_{o\cdot br}=52\times2.6+2\times54.8=244.8 \text{ cm}^2$$

We find the distance from the centre of gravity of the outer branch to the edge of the angle backs

$$z_{bo\cdot br}=\frac{2\times54.8\,(4.63+1.3)}{244.8}-1.3=1.36 \text{ cm}$$

The distance between the centres of gravity of the section is taken equal to $h_1=1{,}250$ mm (this dimension is generally taken up to the backs of the angles),

while the distances from the centre of gravity of the whole column section to the axes of the column branches will be

$$z_1 = \frac{A_{o\cdot br} \times 1.25}{\sum A} = \frac{114 \times 1.25}{114 + 244.8} = 40 \text{ cm}$$

$$z_2 = 125 - 40 = 85 \text{ cm}$$

We find the final values of the forces in the branches induced by the corresponding loads, and check the stresses.

Crane runway branch: the force

$$F_{c\cdot br} = \frac{F_1' z_1}{h_1} = \frac{M_1'}{h_1} = \frac{109.6 \times 0.4}{1.25} + \frac{185.4}{1.25} = 183.5 \text{ t}$$

the slenderness ratio of the whole branch

$$\lambda_y = \frac{L_{y_1}}{r_y} = \frac{1,530}{22} = 70 \qquad \varphi_y = 0.81$$

the slenderness ratio of the part of the branch between lattice joints $\lambda_{br} = \frac{L_{br}}{r_{xo}} = \frac{150}{3.44} = 43.7 < \lambda_y$

the stress $\sigma = \frac{F_{c\cdot br}}{\varphi_y A} = \frac{183,500}{0.81 \times 114} = 1.985 \text{ kg/cm}^2 < 2,100 \text{ kg/cm}^2$

Outer branch: the force

$$F_{o\cdot br} = \frac{F_1 z_2}{h_1} + \frac{M_1}{h_1} = \frac{189.1 \times 0.85}{1.25} + \frac{346}{1.25} = 402 \text{ t}$$

$$I_{o\cdot br_y} = I_p + 2(I_{ang_0} + c^2 A_{ang}) = \frac{2.6 \times 52^3}{12} + 2[1,299 + (27.5 - 4.63)^2 \times 54.8] =$$

$$= 98,800 \text{ cm}^4$$

$$I_{x0} = 2(I_{ang_0} + c^2 A_{ang}) + c_p^2 A_p = 2[1,299 + (4.63 - 1.53)^2 \times 54.8] + 2.83^2 \times$$

$$\times 135 = 4,730 \text{ cm}^4$$

$$r_y = \sqrt{\frac{98,800}{244.6}} = 20.2 \text{ cm} \qquad r_{x_0} = \sqrt{\frac{4.730}{244.6}} = 4.4 \text{ cm}$$

the slenderness ratio of the whole branch $\lambda_y = \frac{L_{y_1}}{r_y} = \frac{1,530}{20.2} = 75.6$; $\varphi_y = 0.78$

the slenderness ratio of the branch between lattice joints $\lambda_{br} = \frac{L_{br}}{r_{x_0}} = \frac{150}{4.4} = 35 < \lambda_y$

the stress $\sigma = \frac{F_1}{A\varphi_y} = \frac{402,000}{244.6 \times 0.78} = 2,100 \text{ kg/cm}^2$

We check the load-carrying capacity of the column as a whole by means of equation (8-22), for which end we compute

$$A = A_{c\cdot br} + A_{c\cdot br} = 114 + 244.6 = 358.6 \text{ cm}^2$$

$$I_x = (I_{x_2} + A_{o\cdot br}\ z_1^2) + (I_{x_1} + A_{c\cdot br}\ z_2^2) = 4,730 + 244.6 \times 40^2 + 1,350 + 114 \times 85^2 =$$

$$= 1,220,080 \text{ cm}^4$$

$$r_x = \sqrt{\frac{1,220,080}{359.6}} = 58.5 \text{ cm}$$

the distance from the centre of gravity to the most compressed branch $z_1 = 40$ cm.

From equation (8-43) we find

$$m = \frac{M_1}{F}\frac{Az_1}{I_x} = \frac{346}{189.1} \times \frac{358.6 \times 40}{1,220,080} = 2.14$$

The slenderness ratio of the lower part of the column

$$\lambda_x = \frac{L_{x_1}}{r_x} = \frac{3,820}{58.5} = 65.5$$

For determining the reduced slenderness ratio of the column it is necessary to select the section of the lattice angles and then check it by calculations.

We design the lattice of angle 100×8 ($A = 15.6$ cm^2, $r_{min} = 1.96$ cm). Hence the reduced slenderness ratio, from equation (8-7), will be

$$\lambda_{red} = \sqrt{\lambda_x^2 + 31\frac{A}{A_d}} = \sqrt{65.5^2 + \frac{31 \times 358.6}{2 \times 15.6}} = 68$$

From Table 8 of Appendix II with $m = 2.14$ and $\lambda_{red} = 68$ we find $\varphi_{ec} = 0.27\lambda$.

The stress $\sigma = \frac{F}{\varphi_{ec} A} = \frac{189,100}{358.6 \times 0.27} = 1,960$ kg/cm^2 $< 2,100$ kg/cm^2

5. *Checking the Column Lattice Diagonals.* The maximum shear force $Q = 18.92$t. The slope of the diagonal is $\tan \alpha = \frac{1.25}{1.5} = 0.833$, whence $\alpha = 39°40'$ and $\sin \alpha = 0.638$.

We find the force in a diagonal of the lattice located in one plane, from equation (8-19)

$$F_d = \frac{Q}{2 \sin \alpha} = \frac{18,920}{2 \times 0.638} = 14,850 \text{ kg}$$

The length of a diagonal $L_d = \frac{h_1}{\sin \alpha} = \frac{1.25}{0.638} = 1.96$ m

$$\lambda_d = \frac{L_d}{r_{min}} = \frac{196}{1.96} = 100 \quad \varphi = 0.6$$

The stress $\sigma = \frac{F}{\varphi A_d} = \frac{14,850}{0.6 \times 15.6} \cong 0.75 \times 2,100$ kg/cm^2 $= 1,575$ kg/cm^2

8-4. SPLICES AND DETAILS OF COLUMNS

Both shop and field column splices are used in practice. Shop splices are made owing to the limited length of rolled shapes (see Chapter 3). Field splices are made because of limitations in transportation possibilities (for example, a maximum length of 13 metres is permissible when transporting members by railway on single eight-wheel flat cars, and of 27 metres when groups of two flat cars coupled together are being used).

The shop splices of elements are ordinarily staggered, and not concentrated at one place, since the separate elements can be connected together before general assembly of the column. Examples of welded shop splices of separate column elements are pictured in Fig. 8-23.

The main condition for the formation of a strong splice is adequate provision for the transferring of the load from one element to the

other. With butt welding this is ensured by selecting the corresponding length of the welds (see Chapter 4), while when straps are used to make the splices, care should also be exercised to ensure the required sectional area of the straps or splice plates. This must not be less than the sectional area of the main elements being spliced.

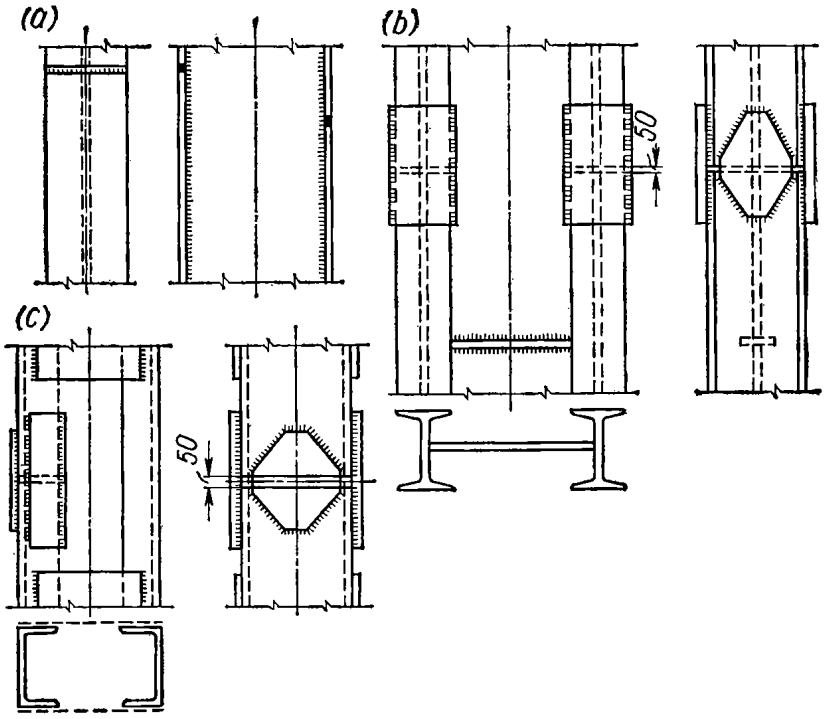


Fig. 8-23. Welded shop column splices:
a—splicing flanges of welded I section; *b*—splicing I section branches of solid column; *c*—splicing branches of open-web column with batten plates

The simplest splice, and hence the one most recommended for use, is a straight splice with butt welding. Such a splice can be designed in all instances, since in eccentrically loaded columns it is always possible to find a section with low tensile stresses.

Field column splices are located at spots that are convenient for erection of the members. In variable dimension columns such a place is the step at the level of the crane girder bearings, where the section of the column changes.

Figure 8-24 illustrates splices of the upper and lower parts of a single-stepped solid column, namely, a shop splice (Fig. 8-24*a*) and a field splice (Fig. 8-24*b*).

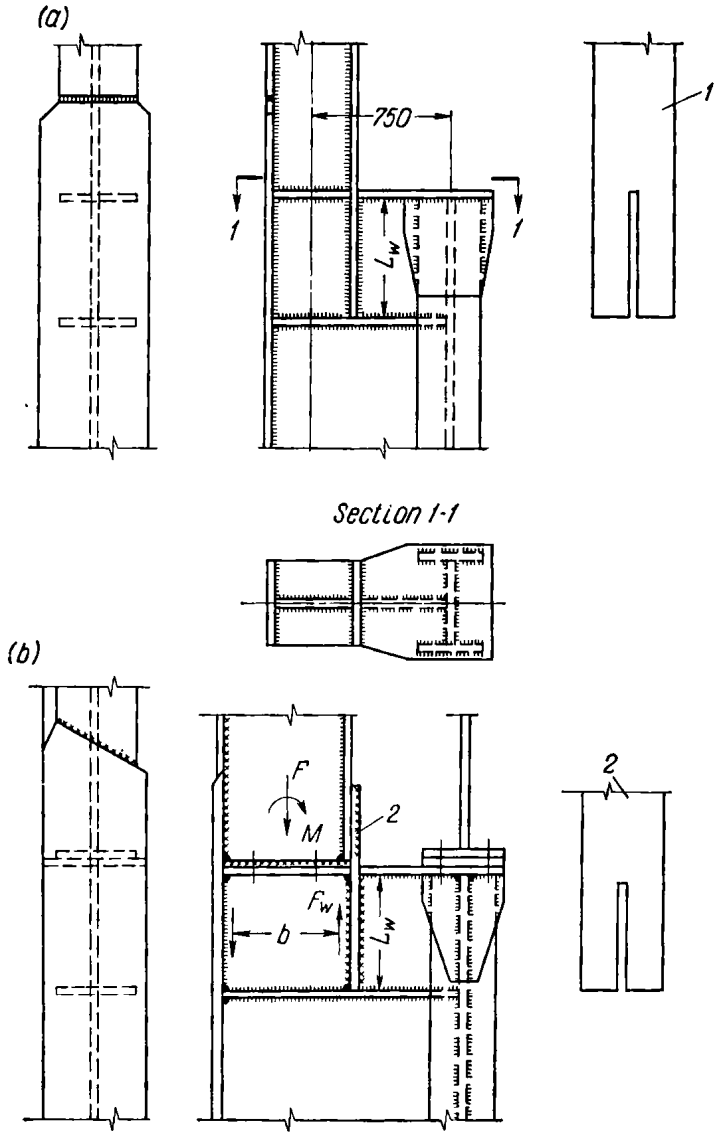


Fig. 8-24. Splicing of upper and lower parts of single-stepped solid column

Figure 8-25 shows the connection of the upper part of a column to the lower open-web part by means of a double-web (Fig. 8-25*a*) and a single-web (Fig. 8-25*b*) traverse or cross-piece.

The length of the welds (L_w in Fig. 8-24) required for connecting the inner flange of the upper part of the column is determined for

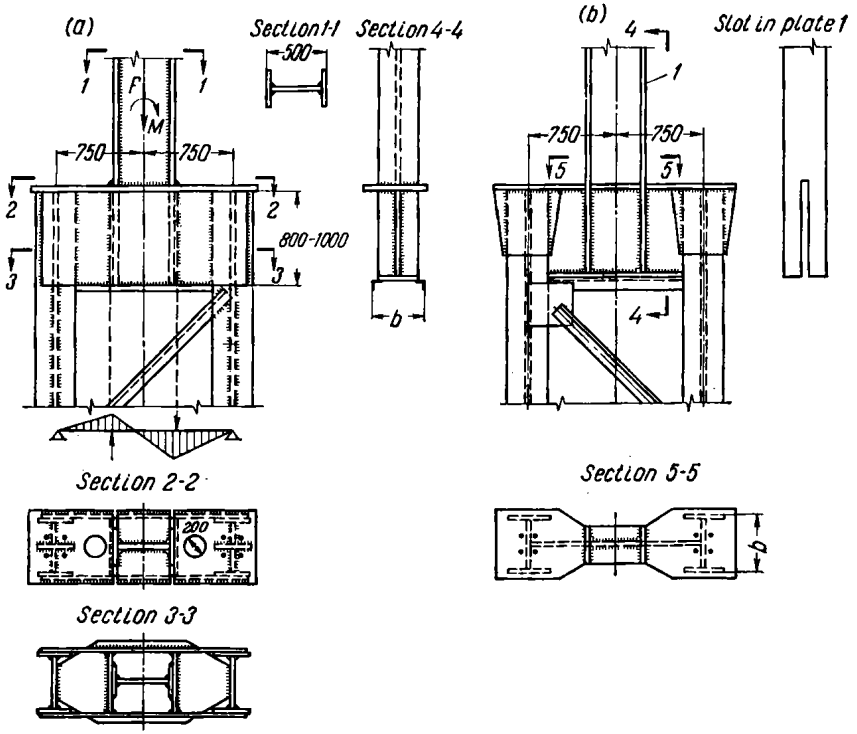


Fig. 8-25. Connection of roof-supporting part of column to open-web crane runway part

the condition that the moment M and the longitudinal force F acting in the upper part of the column, at the place of its connection to the lower part, be resisted by the welds connecting the flanges of the upper part of the column. In this instance the welds connecting the web are not generally taken into consideration.

The force in a flange, equal to

$$F_w = \frac{F}{2} + \frac{M}{b} \tag{8-44}$$

is transmitted through four welds connecting component *1* to the web of the lower part of the column (Fig. 8-24*a*). Component *1* has a slot that permits fitting it onto the web of the lower part of the column

(this slot is made 2-3 mm greater than the thickness of the plate). With a field splice this component is made separately from the flange plate and is welded to the lower part of the column (Fig. 8-24*b*).

In columns with a lower latticed part the upper part is connected with the aid of a component called a *cross-piece* or *traverse* (Fig. 8-25*a* and *b*). The traverse is subjected to bending in the same way as a beam on two supports and must be checked for adequate strength. A diagram of the moments in the traverse is shown in Fig. 8-25*a*. The traverse is connected to the branches of the column by means of continuous welds, and the connection is investigated for the support

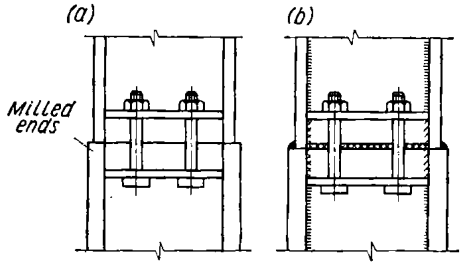


Fig. 8-26. Field splices of solid section columns

reaction of the traverse. Horizontal membranes or stiffeners are installed to ensure general stiffness of the connection between the upper and the lower parts of the column.

The field splicing of solid-section columns that mainly transmit compressive forces can be accomplished with the aid of milled ends (Fig. 8-26*a*). Such a type of splice has been employed in multistoried buildings erected in Moscow. If the column also transmits a moment, use can be made of the welded splice pictured in Fig. 8-26*b*, in which no milling of the ends is required. The employment of a straight butt weld is possible if equal strength of the deposited and the basic metal is ensured. It is generally assumed that in columns subjected mainly to compression the appearance of tension is nevertheless possible at any edge of the section. For this reason it is necessary to ensure in splices and joints resistance to a conditional tensile force which is ordinarily taken equal to 15% of the design normal compressive force (of course, if there are no actual tensile forces exceeding this value).

Crane girders are seated on constant dimension columns (in shops with a light duty) by using a ledge made from a welded I section (of plates, Fig. 8-27*a*) or from two channels (Fig. 8-27*b*). The ledge is investigated for the moment induced by the load of two cranes travelling over the crane girders and both operating in the vicinity

of the support, namely, $M = Pe$, where e is the distance from the axis of the crane girder to the branch of the column (Fig. 8-27).

The welds connecting a single-web ledge (Fig. 8-27*a*) are investigated for the action of the moment M and the shearing force P.

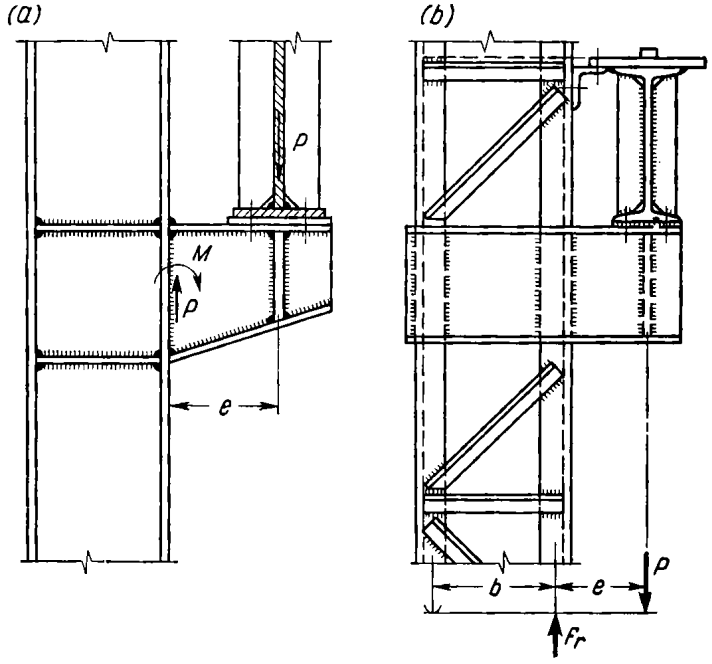


Fig. 8-27. Seating of crane girders on ledge

The welds connecting a ledge consisting of two channels surrounding the column (Fig. 8-27*b*) are analyzed for the reaction F_r found in the same way as in a single-cantilever beam

$$F_r = \frac{P(e+b)}{b}$$

8-5. COLUMN BASES

(1) Types and Designs of Bases

The base (footing) of a column is designed to distribute the concentrated column load over a certain definite foundation area and to ensure connection of the lower column end to the foundation in accordance with the planned design.

Two basic types of bases are distinguished, namely, pinned and rigid ones.

The simplest kind of pinned base for axially loaded columns is a base consisting of a thick steel base plate on which the milled end of the column rests (Fig. 8-28*a*).

The employment of bases with transmission of the load through the milled end of the column is expedient for columns with a considerable load. For light columns (and also when no end-milling machines are available) bases are employed in which the entire load is transmitted to the plate through welds (Fig. 8-28*b*).

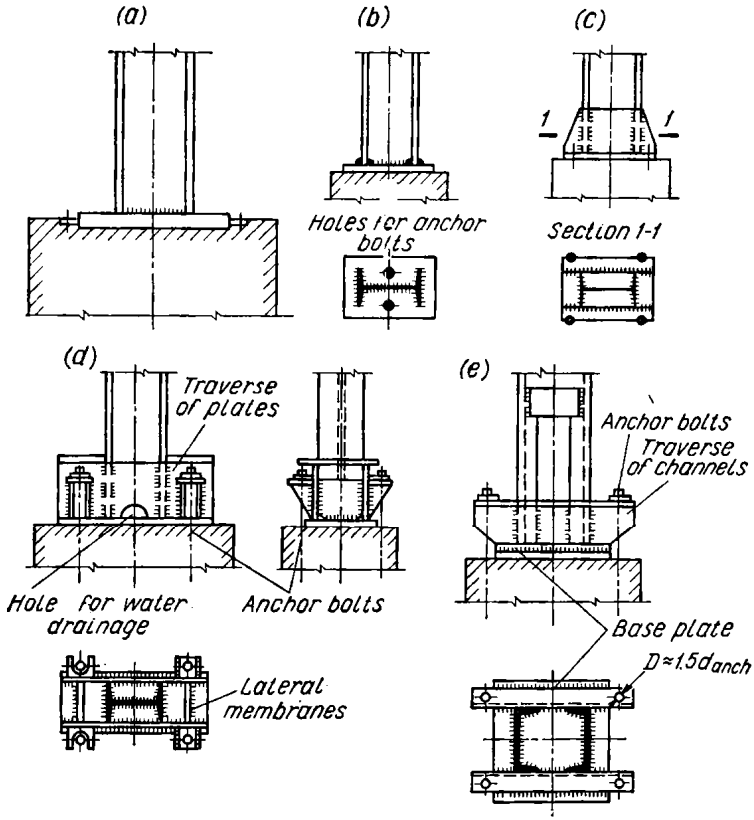


Fig. 8-28. Types of column bases

The load can also be transmitted from the column to the base plate with the aid of a traverse (Fig. 8-28*c*), which serves for more or less uniform transmission of the force field from the column to the base plate. This, with respect to the nature of the load action, makes the design resemble a rigid "stamp" bearing on the foundation. The traverse simultaneously serves as a bearing member for the base plate,

when it is subjected to bending induced by the reaction or upward pressure of the foundation. The traverse itself resists bending as a double-cantilever beam supported on the flanges or branches of the column and loaded with the upward pressure of the foundation.

In eccentrically loaded columns, the rule is to design rigid bases that can transmit bending moments. For this purpose it becomes

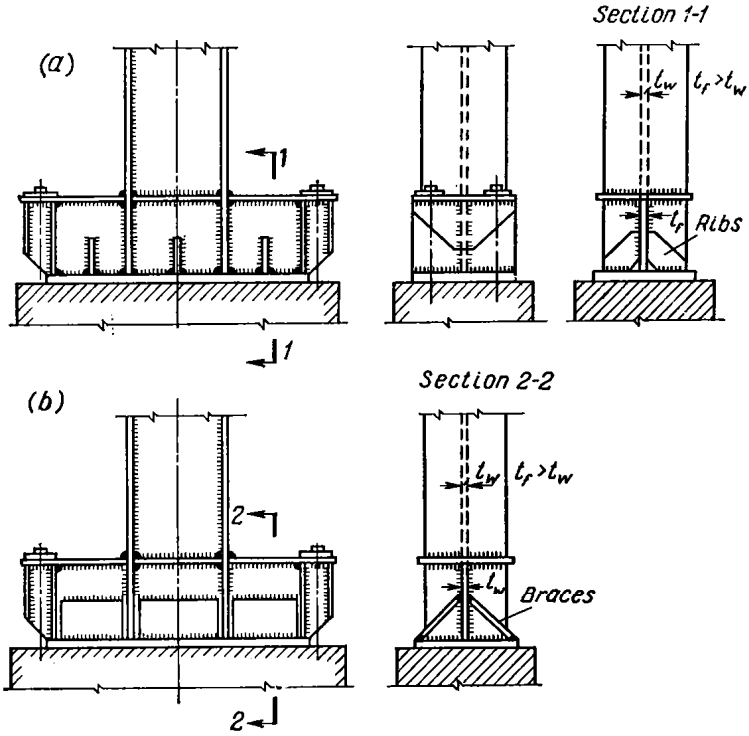


Fig. 8-29. Open single-web footings

necessary to develop the traverses in the direction of action of the moment. With relatively small moments at the bearings, the traverses are made from plates 10-12 mm thick (Fig. 8-28*d*) or channels (Fig. 8-28*e*).

In columns with greater crane loads and with larger moments at the bearings, it becomes necessary to develop the bases and their traverses to a still greater extent.

To make welding more convenient, it will be good to use open single-web footings strengthened with ribs (Fig. 8-29*a*) or plate braces (Fig. 8-29*b*). The latter should be connected with welds having the minimum possible size to avoid warping of the base plate.

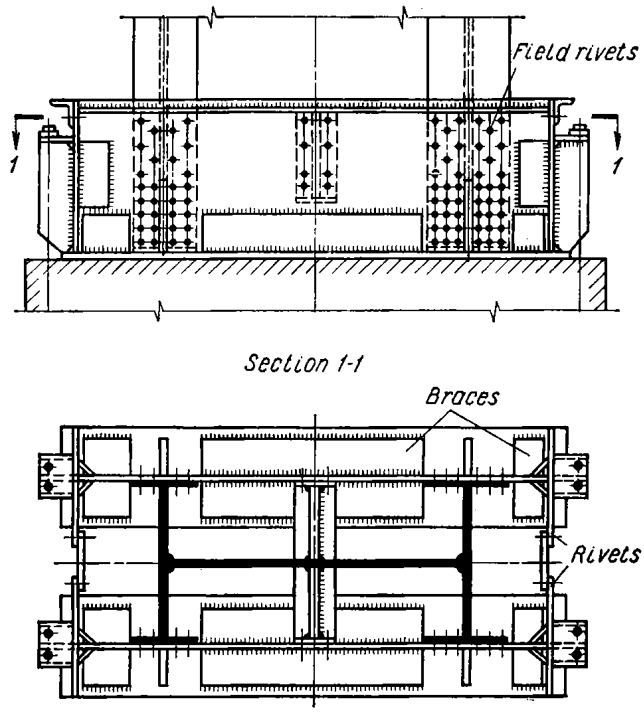


Fig. 8-30. Braced double-web split footing

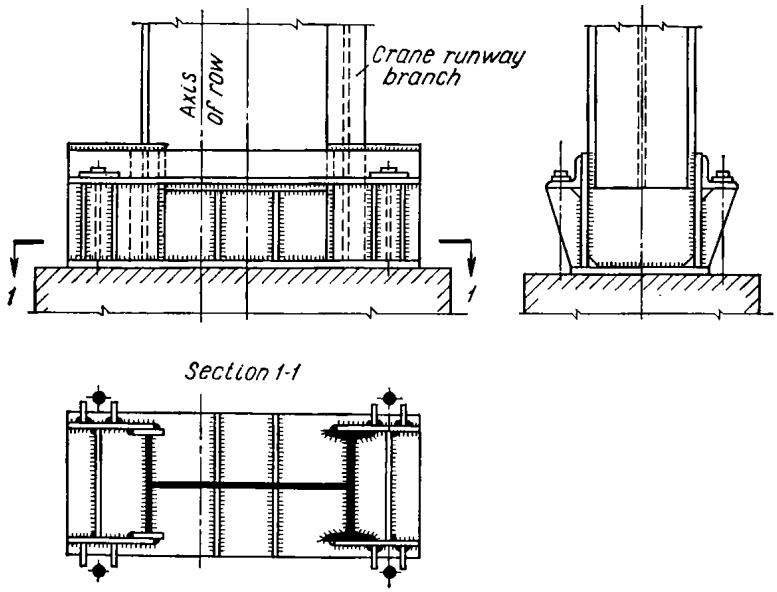


Fig. 8-31. Footing with separate traverses

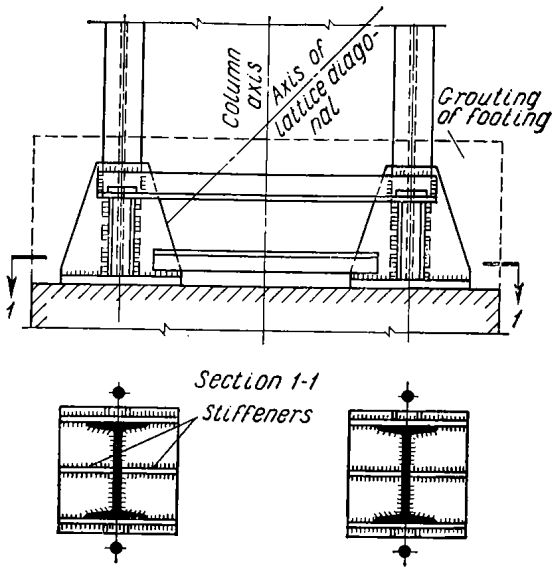


Fig. 8-32. Base of open-web column

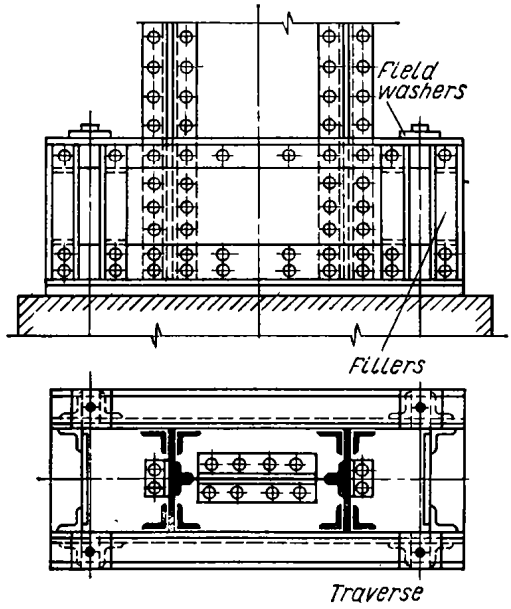


Fig. 8-33. Riveted footing of solid column

Single-web footings are most frequently employed in solid constant dimension columns. Their drawbacks include a low stiffness in a plane perpendicular to that of the frame.

An alternative design in the form of a double-web footing with braces can be used for the heaviest columns (Fig. 8-30). In this instance such footings are sometimes made of the split type, owing to the large dimensions of the base, which prevent transportation of the column together with its base as a single member.

In solid variable dimension columns, a very widely used type of footing is a welded one with separate traverses, namely, a single-web one within the limits of the web and a double-web one at the branches (Fig. 8-31).

In the open-web columns of industrial buildings, use is generally made of footings of the split type, consisting of two independent footings connected by means of angle ties (Fig. 8-32). With a large distance between the column branches, these footings are more economical than their solid counterparts.

Riveted footings are designed only in riveted columns, and are similar in design (Fig. 8-33) to welded ones.

The footings are connected to the foundations with the aid of anchor bolts (anchors) embedded in the foundation in concreting. In axially loaded columns the anchor bolts are not investigated, and their dimensions are established from considerations of design (d = =22-26 mm).

In constrained columns subjected to bending the anchor bolts resist tension originated by the bending moment. Their diameter and length are established from the results of analysis.

The anchor holes should be laid out in a horizontal plane with the employment of rigid jigs, and checked by means of geodetic instruments. As a rule, the holes in a footing for the anchor bolts are made with a diameter greater than that of the bolts; they are closed with field washers, which are welded to the footing after the column has been installed in its proper position for erection. When the columns have been installed, the bases are grouted to prevent corrosion.

(2) Analysis and Design of Bases

Analysis of Base Plate and Traverse of Axially Loaded Column. The dimensions of the base plate of an axially loaded column are determined in accordance with the design resistance of the foundation material to axial compression R_{fd}. The minimum area of the base plate will be

$$A_{pl} \geqslant \frac{F}{R_{fd}} \tag{8-45}$$

in which F is the design force in the column.

The design compressive strength of a concrete foundation is found from the equation

$$R_{fd} = R_{con}\psi = R_{con}\sqrt[3]{\frac{A_{fd}}{A_{pl}}} \tag{8-46}$$

where R_{con} = design compressive strength of concrete (44 kg/cm² for grade 100 concrete and 65 kg/cm² for grade 150)
A_{fd} = area of foundation
F_{pl} = area of base plate
ψ = strength factor whose value should not exceed 2.

Having found the required base plate area, one may commence to design the footing, taking the width of the base plate B somewhat greater than that of the column.

The base plate resists bending induced by a uniformly distributed load (the upward pressure of the foundation)

$$q = \sigma_{con} = \frac{F}{LB}$$

and different parts of the base plate will be in different conditions of bending. Figure 8-34*a* pictures a base plate which may be considered as consisting of three different parts.

The first part *1* of the plate behaves and is investigated as a cantilever (Fig. 8-34*b*). For this purpose the moment in section *I-I* is found acting on a strip 1 centimetre wide

$$M = \frac{\sigma_{con}c^2}{2} \tag{8-47}$$

The second part of the base plate *2* (Fig. 8-34*a*) behaves as a plate supported on four sides and loaded from below with the same uniformly distributed load $q = \sigma_{con}$. Such a rectangular plate with the maximum moment acting at its centre is investigated with the aid of tables compiled by Academician B. Galerkin, using the equations

$$M_a = \alpha_1 q b^2 \qquad M_b = \alpha_2 q b^2 \tag{8-48}$$

Here M_a and M_b = moments calculated for strips 1 centimetre wide in the direction of the dimensions a and b
b = length of short side of rectangle
α_1 and α_2 = coefficients taken from Table 8-11 depending on the ratio between the longer side of the rectangle a and its shorter side b (Fig. 8-34*a*).

If a/b exceeds 2 the moment can be determined for a strip cut out along the short side, as in a single-span beam (see Table 8-11, last column).

Should it be assumed that the plate edges are elastically constrained, the moments obtained from equations (8-48) or as in a single-span beam may be reduced by 20%.

The third part of the base plate behaves as a plate supported on three sides. The critical point in such a plate is the middle of its free

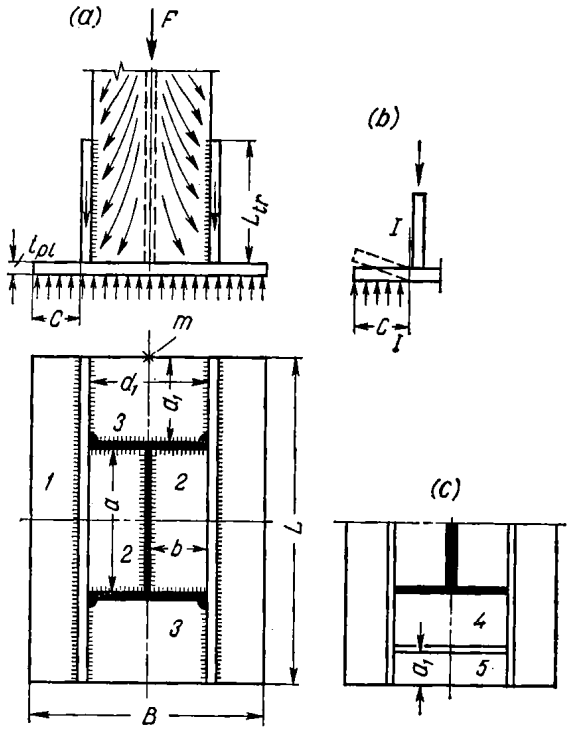


Fig. 8-34. To analysis of base plate of axially loaded column

edge (point m in Fig. 8-34). The moment in this section is determined from the equation

$$M_3 = \alpha_3 q d_1^2 \tag{8-49}$$

in which α_3 = coefficient taken from Table 8-11

d_1 = length of the free edge of the plate.

When a_1/d_1 is less than 0.5, the plate is analyzed as a cantilever.

The thickness of the plate is determined in accordance with the greatest of the moments computed from equations (8-47), (8-48) and (8-49). The plate should have a thickness adequate to ensure uniform transmission of the load to the concrete without bending

TABLE 8-11 **Coefficients α_1, α_2 and α_3 for Analysis of Bending of Rectangular Plates Supported on Four and Three Sides**

Plates supported on four sides	Coefficient	Ratio of sides a/b											
		1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	Over 2
	α_1	0.048	0.055	0.063	0.069	0.075	0.081	0.086	0.091	0.094	0.098	0.100	0.125
	α_2	0.048	0.049	0.050	0.050	0.050	0.050	0.049	0.048	0.048	0.047	0.046	0.037

Plates supported on three sides	α_3	Ratio of sides a_1/d_1									
		0.5	0.6	0.7	0.8	0.9	1	1.2	1.4	2	Over 2
		0.060	0.074	0.088	0.097	0.107	0.112	0.120	0.126	0.132	0.133

d_1 = length of free edge

(as depicted in the exaggerated view in Fig. 8-34*b*), i. e., the footing should behave as a rigid stamp or punch.

The section modulus of a plate with a thickness of t and a width of 1 cm will equal $W = \frac{1t^2}{6}$.

Utilizing the total stress in the plate, equal to the design strength, we have

$$\sigma = \frac{M}{W_{pl}} = \frac{6M}{t_{pl}^2} = R$$

whence

$$t_{pl} = \sqrt{\frac{6M}{R}} \qquad (8\text{-}50)$$

When designing the base of a column care should be taken to have the thicknesses of the different parts of the base plate determined from equations (8-47) through (8-49) close to each other. This can be attained by changing the dimensions a, b and c. Thus, for example, in Fig. 8-34*c* by installing a membrane, part *3* (below) is divided into two, namely, into part *4* supported on four sides and into part *5* supported on three sides, but with a smaller dimension a_1.

The thickness of a base plate is generally taken within the limits of 16 to 40 mm (except for the plates of columns with milled ends, where the thickness can be greater if required by greater loads).

The height of the traverse is determined to make possible arrangement of the welds through which the load will be transmitted from the column to the traverse.

Analysis of Base Plate and Anchor Bolts of Eccentrically Loaded Column. The footing of an eccentrically loaded column transmits an uneven pressure to the surface of the foundation. In the direction of action of the moment, the base plate of the footing applies a compressive load to the foundation, while on the opposite side it tends to tear away from the surface of the foundation (Fig. 8-35).

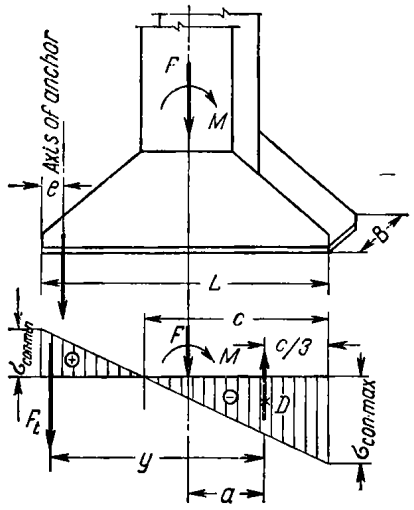


Fig. 8-35. Diagram to analysis of anchor bolts

This tearing away is prevented by the anchor bolts constraining the end of the column.

In the process of designing, the first dimension to be assumed is the width of the base plate B. The length of the plate L is determined from the condition that the maximum stress in the foundation at the edge of the plate $\sigma_{con\cdot max}$ be less than the design compressive strength of the concrete

$$\sigma_{con\cdot max} = \frac{F}{A} + \frac{M}{W} = \frac{F}{BL} + \frac{6M}{BL^2} \leqslant R_{con} \qquad (8\text{-}51)$$

Here the maximum tensile stress at the opposite edge of the plate is

$$\sigma_{con\cdot min} = \frac{F}{A} - \frac{M}{W} = \frac{F}{BL} - \frac{6M}{BL^2} \qquad (8\text{-}52)$$

When performing these calculations, the most disadvantageous combination of loads is selected for determining the values of F and M.

By solving expression (8-51) for L, the required length of the base plate can be found as a function of the assumed width of the plate B and the given design strength of the concrete R_{con}

$$L = \frac{F}{2BR_{con}} + \sqrt{\left(\frac{F}{2BR_{con}}\right)^2 + \frac{6M}{BR_{con}}} \qquad (8\text{-}53)$$

After having determined the dimensions of the base plate L and B, the next step is to design the base and determine the thickness of the base plate.

When determining the thickness of the base plate, it is assumed (with a certain increase in the margin of safety) that the plate is loaded with a uniformly distributed load $q = \sigma_{con\cdot max}$. An exception

is allowed only for the middle parts of the plate, which can be investigated for a uniformly distributed load equal to the maximum stress corresponding to the edge of this part of the plate.

The anchor bolts are investigated on the assumption that the tensile force F_t determined by the tension zone of the stress diagram (Fig. 8-35) is completely resisted by the anchor bolts.

For this reason, when drawing up the equation of equilibrium with respect to the centre of gravity D of the compression triangular zone of the stress diagram, i. e., the point of application of the resultant compressive force, we obtain

$$M - Fa - F_t y = 0$$

Hence the total force F_t in all the anchor bolts located at one side of the footing is

$$F_t = \frac{M - Fa}{y} \tag{8-54}$$

and the total sectional area of these bolts (area at roots of the threads) will correspondingly be

$$A_{anch \cdot n} \geqslant \frac{F_t}{R_{anch \cdot t}} = \frac{M - Fa}{yR_{anch \cdot t}} \tag{8-55}$$

where $R_{anch \cdot t}$ = design tensile strength of anchor bolts, taken equal to 1,400 kg/cm² for bolts made from steel 3.

The value of a is determined from the geometrical relation

$$a = \frac{L}{2} - \frac{c}{3} \tag{8-56}$$

in which $c = \frac{\sigma_{con \cdot max}}{\sigma_{con \cdot max} + \sigma_{con \cdot min}}$

When determining the value of c the absolute values of σ_{con} should be taken, without regard to their signs.

The arm of the anchor bolts, i. e., the dimension y, is determined as follows. First the part to be used for connecting the anchor to the footing of the column is designed, and thus the dimension e (Fig. 8-35) is established, which is generally equal to 75-100 mm. The dimension y will then be obtained from the equation

$$y = L - \frac{c}{3} - e \tag{8-57}$$

When investigating anchor bolts it is necessary to take into account the combination of loads that gives with a minimum F the maximum value of M (for example, with a wind, but without cranes and snow).

The sectional area of one anchor will obviously be obtained if the total area found from expression (8-55) is divided by the number of anchors located on one side of the footing. The anchor bolts on both

sides of the footing are generally arranged symmetrically. The diameter of the anchors is taken within the limits of 20 to 76 mm, since thicker anchor bolts are complicated to fabricate. Anchors can be secured in a foundation by bonding them to the concrete (Fig. 8-36*a* and *b*), which determines their depth (this must be sufficient to ensure an adequate grip), or by means of bearing washers (Fig. 8-36*c*), which resist the pressure of the concrete over their area.

Table 8-12 can be used to determine the depth of embedding anchor bolts L_i. The bolts are usually threaded over a length of 120 to

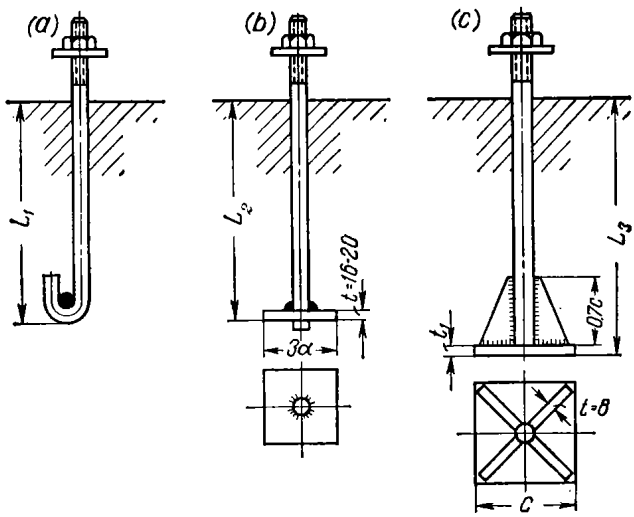


Fig. 8-36. Types of anchorages

150 mm. When designing the base it should be made possible to freely turn the nuts when tightening the bolts. For this reason it is desirable to have a minimum distance from the axis of the bolt to the traverse equal to $1.5d$ (where d=diameter of bolt). The anchor bolts are located beyond the edges of the base plate to permit the column to be moved to all sides (by about 20 mm) during erection when centering it.

The height of the traverse is selected with a view to proper accommodation of the welds or rivets used to connect the column to the traverse.

Example 8-4. Analyze the design of the latticed column footing shown in Fig. 8-37. Assume that the maximum design loads in the column are the same as in example 8-3, namely

$M = -185.4$ t-m $\quad F = 109.6$ t
$M = +346$ t-m $\quad F = 189.1$ t

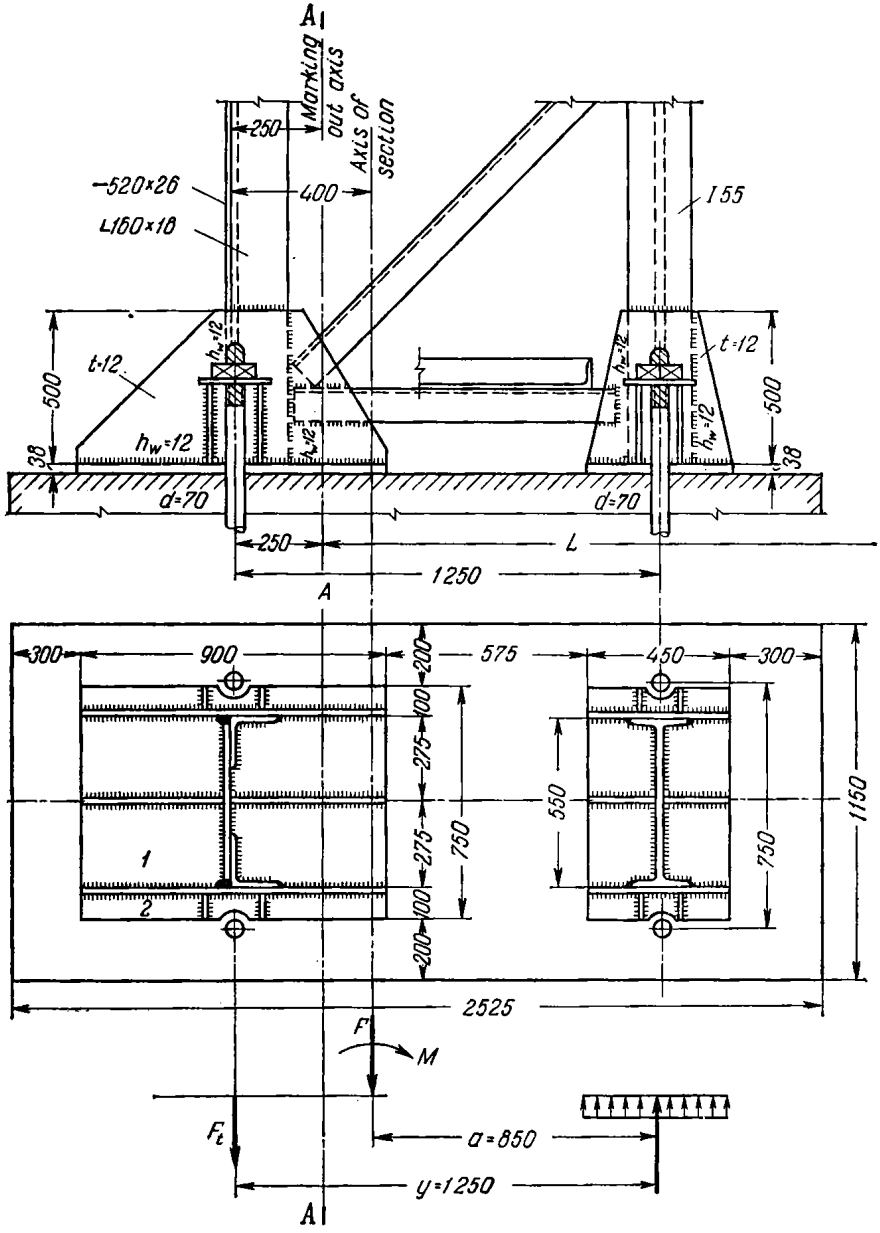


Fig. 8-37. To example 8-4

TABLE 8-12 **Normal Dimensions of Anchor Bolts**

External diameter of bolt, d mm	Net area (within root of thread), cm²	Length of anchor embedment, mm			Dimension of washer c	Maximum design force allowed per bolt, kg (with R_t=1,400 kg/cm²)
		without bearing washer (Fig. 8-36a and b)		with bearing washer (Fig. 8-36c) L_3		
		L_1	L_2			
20	2.25	700	—	—	—	3,150
22	2.81	750	—	—	—	3,940
24	3.24	850	—	—	—	4,540
27	4.27	950	—	—	—	5,980
30	5.18	1,050	—	—	—	7,250
36	7.58	1,250	—	700	160×16	10,600
42	10.0	1,450	—	800	200×20	14;000
48	13.4	—	1,450	850	240×25	18,700
56	18.75	—	1,650	1,000	240×25	26,200
64	24.65	—	1,850	1,100	280×30	34,500
72	31.6	—	2,000	1,250	280×30	44,200
76	35.8	—	2,100	1,350	320×30	50,100

In the example cited the maximum forces in the branches of the column were found, viz., $F_{c\cdot br}=183.5$ tons for the crane runway branch and $F_{o\cdot br}=402$ tons for the outer branch.

The design combination of loads on the column for analyzing the anchor bolts (caused by the dead and the wind loads) are taken equal to

$$M=-170.9 \text{ t-m and } F=86.9 \text{ t}$$

The design resistance of grade 100 concrete to axial compression is $R_{con}=44$ kg/cm². The material of the footing is grade ВСт.3кп steel, and type Э42 electrodes are used for welding.

Solution. 1. We plan the approximate dimensions of the plate under the branches of the column on the basis of the width of the branches and overhangs of 10 cm each, altogether $55+2\times10=75$ cm, and assuming the approximate value of $R_{fd}=60$ kg/cm². We plan a width of the foundation exceeding that of the plate by 40 cm, or 400 mm, and a length greater than that of the plate by 60 cm or 600 mm (see Fig. 8-37).

$$A_{req_1}=\frac{F_{c\cdot br}}{R_{fd}}=\frac{183,500}{60}=3,100 \text{ cm}^2$$

$$A_{req_2}=\frac{F_{o\cdot br}}{R_{fd}}=\frac{402,000}{60}=6,700 \text{ cm}^2$$

We appoint the dimensions of the plates and the foundation

$$A_{pl\cdot c\cdot br}=45\times75=3,350 \text{ cm}^2 \qquad A_{pl\cdot o\cdot br}=90\times75=6,750 \text{ cm}^2$$

The part of the foundation area under each plate (assuming that the foundation is divided into two equal parts on the section between the plates) with a width of 1.15 m will be

$$A_{1\cdot fd}=1.15\times1.025\cong1.15\ \text{m}^2 \qquad A_{2\cdot fd}=1.15\times1.5=1.7\ \text{m}^2$$

The strength factors will be

$$\psi_1=\sqrt[3]{\frac{A_{1\cdot fd}}{A_{c\cdot br}}}=\sqrt[3]{\frac{1.15}{0.335}}=1.51 \qquad \psi_2=\sqrt[3]{\frac{1.7}{0.675}}=1.37$$

The design strength of the concrete under the plate will be

$$R_{1\cdot fd}=\psi R_{con}=1.51\times44=66.5\ \text{kg/cm}^2$$
$$R_{2\cdot fd}=1.37\times44=60.4\ \text{kg/cm}^2$$

The pressure on the concrete will be

$$\sigma_{c\cdot br}=\frac{F_{c\cdot br}}{A_{c\cdot br}}=\frac{183{,}500}{3{,}350}=55\ \text{kg/cm}^2<66.5\ \text{kg/cm}^2$$

$$\sigma_{o\cdot br}=\frac{F_{o\cdot br}}{A_{o\cdot br}}=\frac{402{,}000}{6{,}750}=60\ \text{kg/cm}^2<60.4\ \text{kg/cm}^2$$

2. We find the required thickness of the plate under the outer branch, considering that it resists bending. Part 2 of the plate behaves as a cantilever under a uniformly distributed load in the form of the upward pressure of the concrete $q=\sigma_{con}$ (Fig. 8-37).

The moment in the cantilever

$$M=\frac{\sigma_{con}c^2}{2}=\frac{60\times10^2}{2}=3{,}000\ \text{kg-cm}$$

Part 1 of the plate is supported on three sides and is also loaded with a uniformly distributed load $q=\sigma_{con}$. With a ratio between the sides of

$$\frac{a_1}{d_1}=\frac{450}{275}=1.63$$

we find from Table 8-11 the coefficient $\alpha_3=0.128$ (by interpolation). The maximum bending moment at the middle of the free side (with use made of a factor of 0.8 that takes into account the constraining of the plate) will be

$$M=0.8\ \alpha_3\sigma_{con}d_1^2=0.8\times0.128\times60\times27.5^2=4{,}640\ \text{kg/cm}$$

This moment is greater than the cantilever one, and therefore it is used to select the thickness of the plate. From equation (8-50)

$$t_{pl}=\sqrt{\frac{6M}{R}}=\sqrt{\frac{6\times4{,}640}{2{,}100}}=3{,}64\ \text{cm}$$

We take $t_{pl}=3.8$ cm or 38 mm.

3. We investigate the traverse and the ribs of the base. We design the traverse from plates 500×12, with the size of the welds securing the branch to the traverse $h_w=12$ mm. Assuming in the investigation of the welds that the force is transmitted from the branch to the base plate through the plates of the traverse (connected to the column branch by means of four welds) and the middle

ribs (also connected with four welds), and taking the effective length of a weld equal to $L_w = 50-2=48$ cm (where 2 cm is a deduction for lack of penetration at the ends of the welds), we find the stress in the welds

$$\tau = \frac{F_{o\cdot br}}{8L_w 0.7h_w} = \frac{402{,}000}{8\times 48\times 0.7\times 1.2} = 1{,}250 \text{ kg/cm}^2 < 1{,}500 \text{ kg/cm}^2$$

The stress in the welds connecting the plates of the traverse to the base plates, with $h_w = 10$ mm, will be

$$\tau = \frac{F_{o\cdot br}}{0.7h_w \sum L_w} = \frac{402{,}000}{0.7\times 1\,[2\,(90-2)+2\,(90-2-19)+2\,(90-5)]} =$$
$$= 1{,}190 \text{ kg/cm}^2 < 1{,}500 \text{ kg/cm}^2$$

We check the middle rib strengthening the base plate. This rib $500\times 450\times$ $\times 12$ mm in size resists the upward pressure of the concrete σ_{con} from a tributary area $550/2 = 275$ mm wide. The load acting on the rib will be (Fig. 8-37)

$$q_r = \sigma_{con}\times 27.5 = 60\times 27.5 = 1{,}650 \text{ kg/cm}$$

For a rib behaving as a cantilever constrained in a wall we find

$$M_r = \frac{q_r a^2}{2} = \frac{1{,}650\times 43.75^2}{2} = 1{,}570{,}000 \text{ kg-cm}$$

$$W_r = \frac{th^2}{6} = \frac{1.2\times 65^2}{6} = 844 \text{ cm}^3$$

$$\sigma = \frac{M_r}{W_r} = \frac{1{,}570{,}000}{844} = 1{,}870 \text{ kg/cm}^2 < 2{,}100 \text{ kg/cm}^2$$

The support reaction of the cantilever F_r tending to shear the rib with respect to the web is

$$F_r = q_r b = 1{,}650\times 43.75 = 72{,}200 \text{ kg}$$

We analyze the welds connecting the cantilever to the web. There are two welds each with a size of $h_w = 12$ mm. Each weld is subjected to the action of a shearing force F_r and a moment M. For this reason we check for the resultant stress

$$\sigma_{res} = \sqrt{\sigma_w^2 + \tau_w^2} \leqslant R_{w\cdot f}$$

here

$$\sigma_w = \frac{M_r}{W_w} = \frac{1{,}570{,}000}{1{,}180} = 1{,}330 \text{ kg/cm}^2$$

where

$$W_w = \frac{2\times 0.7h_w L_w^2}{6} = \frac{2\times 0.7\times 1.2\times 65^2}{6} = 1{,}180 \text{ cm}^3$$

$$\tau_w = \frac{F_r}{A_w} = \frac{72{,}200}{106} = 680 \text{ kg/cm}^2$$

in which

$$A_w = 2\times 0.7h_w L_w = 2\times 0.7\times 1.2\times 63 = 106 \text{ cm}^2$$

$$\sigma_{res} = \sqrt{\sigma_w^2 + \tau_w^2} = \sqrt{1{,}330^2 + 680^2} = 1{,}495 \text{ kg/cm}^2 < 1{,}500 \text{ kg/cm}^2$$

4. We analyze the anchor bolts. The required total sectional area of the anchor bolts connecting the outer branch of the column is found from equation (8-55)

$$A_{anch \cdot n} = \frac{M - Fa}{yR_{anch \cdot t}} = \frac{170.9 \times 10^5 - 86,900 \times 85}{125 \times 1,400} = 55 \text{ cm}^2$$

Here $a = 85$ cm = distance from the axis of the column to the middle of the base plate of the crane runway branch

$y = 125$ cm = distance from the centre line of the anchors being investigated to the middle of the same plate

$R_{anch \cdot t} = 1,400$ kg/cm^2 = design tensile strength of the anchor bolts.

The values of a and y are determined not from the expressions deducted for a solid base plate, but from the conditions of equilibrium, equating to zero the sum of the moments of all the forces with respect to the centre of the compressive stress diagram.

We divide the total sectional area of the bolts found in this way by 2 (the number of bolts)

$$A_{anch \cdot 1} = \frac{55}{2} = 27.5 \text{ cm}^2$$

We adopt bolts with a diameter of $d = 70$ mm and an embedded length in the concrete of $L_{emb} = 2,000$ mm.

CHAPTER NINE

Steel frames of single-storey industrial buildings

9-1. GENERAL

The steel skeleton or frame of an industrial building is the main load-carrying member supporting the roof and the walls, as well as the runways of overhead and other types of cranes serving the production process. Sometimes various technological equipment and working areas are supported directly on the frame.

Before designing an industrial building, the expediency of making the building of steel should be ascertained, since at present, in connection with the mastering of prestressed reinforced concrete members, the possibility of using such members in industrial construction is continuously growing.

The field of using steel and reinforced concrete in industrial construction in the USSR is determined in the main by the "Rules for the Economical Consumption of Metal, Lumber and Cement in Construction" (ТП 101-61) published by the State Building Committee of the Council of Ministers of the USSR.

When commencing to design an industrial building, it is essential already when drawing up the preliminary project report to obtain some information both of a technological and a general construction nature concerning the contemplated service of the structure.

The technological information concerns data on the location of railway sidings in the shop, the arrangement of the crane runways and the lifting capacity of the cranes, on special overall dimensions of machines, on different live loads and their dynamic action, on underground facilities, special working and repair areas, passageways and stairs, on the sequence of construction and the possibilities of development and expansion, on the location of service facilities, etc.

Information of a general construction nature includes the location of the shop on the general plan, appointment of the floor level elevation, data on the soils and groundwater table at the construction site, as well as the design strength of the soil to be used, data relating to local building materials, data on illumination, ventilation, heating and a number of other special requirements.

As has been indicated in a previous chapter, any structure being designed must comply with the service requirements, must be strong,

stable and have the required space stiffness. In every industrial building there should be created normal conditions of work for the employees (the shop should be light, warm or cool, as the case may be, and well ventilated), and also normal conditions for operation of the shop (the required stiffness of the crane runways, access for cleaning glazed surfaces, good drainage, reliable functioning of the gates, windows and doors, a proper selection of the floors, etc.).

Furthermore, a requirement that must be met in designing every structure is the greatest possible reduction of its weight, with minimum labour requirements for its erection. For this purpose a layout of the structure should be selected in which the force fields induced by the loads acting on the frame of the structure would pass into the ground along the most rational path. It is also essential that the material be utilized in the best possible manner which, in particular, leads to the requirement of a certain concentration thereof.

To ensure greater industrialization and reduce the labour requirements for the fabrication and erection of the members, care should be taken to ensure the greatest possible repetition of elements and their simple form.

All industrial buildings can be divided into single-storey and multistorey ones. The buildings encountered most frequently are single-storey ones, which are divided in turn into single-span and multispan buildings.

The main elements of the load-carrying steel framework of an industrial building (Fig. 9-1), which resist almost all the loads acting on a shop, are the flat lateral frames formed by the columns and the roof trusses (collar beams). These frames are installed one after the other with a certain distance between them. The lateral frames carry the longitudinal elements of the framework such as the crane girders, the collar beams of the wall framework, the purlins of the roofing, and the skylights. The framework of a building must have space stiffness, which is attained by designing braces and ties in a longitudinal and lateral directions, as well as by rigid connection of the frame beam to the columns.

When it is necessary to space the columns far apart along the middle rows in multispan shops, the intermediate roof trusses are supported on secondary trusses installed along the longitudinal rows of columns (Fig. 9-2).

The roofing, designed for protecting the building against the weather, together with the members supporting it, namely, the roof and secondary trusses, purlins, skylights, etc., is called the *roof*. The main design loads for the members of the roof are the snow load and the own weight of the roof. Members that reinforce the wall or support separate sections of the wall are referred to as the *wall framework*. The main loads for the elements of the wall framework are

the weight of the wall in a vertical direction and the wind load in a horizontal one.

The main loads on a lateral frame are the load of the roof and the wall framework, as well as the action of the crane load, consisting of the vertical pressure of the cranes and the horizontal braking forces acting in a transverse and a longitudinal directions. For resisting the lateral braking forces acting on the crane girders, horizontal

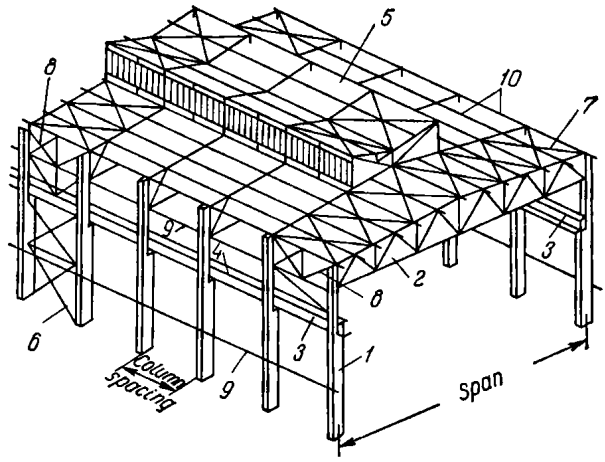
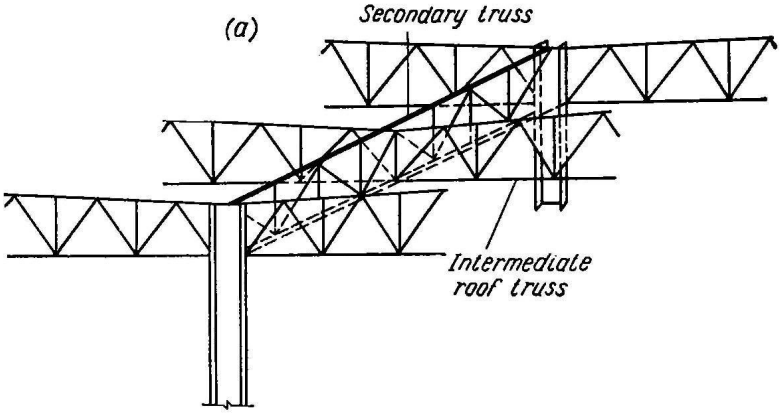


Fig. 9-1. Principal elements of steel framework of industrial building:
1—column of frame; *2*—roof truss (truss of frame); *3*—crane girders; *4*—bracing beam; *5*—skylight; *6*—vertical ties between columns; *7*—horizontal ties of roof; *8*—vertical ties of roof; *9*—wall framework; *10*—purlins

bracing beams are installed, while the longitudinal braking forces are resisted by vertical braces or ties between the columns. An additional load on the frame is the wind load (see page 30).

The general dimensions of a shop, namely, its span (Fig. 9-1), height to the elevation of the top of the crane runway rail h_1 (Fig. 9-3), as well as the total height of the shop up to the bottom of the frame collar beam H are established depending on the size of the equipment to be accommodated and the nature of the production process to be followed in the shop. In accordance with the general rules for standardization in buildings with travelling cranes, the height of the premises from floor level to the bottom of the load-carrying members of the roof should be a multiple of 1.8 (the width of a precast wall panel) as illustrated in Fig. 9-3. The magnitude of the clearance for the crane above the top of the rails of the crane runway H_c is taken according to the USSR Standard for overhead travelling cranes.

The total height of the shop to the bottom of the collar beam H (Fig. 9-3), which is a multiple of 1.8, is made up of the height from the floor level to the head of the crane runway rails h_1, and the height h_2 including the crane clearance H_c with an additional 200-250 mm



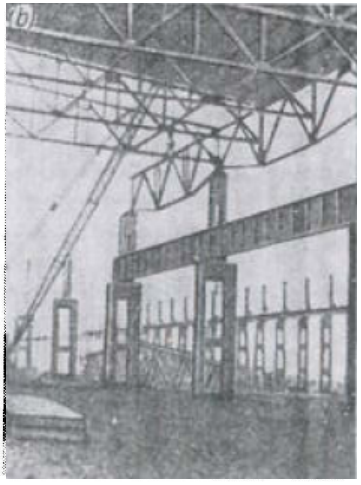


Fig. 9-2. Supporting of roof trusses on secondary ones

allowing for possible deflection of the roof trusses and braces along their bottom chords, and also the usual design of these braces with the angle legs outstanding downward (see Fig. 9-12).

The selection of the design and layout of both single-span and multispan shops depends to a considerable extent on a number of factors, the most important of which are:

1. The magnitude of the crane loads, as well as the duty of the cranes and the operating conditions in the shop as a whole.

2. The height of the shop.

3. The roof load (the type of roofing, the magnitude of the snow load).

With a view to the above factors, all shops can be divided into three groups, viz.; light, medium capacity and heavy shops.

Of special importance, mainly for iron and steel works, are the conditions of work of the shop. When these are heavy, there appear a number of additional requirements which the design of the structure must meet. Buildings with heavy service conditions include shops with continuous three-shift operation of cranes with a very heavy and very heavy continuous duties; the cranes and crane runways of such shops must be repaired without interrupting the production process. These shops include, for example, the main buildings of steel foundries (open-hearth, etc.), the bedding plant, the ingot pit building, certain bays of rolling mills, ingot and pig iron stores.

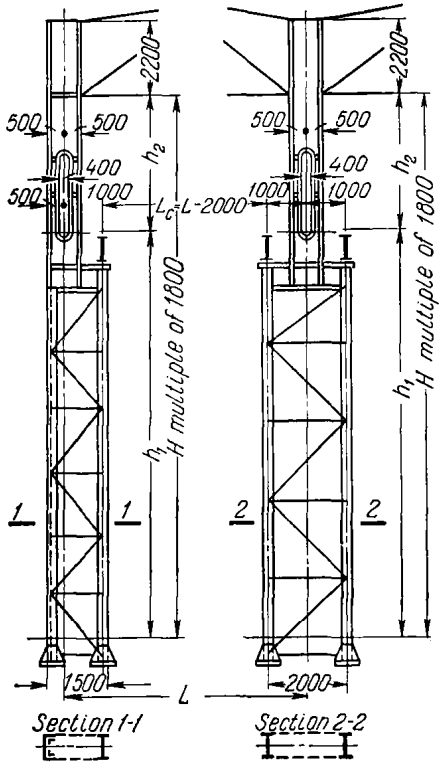


Fig. 9-3. General dimensions in height of shop. Horizontal coordination of columns with passages in shops with heavy service conditions

As has already been mentioned, the steel members in shops with heavy service conditions must meet somewhat stricter design requirements. This especially relates to fastenings and connections, which must have a higher reliability (see Table 6-11).

In shops with heavy service conditions, openings (400 mm wide) must be provided in the upper part of columns along the line of passageways provided at the level of the crane girders (Fig. 9-3). These passageways are guarded with railings on the crane side along the entire length of the crane runway (see Fig. 6-49).

The most stable index that can be used to judge whether the design of the steel framework of an industrial building is sufficiently rational is the amount of metal required per square metre of building area.

Table 9-3 (page 391) gives the approximate weights of members for different groups of shops.

9-2. COLUMN LAYOUT

(1) Column Spacing and Span

When designing the steel framework of a shop, it is necessary first of all to plan the so-called *column grid*, i.e., the layout of the columns, which includes the spans (bays) and the spacing of the columns (the distance between the columns along the shop). To ensure the greatest possible repetition of members, a constant spacing of the columns should be employed, which is a multiple of a definite predetermined quantity known as the *module.*

The *modular system* provides for integer relations between the principal dimensions of structures and their elements and serves as the basis for unification and standardization of members.

The basic module used in single-storey industrial buildings is three metres. In this connection it is recommended to design spans up to 18 metres long with a length that is a multiple of three metres, and over 18 metres long—a multiple of six metres. It will be good practice to design the distances between column centres in a longitudinal direction (the column spacing) as multiples of six metres. Most frequently the spacing of columns is taken equal to 6 or 12 metres (if the technological conditions do not require a greater spacing). With an increase in the height of the building the optimal spacing grows.

The size of the spans and the spacing of the columns depend primarily on the production process of the shop, and on the required manoeuvrability of the cranes and other kinds of intra-shop transportation facilities serving this process. At present production processes are being rapidly perfected and often a corresponding reconstruction is required inside an already erected structure. This has led to the tendency of increasing the column spacing and spans, which ensures greater flexibility in meeting the demands of various production processes.

Of special importance in laying out a column grid is its coordination with the production process in the shop being designed, bearing in mind the necessity of retaining the selected span and spacing modules. Attention must be given to the location of the column foundations in respect to the underground facilities (ducts, foundations for equipment, furnaces, etc.), and also to the necessity of designing railway entrances that require definite clearances.

(2) Expansion Joints

The total length and width of a shop are determined by the technological conditions in it. If a building has a considerable length or width, the danger appears of significant deformations of its separate

elements owing to summer and winter temperature changes. The magnitude of the temperature strain is $\Delta = \alpha L t$, where α is the coefficient of linear expansion of steel ($\alpha = 0.000012$), L is the length and t is the temperature difference. Figure 9-4 depicts the growth of strains in the columns of a shop from the centre of the building to its edges caused by an increase in the temperature.

Long shops should be divided into separate sections (compartments or bays) with expansion joints between them. The distance between

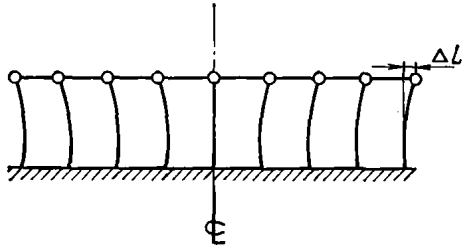


Fig. 9-4. Temperature strains of columns

expansion joints in steel structures, for which the standards and regulations permit no account to be taken of the temperature stresses, should not exceed the values indicated in Table 9-1.

TABLE 9-1. **Maximum Permissible Dimensions of Expansion Sections of Buildings and Structures, m**

Category of building or structure	Maximum distance from end of section to centre line of nearest vertical brace	Maximum length of section (along building)	Maximum width of section (building)
Heated buildings	90	230	150
Unheated buildings and hot shops	75	200	120
Open trestles	50	130	—

When within the limits of an expansion section of a building two vertical ties or braces are used between columns (see page 383), the distance between their centres should not exceed 50 metres for buildings and 30 metres for open trestles.

When precast reinforced concrete columns are used, the expansion joints are spaced not more than 60 metres apart, and with self-bearing brick walls—40 to 60 metres apart.

The best way of making the expansion joints in the framework of industrial buildings is to install double lateral frames (on a common foundation), i.e., double columns in each row and correspondingly two roof trusses, etc., in other words to design what may be called two separate building sections. In this case, in accordance with the basic rules for the unification of industrial building members, the centre line of the expansion joint is made to coincide with the marking out line, while the axes of the columns are displaced from the expansion joint centre line by 500 mm (Fig. 9-10). Such a design makes it possible to use walls made from standard precast wall panels. Sometimes, however, with a view to the arrangement of the equipment, the expansion joints may be designed with an additional short section inserted between the double frames. In this case the axes of the columns will coincide with the centre lines of the rows.

The design of expansion joints by employing connections of the member elements which permit movement in a longitudinal direction (for example, with the aid of oval holes) has been proved by practice to be insufficiently reliable.

9-3. LATERAL FRAMES

(1) Single-Span Shops

Some examples of single-span shops are forge and smith's, press, heat treatment shops, ingot pit buildings, ingot mould yards, and mixer buildings. Generally these are shops that require a high lateral stiffness owing to the necessity of installing crane runways which are rigid in a horizontal direction. Such a requirement will be best met by a lateral design of the shop structure in the form of a frame. The frames can be of three types, viz., two-hinged with the hinges at the corners (Fig. 9-5*a* and *d*), two-hinged with hinges at the supports (Fig. 9-5*b*) and fixed (Fig. 9-5*c* and *e*).

The fixed type of frame is the most rigid and economical one, and for this reason it is the basic type of lateral member used for the steel frameworks of single-span industrial buildings.

The most widely used type of collar member is a truss, which is lighter and stiffer than a beam or girder.

The roof truss, which serves, as has been mentioned previously, as an element of the frame in industrial buildings with steel columns, where a high lateral rigidity of the shop is required, is rigidly connected to the columns. With such a design the truss will have fixed end conditions, which will result in a support moment M_s appearing at the supports besides the support reaction (Fig. 9-6*a*). Upon dividing the support moment M_s by the depth of the truss at the support h_s, we shall obtain the horizontal forces F_h (a pair of forces) which act

both on the truss and on the columns (Fig. 9-6*b*)

$$F_h = \frac{M_s}{h_s} \tag{9-1}$$

The support moments are found when investigating the frame by the general methods of structural mechanics. Depending on the direction of the horizontal forces F_h, some of the elements of the truss will be additionally loaded by these forces (in addition to the vertical loads), while others will have their loads reduced.

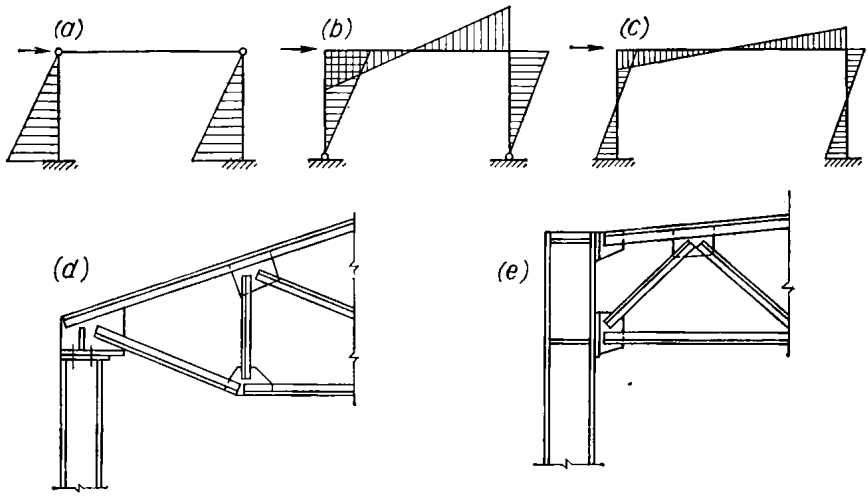


Fig. 9-5. Diagrams of lateral frames

Usually the forces in the elements are determined for the two combinations of the support moments creating the worst possible conditions for the behaviour of the chords (Fig. 9-6*c*) and for that of the diagonals (Fig. 9-6*d*).

Plotting of Cremona's force plan for the computed horizontal forces F_h yields the additional forces in the elements of the trusses.

The design force in each of the elements of a truss will be the sum of the force induced by the vertical loads found as for a statically determinate truss, and the force originated by the support moment. When the support moment leads to relieving of the truss elements, this reduction in the forces is not ordinarily taken into account, since it is considered that the truss may behave as a statically determinate system, without the relieving influence of the support moments (i.e., only subjected to vertical loads).

A truss is rigidly connected to a column in the way shown in Fig. 9-7*a*. Here in the lower support joint the support pressure F_s and

the horizontal force F_h (appearing as the result of the angular moment of the frame) are transmitted separately. The support gusset plate, made in the form of a welded T section, transmits the support pressure F_s to the seat (via the milled surfaces) the bearing plate protruding by 10-20 mm to ensure better transmission. The thickness of the plate is taken equal to 16 mm. The seat, made from an angle stub or a thick plate (t =30 mm) welded to the column, is designed somewhat wider than the bearing plate. Each of the vertical welds used to connect the seat to the column is computed for 2/3 F_s owing to possible unevenness in load transmission. The horizontal forces F_h may

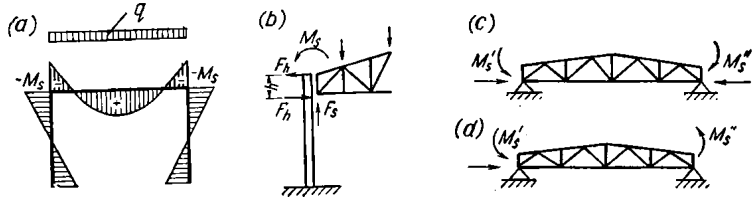


Fig. 9-6. Support moment in frame with collar truss and design combinations of moments in support sections of trusses

originate in the joint compression and tension (tearing away of the joint from the column). The tension is resisted by unfinished bolts. The force F_h applied at the level of the truss chord centre line does not pass, as a rule, through the centre of the bolted connection. It is conditionally assumed here that the resulting rotation of the joint takes place about a line passing through the axis of the bolts which are the remotest from the point of application of the force F_h (about 40-80 mm below the top of the gusset). Hence the force applied to the most loaded bolt located at the bottom (Fig. 9-7*b*) will be

$$F_{max} = \frac{1}{2} \frac{F_h z L_1}{\sum L_i^2} \qquad (9\text{-}2)$$

where z = distance from the centre line of the bottom truss chord (line of application of force F_h) to the axis of the remotest bolts (conditional axis of rotation of the joint)

L_1 = distance between extreme bolts

$\sum L_i^2$ = sum of squares of distances between axes of bolts and axis of rotation of joint (in the example shown in Fig. 9-7 we have $\sum L_i^2 = L_1^2 + L_2^2$)

$^1/_2$ = factor indicating the presence of two bolts in each horizontal row of the connection.

The holes for the bolts in the bearing plate are made 3-4 millimetres greater than the diameter of the bolts. This is done to prevent the unfinished bolts from taking the support reaction in case of slight

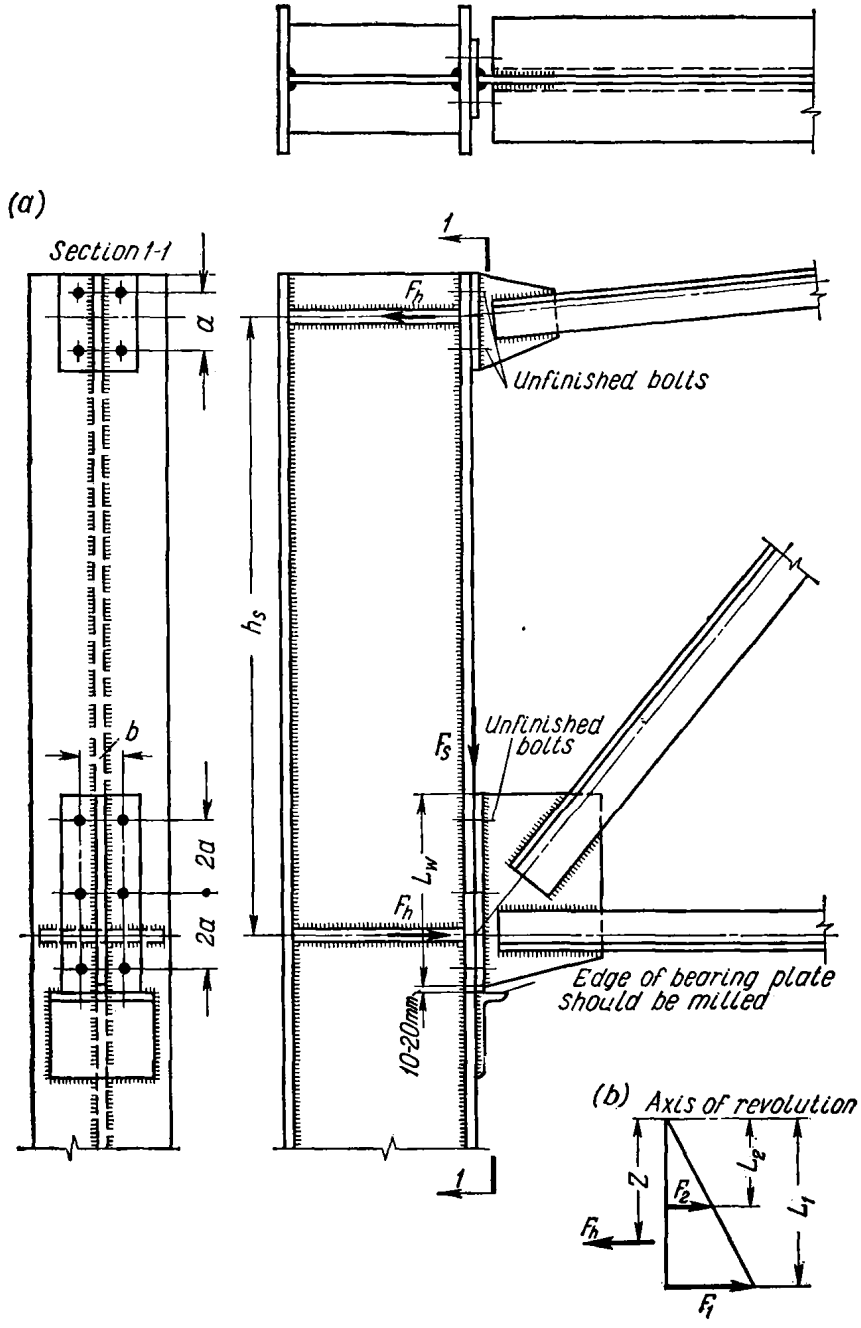


Fig. 9-7. Rigid truss-to-column connection

deviations in the connection of the seat in height. These bolts are designed to withstand only tension.

The weld *c* (Fig. 9-7*a*), by means of which the gusset is connected to the bearing plate, is in a combined stress state. It transfers to the column the support pressure F_s and the eccentrically applied force F_h. The bottom point of the weld will be subjected to the greatest stress. The following conditional formula is used for analysis

$$\sqrt{\sigma_w^2 + \tau_w^2} \leqslant R_{w \cdot f} \tag{9-3}$$

Here

$$\sigma_w = \frac{F_h}{1.4 h_w L_w} + \frac{6 F_h z_1}{1.4 h_w L_w^2}$$

$$\tau_w = \frac{F_s}{1.4 h_w L_w}$$

in which h_w = size of weld
z_1 = eccentricity of force F_h with respect to middle of weld
L_w = length of weld.

(2) Ties

In order to ensure space rigidity of a shop framework, as well as to ensure stability of the frame elements, ties or braces are installed between the frames. There are distinguished horizontal ties in the plane of the top and bottom truss chords, and vertical ones both between the trusses and between the columns.

The main purpose in installing ties is to:

1. Ensure the absence of changes in the structure both in service and in the process of erection.
2. Ensure stability of the compression elements of the structure.
3. Resist and distribute all the horizontal loads (wind and inertial ones such as, for example, the braking forces of cranes).

The designation of horisontal ties along the top chords of trusses was considered in Sec. 7-4. These ties and braces ensure stability of the top chords of trusses in a direction perpendicular to their plane. Figure 9-8 contains an example of the location of ties along the top chords of trusses in a roof with purlins.

In roofs without purlins, where large-size reinforced concrete slabs are welded to the top chords of the trusses, the rigidity of the roof is so great that there seems to be no necessity of installing ties. Taking into account, however, the requirement of ensuring adequate rigidity of the members during installation of the slabs, it is necessary to provide ties along the top chords of the trusses at the edges of the expansion sections. Braces must also be installed at the ridge of the trusses, at the supports and sometimes under the extreme skylight verticals. These braces serve for connecting together the top chords of all the intermediate trusses. The slenderness ratio of the top chord

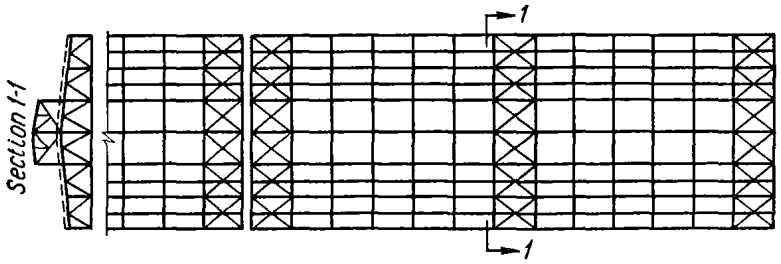


Fig. 9-8. Ties along top chords of trusses

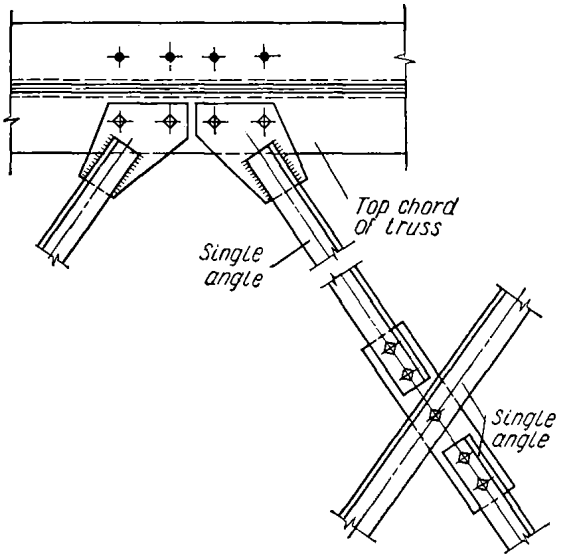


Fig. 9-9. Connection of ties to top truss chord

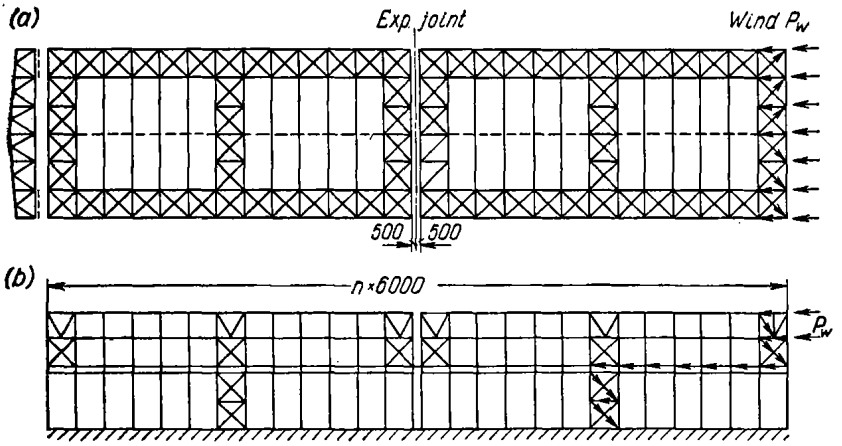


Fig. 9-10. Ties along bottom chords of trusses and between columns

between the points braced during the period of installation of the slabs should not exceed 220. The ties along the top chords of roof trusses are secured to the chords with unfinished bolts (Fig. 9-9).

The horizontal ties along the bottom chords of the trusses (Fig. 9-10*a*) are arranged both across the shop (lateral ties) and along the shop (longitudinal ties). The lateral ties located at the ends of the shop are used in the capacity of wind trusses. They carry the struts of the end wall framework that take the pressure of the wind (Fig. 9-10*a*). The bottom chords of the roof trusses serve as the chords of the wind truss. The same kind of lateral ties along the bottom chords of the

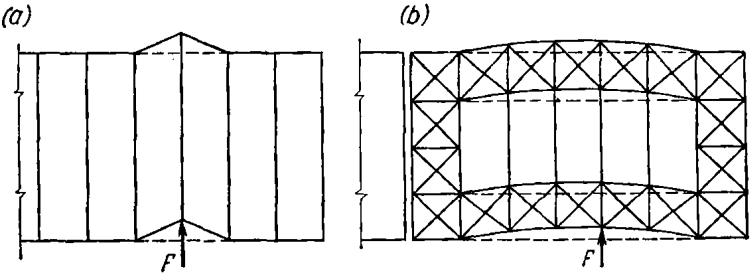


Fig. 9-11. Lateral deformation of framework under local (crane) load

trusses are installed at the expansion joints (in order to form a rigid roof). With a large length of an expansion section the lateral ties are also installed at the middle part of the section (Fig. 9-10*a*) to ensure that the distance between lateral ties does not exceed 50-60 metres. This is necessary because the ties are often connected to the trusses by means of unfinished bolts that permit considerable displacements, and therefore the influence of the ties does not spread over large distances.

With a 12-metre truss spacing, the horizontal tie trusses are made with a width of 6 metres, arranging them on braces connected at the joints of the roof trusses.

The horizontal longitudinal ties along the bottom chords of the trusses are chiefly designed to make the adjacent frames participate in space load resistance under the action of local loads, such as crane loads. This reduces the deformations of the frame and improves the lateral rigidity of the shop (Fig. 9-11*a* and *b*). Longitudinal ties are of special importance in shops with heavy service conditions, as well as with light and nonrigid roofings (of corrugated steel, asbestos-cement sheets, etc.).

In the absence of such ties in rigid roofings the horizontal crane loads are distributed by the large-size reinforced concrete slabs, which may lead to disarrangement of the roofing, and sometimes

of the slabs themselves, since the latter are not designed to resist shearing forces. Besides, in this instance the welded connection of the slabs to the trusses may fail.

With two or more spans there is no necessity of installing longitudinal ties on the bottom chords of the trusses along the middle rows of columns in both spans. The first panel of the bottom chord, however, must be reinforced with braces to permit the chord to serve as the support of the second compression diagonal of the trusses perpendicular to its plane.

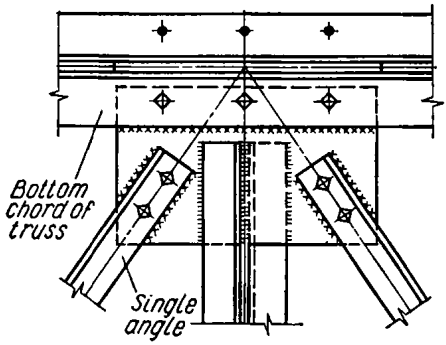


Fig. 9-12. Connection of ties to bottom chord of truss

In buildings with heavy service conditions, the ties should be welded to the bottom chord (Fig. 9-12); unfinished bolts will be sufficient in all other instances.

When bracing trusses the rule is to use a cross lattice, it being assumed that loads acting on any side will be resisted only by the system of tension diagonals, while the other part of the diagonals (the compression ones) will be idle. Such an assumption is true if the diagonals are slender ($\lambda > 200$). For this reason the elements of cross ties, as a rule, are designed of single angles. When checking the slenderness ratio of cross lattice tension diagonals of ties made from single angles, the radius of gyration of the angle is taken with respect to the axis parallel to the leg. With a triangular lattice of the tie trusses, compressive stresses may appear in all the diagonals, and therefore they must be designed with a slenderness ratio of $\lambda \leqslant 200$, which is less economical.

In spans over 24 metres long, owing to limiting of the lateral slenderness ratio of the bottom truss chords, it often becomes necessary to install additional braces at the middle of the span (Fig. 9-10*a*). This eliminates vibration of the trusses during operation of the cranes.

The vertical ties between trusses are generally installed at the truss supports (between the columns) and at the middle of the span (or under the verticals of the skylight), locating them along the length of the shop in the rigid panels, i. e., where the horizontal lateral ties are installed along the chords of the trusses.

The principal purpose of installing vertical ties is to ensure a rigid unchangeable state of a space member consisting of two roof trusses and lateral ties along the top and bottom chords of the trusses. The vertical ties are designed in the form of a cross of single angles, always with a horizontal closing element, or in the form of a

small truss with a triangular lattice. Unfinished bolts are used to connect the vertical ties to the roof trusses (Fig. 9-13).

In view of the low magnitude of the stresses acting in the elements of roof ties, a slight eccentricity may be tolerated when designing their connections.

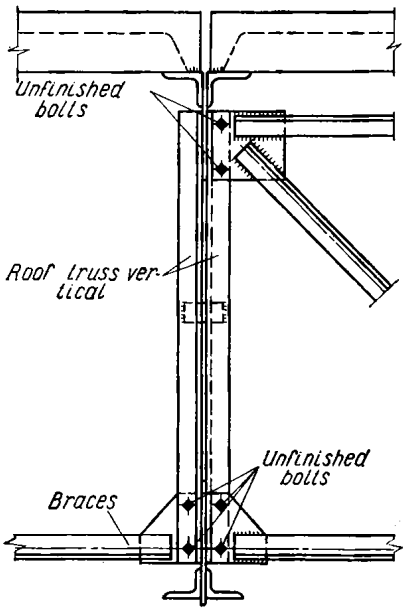


Fig. 9-13. Connection of vertical ties to roof truss

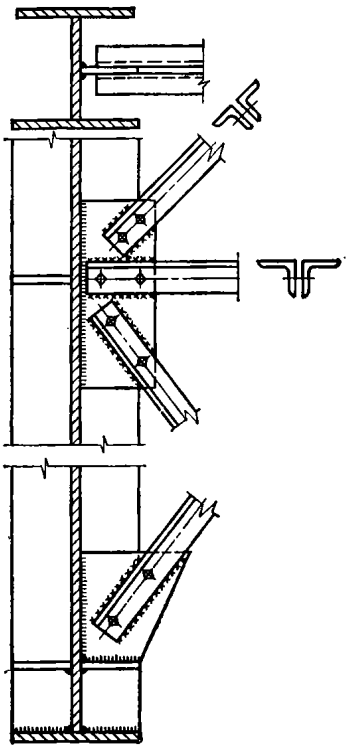


Fig. 9-14. Design of vertical ties and their connection to a column

Vertical ties are installed along a shop between columns to ensure stability of the shop in a longitudinal direction, and also to resist the longitudinal braking forces and the pressure of the wind on the end of the building (Fig. 9-10*b*). In a lateral direction frames constrained in their foundations are an unchangeable system, but in a longitudinal direction a number of frames pin-connected by crane girders represent a changeable system, which in the absence of vertical ties between the columns may collapse (the supports of the columns should be considered as pinned ones in a longitudinal direction). For this reason the elements of the ties between columns (below the crane girders), which are quite important for the stability of the

structure as a whole, are designed with adequate stiffness to avoid their vibration. This is achieved by limiting the maximum slenderness ratio of such elements to $\lambda=150$ for compression elements and to $\lambda=300$ and 200 (the latter in buildings with heavy service conditions) for tension ones.

For other tension elements of the ties between columns, the slenderness ratio should not exceed $\lambda=400$, and for compression elements $\lambda=200$. The elements of cross ties between columns are generally made from angles (Fig. 9-14). Especially heavy cross ties are made from pairs of channels connected by a lattice or batten plates.

When determining the slenderness ratio of crossing elements (in a cross lattice) their effective length in the plane of the lattice is taken from the centre of the joint to their intersection, while the effective length of elements in a direction perpendicular to the plane of the truss is taken from Table 9-2.

TABLE 9-2 **Effective Length of Cross Lattice Elements Perpendicular to Plane of Truss**

Nature of intersection joint of lattice elements	Tension in supporting element	Idle supporting element	Compression in supporting element
Both elements are continuous . . .	$0.5L$	$0.7L$	L
The supporting element is interrupted and covered with a gusset plate	$0.7L$	L	L

Vertical cross ties are also ordinarily investigated with the assumption made that only the tension elements withstand the total load. If the behaviour of the elements of a cross lattice in compression is also taken into account, then the load is distributed equally between the diagonals.

To ensure free longitudinal expansion or contraction of the framework in a longitudinal direction upon changes in temperature, it is good to arrange the vertical ties between the columns at the middle of an expansion section or near it (Fig. 9-10*b*). Since erection of a structure generally begins from its ends, however, it is desirable to connect the first two columns into a frame in such a way as to ensure their stability. This makes it necessary to design the ties as illustrated in Fig 9-10*b*, i. e., to install ties only within the limits of the upper part of the column in the end panels. Such ties make possible bending deformation of the lower parts of the columns upon changes in temperature. At the same time one of the diagonals, tensioned

by the wind load, transmits the resulting forces to the crane girder. The further path of the wind load forces is shown in Fig. 9-10*b*; they are transmitted along the rigid crane girders to the middle ties, and along the latter travel down into the ground. It is desirable to select such a layout of the ties in which they will be connected to the columns at an angle close to 45 deg (Fig. 9-14), as otherwise too greatly extended and heavy gussets will have to be used.

If for technological reasons not a single span can be completely used for the installation of ties, and also with large column spacings, use is made of frame ties (Fig. 9-15). In this instance it is considered

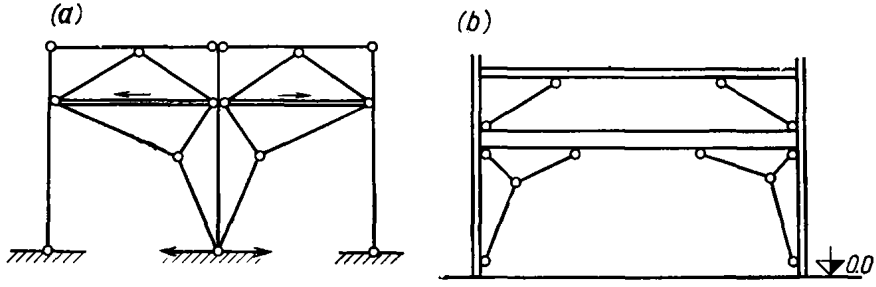


Fig. 9-15. Vertical frame ties:
a—with column spacing of 6 metres; *b*—with column spacing of 12 metres and above

that upon application of a one-sided load the ties of one corner resist tension, while the elements of the other corner, owing to their high slenderness ratio ($\lambda = 200\text{-}250$), are idle. With such behaviour of the member we obtain a "three-hinged arch".

Vertical ties between columns under the crane girders are installed in the plane of the crane runway branch of the column, while those above the crane girders are located in the plane containing the axis of the column section.

Unfinished bolts are used to secure the elements of the ties to the columns, while in buildings with heavy service conditions the ties below the crane girders must be welded to the columns (Fig. 9-14).

(3) Multispan Shops

The selection of the cross section of multispan shops depends not only on the required clear dimensions of the shop and the size of the overhead cranes, but also on a number of general construction requirements, first of all on the method to be used for draining water from the roof and on the system of illumination of the middle bays. The water drainage system can be either an external or an internal one. External water drains are installed in narrow shops, as well as in

unheated hot shops with cold roofs. The maximum width of a building with a ridge roof and with external water drainage is 60 metres for heated buildings and 100 metres for unheated ones.

In wide multispan buildings the drains are of the internal type (the water is drained off through pipes into the internal sewerage system).

Figure 9-16 presents an example of a multispan building with internal water drainage and with skylights used for lighting.

In multispan shops the simplest possible cross section (without any changes in height) should be employed, and the maximum repetition of elements should be ensured. It is permitted, when necessary, to design differences in height of at least 2 metres.

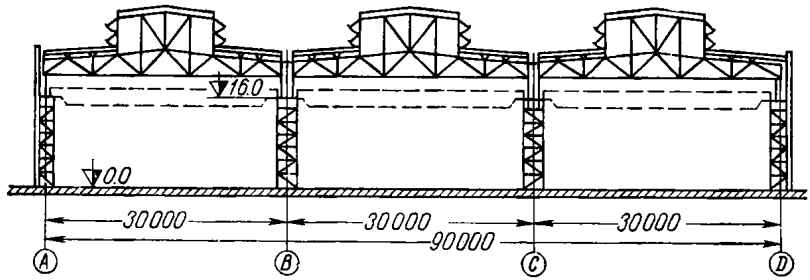


Fig. 9-16. Multispan shop

Heavy two-span shops, unlike light ones, require a high lateral rigidity, which is attained by designing stiff frame systems.

The various layouts of the lateral members of such shops can be divided into two main groups. Layouts of the first group consist of a number of parallel frames supporting the longitudinal members (Fig. 9-17*a*). In two-span shops it is not always possible to arrange all the columns in one cross section of the shop, and therefore the lateral frames may have the form of inverted U- and L-shaped frames with intermediate supporting of the roof trusses on secondary ones (Fig. 9-17*b*).

The layout under consideration can be solved in the form of separate plane frames functioning independently, i. e., resisting all the loads originating only on their own tributary area. Hence those columns whose number is less in a cross section will have to be considerably heavier than the columns of the main frames. Such a design (in the form of separate plane frames) is not rational. With a different solution of this layout in the form of a group of frames the space behaviour of the framework is taken into account. With the provision of horizontal ties and other longitudinal elements that can play a reliable part in the resistance of the framework to a local lateral load, the frames can be considered not as separate ones, but as the

elements of a space system including both main and intermediate frames. In this case the displacements of the frames at the level of the longitudinal members connecting them will not be independent of each other, but will be bound by definite relationships depending on the stiffness of the longitudinal members.

Upon the provision between the bottom chords of wide horizontal ties that are reliably connected to the trusses, it can be assumed that these ties and the roof form a very rigid construction, so that

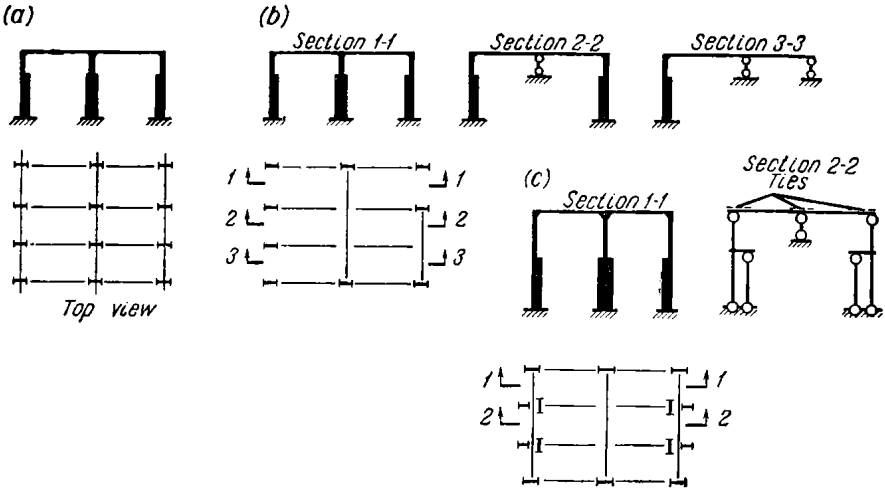


Fig. 9-17. Layouts of lateral members of heavy two-span shops

the displacements of the main frame and the adjacent ones can be considered to be the same. This determines the part played by each frame in the general functioning of the framework and, correspondingly, the section of the columns. Thus, when a system of frames functions simultaneously, the load is distributed between the columns in proportion to their stiffnesses, and they can be designed with a lighter weight. Such a solution (in the form of a system or group of frames) is more rational.

In the second group of layouts the members of the framework are divided with a view to their functional designation. The crane runway column branches, as well as the branches supporting the roof trusses, are designed as separate members and, all of them being pin-connected, are loaded with an axial vertical load (Fig. 9-17*c*). To ensure rigidity of the system when subjected to displacements in a horizontal plane, as well as for resisting horizontal loads, use is made of longitudinal horizontal ties and wide crane runway bracing beams. These transmit all the horizontal forces to the main frames

located at a considerable distance from each other and, therefore, of a heavier design. Such a functional division of the members makes it possible to attain a high degree of element repetition and, consequently, a reduction in the number of different standard shapes required. With heavy cranes, layouts of the second group result in a somewhat lower consumption of steel than those of the first one.

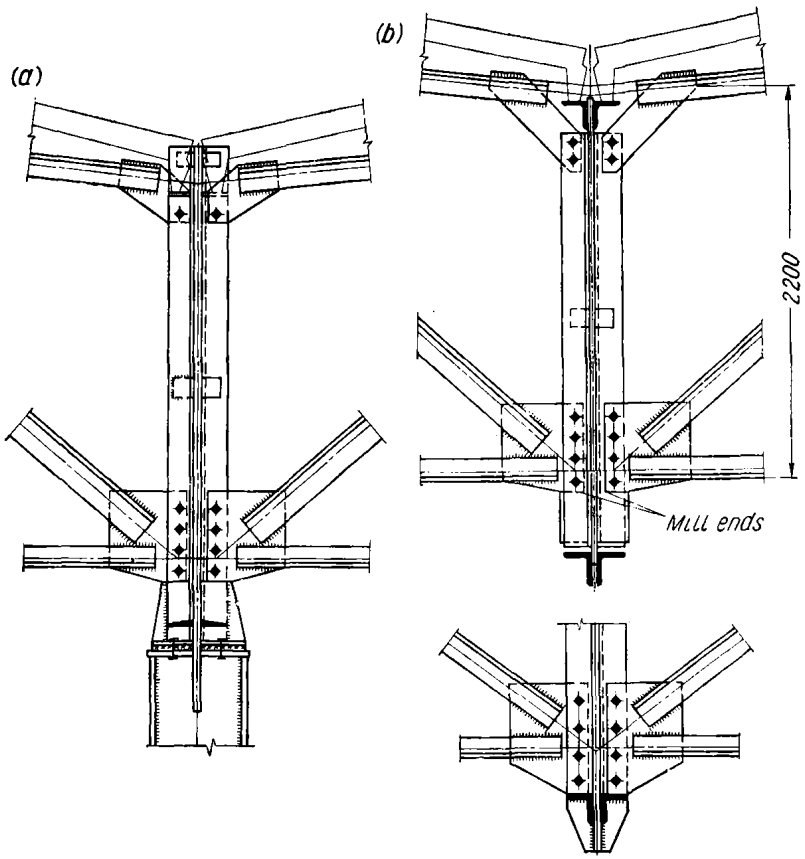


Fig. 9-18. Pinned truss-to-column connection

In multispan shops horizontal rigidity is mainly ensured by the several rows of columns, and for this reason pinned connection of the collar trusses to the columns can be tolerated. This permits designers to make wide use of standard roof trusses (see Fig. 7-14).

The lateral frames of multispan heavy shops can sometimes be designed with fixed frame joints only along the extreme rows of columns, with pinned connections along the middle rows.

An example of a pinned connection of trusses to steel columns is illustrated in Fig. 9-18*a*. A similar design can be used for supporting roof trusses on reinforced concrete columns (see page 299), as well as on secondary trusses (Fig. 9-18*b*). Its essence consists in the roof trusses resting with their bearing gussets on an upright having a cross-shaped section through a specially welded on seat. If a secondary truss is connected to the column, a similar joint is made on the other chords of the cross-shaped section. The milled surfaces of the bearing gussets should be investigated for crushing.

The pinned connection of a roof truss to a column can be designed similar to a rigid one (see Fig. 9-7*a*), but with the installation of a yielding fastening in the upper joint. This is attained by employing a thin easily deformable batten plate (flange) that is bolted to the column (see page 398).

9-4. SPECIAL FEATURES OF LATERAL FRAME ANALYSIS

(1) Design Loads

The lateral frames of industrial buildings are investigated for the following loads:

1. The dead load of the roofing and roof members.
2. The snow load.
3. The dead load of the walls (when supported on the framework).
4. The vertical and horizontal crane loads.
5. The wind load on the walls and skylights of the building.

The first four kinds of loads are included in the main combination of loads when cranes with heavy duties are to be installed. With light or medium duty cranes the main combination should include either the crane or the snow load.

The wind load forms part of the additional load combination.

The dead load of the roofing and roof members, together with the snow load, is transmitted to the column in the form of the support load of the truss P_t (Fig. 9-19*a*), which is applied to the upper part of the column eccentrically. Besides, the upper part of the column also supports eccentrically part of the wall. The load originated by the latter is transmitted through the elements of the wall framework in the form of separate concentrated forces P_c.

The design vertical load of cranes on one column is determined by adding to the corresponding load influence line the load caused by the crane wheels P, which is taken in accordance with the standards for cranes (Fig. 9-19*c*). As previously indicated, the investigations are generally carried out for two cranes, and the maximum and minimum pressure on each column are determined

$$F_{max} = n\sum P_{max} y \quad F_{min} = n\sum P_{min} y \tag{9-4}$$

where $n = 1.2$ = load factor for crane loads
y = ordinates of influence line (Fig. 9-19*c*)
P_{max} = maximum load on crane wheel (according to standards)
P_{min} = minimum load on crane wheel determined from the equation

$$P_{min} = \frac{Q+G}{0.5N_{cw}} - P_{max} \tag{9-5}$$

in which Q = lifting capacity of crane
G = total weight of crane with truck
N_{cw} = total number of crane wheels

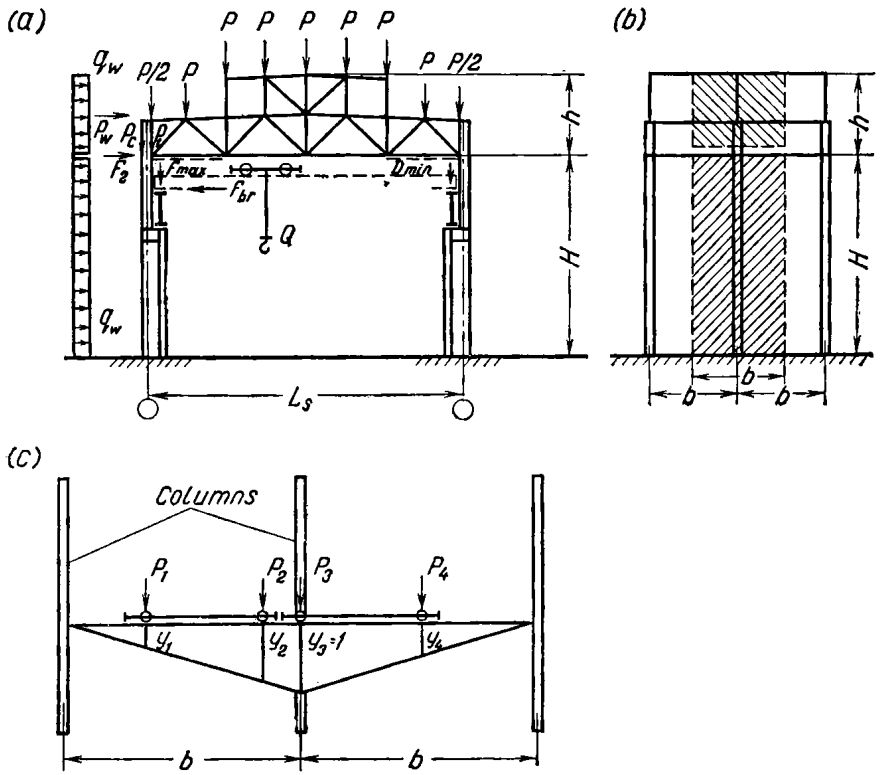


Fig. 9-19. Loads on lateral frame and influence line of crane load on columns

To determine how lateral braking of the cranes acts on the frame, it is first necessary to find the force applied to each wheel of the crane

$$F_{br.cw} = \frac{Q+g}{20N_1} \tag{9-6}$$

where g is the weight of a crane trolley, and N_1 is the number of wheels of each crane on one side.

The total action of the braking forces on the frame is computed in the same way as for the vertical load, and using the same line of influence

$$F_{br} = N \sum F_{br \cdot cw} y$$

The lateral braking force F_{br} is transmitted to only one of the frame columns and may be directed to any side.

The longitudinal braking force transmitted to the vertical ties between the columns is determined from the equation

$$F_{br \cdot l} = 0.1 n P_{max} N_{br} \tag{9-8}$$

in which N_{br} is the number of braking wheels on one girder (equal to half the total number of wheels on the girder).

The wind load is taken as prescribed by the Building Standards and Regulations, assuming that it acts as a uniformly distributed load

$$q_w = n q_r C_a b \tag{9-9}$$

TABLE 9-3 **Approximate Weights of Elements of Steel Industrial Building Frameworks in kg per sq m of Building Area**

Elements of steel framework	Group of shops		
	Light	Medium	Heavy
Roof:			
roof trusses	16-25	18-30	20-40
secondary trusses	0-6	4-7	8-20
purlins	10-12	12-18	12-16
skylights	0-10	8-12	8-12
ties	3-4	3-5	8-15
Total	30-40	45-70	50-80
Columns with ties and platforms	10-18	18-40	70-120
Crane girders with bracing beams and repair platforms	0-14	14-40	50-150
Wall framework	0-3	5-14	12-20
Miscellaneous	—	0-10	3-12
Grand total	35-80	75-170	200-400

where $n = 1.2$ = load factor for the wind load
q_r = rated velocity head
C_a = aerodynamical coefficient
b = spacing of frames.

The wind load acting on the skylight and on the part of the wall within the limits of the collar beam depth is transmitted to the frame in the form of a concentrated force P_w, applied at the level of the bottom chord of the collar beam. This force is $P_w = nq_rC_abh$ (Fig.9-19*a* and *b*).

For estimating the dead load or own weight of the steel members use can be made of the approximate data contained in Table 9-3, which characterize the distribution of the metal between the separate elements of steel industrial building frameworks.

(2) Special Features of Statical Analysis of Frames

The lateral frames of industrial buildings are investigated as plane statically indeterminate systems. In these investigations a number of assumptions are generally introduced that simplify them without greatly influencing the final results. The main simplifications consist in the following.

1. When investigating a frame with a collar truss for all the loads applied to the columns of the frame, the truss is assumed to have an infinitely high stiffness ($I_{cb} = \infty$). This assumption permits the frame to be investigated according to the deflection method, only the horizontal displacements being unknown. By using specially prepared tables compiled for variable dimension columns constrained in the foundation and in the collar beam (at the level of the bottom chord), it is easy to investigate the frames of single-storey shops. Such tables are contained in various reference books and manuals.

2. Frames are analyzed for the vertical loads applied to the collar beam in accordance with the general rules of structural mechanics. Account is taken of the final stiffness of the collar beam, but on the assumption of symmetrically arranged loads in symmetrical frames. This reduces the problem to the analysis of a frame with immovable joints (owing to symmetry) having one unknown—the angle of rotation of the collar beam (for a single-span frame).

In this instance a collar truss is conditionally replaced with an equivalent beam (with respect to the deflection or the angle of rotation on the supports), making the axis of the latter coincide with the bottom chord of the truss. The moment of inertia of an equivalent collar beam (with respect to the deflection) is approximately determined from the equation

$$I_{cb} = k_h (A_t z_t^2 + A_b z_b^2) \qquad (9\text{-}10)$$

where A_t and A_b = sectional areas of top and bottom truss chords
z_t and z_b = distances from centre of gravity (axis) of truss to axes of top and bottom chords at the middle of the span
k_h = factor allowing for the variable depth of the truss, as well as for deformation of the web elements. For trusses with a pitch of the top chord of $^1/_8$ to $^1/_{12}$ this factor can be taken equal to 0.7-0.8.

The ratios between the moments of inertia of the separate parts of the columns, or of the collar beam or truss, required for purposes

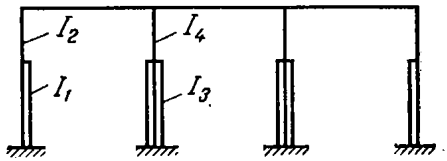


Fig. 9-20. Relation between moments of inertia of frame elements

of analysis are taken on the basis of similar designs already completed. Generally these ratios (Fig. 9-20) are within the following limits

$$\frac{I_1}{I_2}=5 \text{ to } 12 \quad \frac{I_3}{I_4}=8 \text{ to } 15 \quad \frac{I_3}{I_1}=1.2 \text{ to } 4$$

$\frac{I_3}{I_1}=4$ to 12 (with a double spacing between columns along the middle rows).

The smaller values given above relate to light shops, and the higher ones—to heavy shops. A change in the ratio between the moments of inertia within certain limits (about 30%) only slightly affects the magnitude of the design moments.

The introduction of these simplifications makes the analysis of a frame much easier. For example, a single-span frame with rigid corners can be investigated for the loads applied to the columns in the following order.

1. One additional (redundant) constraint is applied that prevents horizontal displacement, as a result the load will cause a reaction r_1 to appear in the constraint (basic system, Fig. 9-21*a*).

2. The collar beam of the unloaded frame is given a displacement in the direction of this constraint equal to unity, i. e., $\Delta=1$ (Fig. 9-21*b*). The total reaction in the constraint $\sum r_{11}$ caused by the displacement of all the columns over a distance of $\Delta=1$, as well as the mag-

nitudes of the moments at the ends of the columns ($\overline{M}_A$, $\overline{M}_B$, $\overline{M}_C$, etc.) are determined from the appropriate tables. The reaction r_{11} is always in the direction of the displacement.

3. The reactions in the redundant constraint induced by the external loads are determined for each load separately, since it is necessary to have total resultant moment diagrams for the columns. Thus, for example, when a crane load is applied to the frame there are simul-

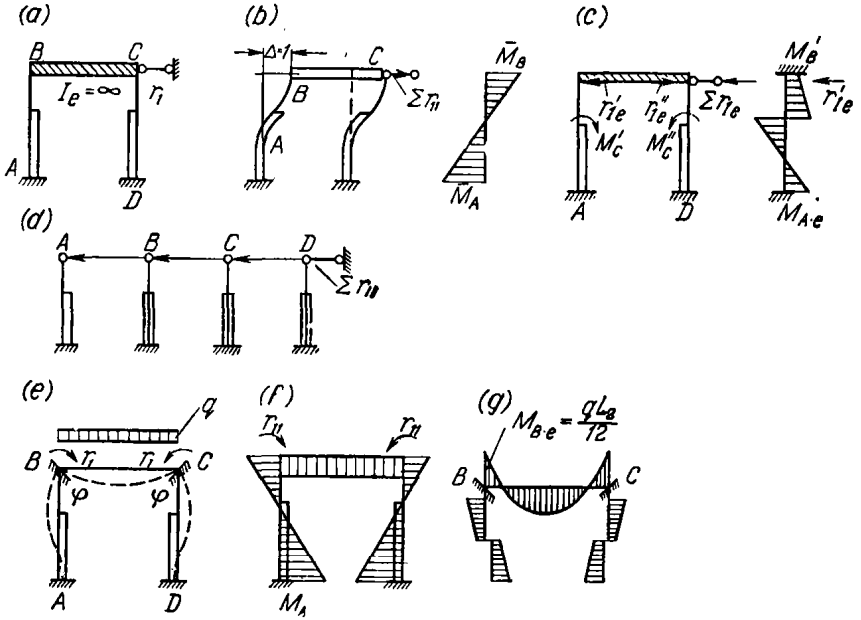


Fig. 9-21. To analysis of frames with equal spacing of columns

taneously induced a maximum moment on one column and a minimum one on another column (Fig. 9-21*c*). The total reaction in the redundant constraint will be

$$\sum r_{1e} = r'_{1e} - r''_{1e}$$

In a similar way, with the aid of tables, moment diagrams are computed and plotted for each load in the fixed columns ($M_{A\cdot e}$, $M_{B\cdot e}$, $M_{C\cdot e}$, etc.).

4. The actual displacement Δ_1 originated by each load in the given frame is found from the basic equation expressing the condition that the reaction in the redundant constraint induced by each load is equal to zero

$$r_1 = \sum r_{11}\Delta_1 + \sum r_{1e} = 0, \qquad \text{whence } \Delta_1 = -\frac{\sum r_{1e}}{\sum r_{11}}$$

5. The final values of the moments in the characteristic sections of the columns are found and the moment diagrams are plotted

$$M_A = M_{A\cdot e} + \overline{M}_A\Delta_1 \quad M_B = M_{B\cdot e} + \overline{M}_B\Delta_1, \text{ etc.}$$

The values of the moments are entered in a table (see example 9-1).

In multispan frames the total reaction caused by a unit displacement will be (Fig. 9-21*d*)

$$\sum r_{11} = r_{1B} + r_{1C} + r_{1D} + r_{1E}$$

Should no tables be available for determining the reactions and the moments in constrained columns, they can be found by employing the general method of structural mechanics—the method of redundant reactions. When the number of spans exceeds three, the horizontal displacement may be disregarded, assuming that $\Delta_1 = 0$.

A frame is investigated for the vertical loads applied to the collar beam, taking into account the final stiffness of the latter, in a similar way.

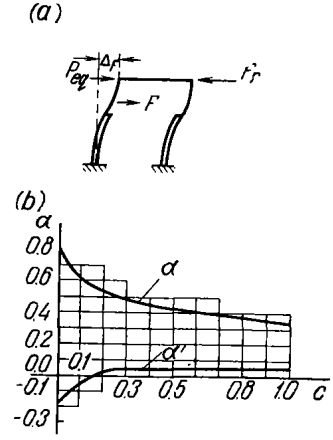


Fig. 9-22. To consideration of three-dimensional behaviour of framework

Owing to symmetry, we have only one unknown when the joints are restrained (Fig. 9-21*e*), namely, the angle of rotation φ_1 ($\varphi_B = \varphi_C = \varphi$). Upon rotation of a joint through an angle of $\varphi = 1$ the following reactive moments appear (Fig. 9-21*f*)

$$\sum r_{11} = M_{B\cdot col} + M_{B\cdot cb} = M_{11}$$

in which the subscripts *col* and *cb* denote column and collar beam, respectively.

When determining the reactive moments in the joints of a frame induced by an external load ($M'_{B\cdot e} = M'_{C\cdot e} = \frac{qL^2}{12}$) account should also be taken of the moment resulting from the presence of a step in the axis of the column $M''_{B\cdot e}$

$$\sum r_{1e} = \frac{qL^2}{12} + M''_{B\cdot e} = M_{1e}$$

The actual angle of rotation of a joint is found from the equation

$$\sum r_{11}\varphi_1 + \sum r_{1e} = 0, \quad \text{whence } \varphi_1 = -\frac{\sum r_{1e}}{\sum r_{11}} = \frac{M_{1e}}{M_{11}}$$

As a result the final values of the moments are obtained, which are entered in the table.

In frames with pinned beam-to-column connections the unknown force can be found quite easily without the use of tables, solving the problem by means of the method of redundant reactions (see example 9-1).

When investigating plane frames for crane loads, the solution can be based on space behaviour of the framework, taking into account that the adjacent frames will be involved in load resistance via the horizontal longitudinal ties along the bottom chords of the trusses (Fig. 9-11). In this case, with an equal column spacing along all the rows, a group of five to seven frames connected to each other by means of longitudinal ties is considered. The stiffness of these ties is determined by their moment of inertia found from equation (9-10), it being assumed here that for ties welded to the bottom chord the factor $k_h=0.7$, while when bolted tie connections are used $k_h=0.3$.

Analysis consists in determining the magnitude of the elastic reaction F_r, considering the ties as a continuous beam on elastic supports. Hence the basic equation for determining the unknown displacement will be

$$\sum r_{11}\Delta_1+\sum r_{1e}-F_r=0$$

For convenience of calculations it will be feasible to substitute for the displacement of the frames in a group induced by the applied loads a displacement originated by the equivalent load P_{eq} applied at the level of the ties, and causing the same displacement of the frame as the initial loads, for example (Fig. 9-22*a*)

$$P_{eq}=\frac{\Delta_f}{\delta} \tag{9-11}$$

where Δ_f = displacement of collar beam caused by crane load, for example by force F

δ = unit displacement of collar beam (cm/t) caused by unit force.

The reactive force F_r can be found from the equation

$$F_r=\alpha P_{eq}-\alpha' P'_{eq}$$

in which the coefficients α and α' establish the relation between the equivalent force P_{eq} and the elastic reaction, the latter depending on the height of the frame H, the ratio between the linear stiffnesses of the collar beam and the columns, the spacing of the frames b and the moment of inertia of the horizontal ties $I_{h.t}$.

The coefficients α and α' are found from the chart contained in Fig. 9-22*b* depending upon the quantity c'

$$c'=\frac{b^3\sum I_{bot}i^{1/2}}{H^3\sum I_{h.t}}$$

In the above expression $i=\frac{I_{top}}{I_{bot}}$ is the ratio between the moments of inertia of the top part of the columns and of the bottom one.

The equivalent force P'_{eq} represents the influence of the displacement of two frames adjacent to the one under consideration and is determined from the force P_{eq} [see equation (9-11)]. The latter is multiplied by the ratio between the loads applied to the adjacent frames and to the one under consideration.

In buildings with a roof consisting of large-size reinforced concrete slabs without purlins the roof structure also leads to space behaviour

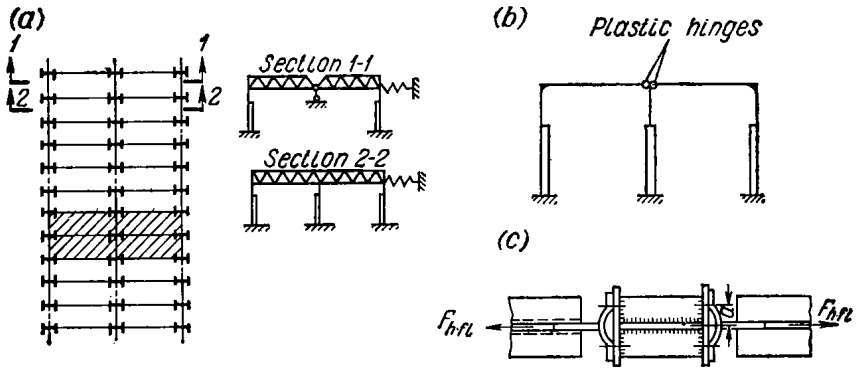


Fig. 9-23. To analysis of frames with different column spacings:
a—to analysis of frames; *b*—diagrammatic view of a frame with a plastic hinge along the middle row of columns; c—plastic hinge

of the framework similar to the longitudinal ties. However, transferring of the distributing functions only to the roofing, as has been noted above, may lead to disarrangement of the slabs.

Should the spacing of the columns along the middle row be twice that along the extreme rows, the investigations can be carried out for a conventional frame in which the moments of inertia of the section of the extreme row columns are equal to the sum of the moments of inertia of two columns, i.e., the group of members hatched in Fig. 9-23*a* is taken in the investigations. Such analysis is possible when horizontal ties are used along the bottom chords of the trusses, or with a rigid roof structure ensuring equal displacement of all the columns of the group. In this instance the horizontal reactive force in the conventional frame induced by the load in the column fixing the frame against displacement is determined as the sum of the reactive forces in the separate plane frames. After this the displacement of the conventional frame is found in the usual way. This displacement is used to determine the forces separately for each plane frame.

In a general case, with different column spacings along different rows, the length of the group of frames investigated is taken equal to the spacing of the main frames, the displacement of all the frames in the group being assumed equal to one another.

After diagrams of the moments in a frame have been plotted for each load separately, a table of the values of these moments for a number of column sections is compiled and the most disadvantageous combination of the total moment M and the longitudinal force F corresponding to it is established. Here account is taken both of the main and the additional combination of loads (including the wind load), for which a load combination factor of 0.9 is introduced (for all loads except the dead ones).

Large support moments appear in stiff (fixed) joints connecting collar beams to columns (especially along the inner rows) induced by the vertical load acting on the beam. These moments can be avoided by employing such an elastoplastic design of the connection of the top truss chord to the column which would be able to transmit the load only up to a certain magnitude, after which, upon reaching the yield point, it would be deformed and not be able to resist a greater moment.

Figure 9-23*b* shows a frame with such plastic hinges used in the connection of the roof trusses to the middle column.

The maximum force that can be transmitted by a flange (Figs. 9-7*a* and 9-23*c*) $F_{h.fl}$ is determined for the condition of a plastic hinge being formed in bending of the flange

$$M_{fl} = \frac{F_{h.fl}a}{4} = RW_{fl.p}$$

seeing that

$$W_{fl.p} = \frac{bt^2}{4}$$

we have

$$F_{h.fl} = \frac{Rbt^2}{a} \tag{9-12}$$

where b and t = height and thickness of flange
a = distance from centre line of flange to bolt gauge line
R = upper yield point for steel 3, which can be taken equal to 30 kg/mm^2.

With such a design of the joint, the corner of the frame can resist a moment of only one sign. This moment may be sufficient to ensure the required stiffness of the shop under the action of horizontal forces.

Thus the possibility appears of investigating frames for vertical loads, assuming a pinned connection of the collar beam to the column;

this makes it possible in the following stage of analysis to apply to the top of the column a moment equal to $M_s = F_{h \cdot fl} h_s$ (h_s = depth of truss at support).

Testing of the actual behaviour of steel frameworks of industrial buildings (N. Aistov, M. Berdichevsky, E. Belen, A. Kikin, G. Shapiro) confirmed the appreciable influence of the space behaviour of members. At the same time these tests proved that the weakest place in these frameworks are the connections of the elements of members and their components, especially the places where crane girders are connected to columns, which are the first to be subjected to the dynamic load of cranes (impacts). This explains why the building standards and regulations contain higher demands to the members of buildings and structures with heavy service conditions with respect to both the connections and the lateral stiffness measured by horizontal deflections of the columns.

Example 9-1. Design and investigate a steel lateral frame for a single-span shop with a span of $L = 30$ metres having two heavy-duty cranes, each with a lifting capacity of $Q = 50/10$ tons. The elevation of the crane rail top is $+16$ metres above floor level, the latter being taken as the zero elevation. The elevation of the top of the foundation is -1 metre.

The spacing of the columns is 12 metres. The shop is to be erected in Moscow.

I. SELECTION OF STRUCTURE LAYOUT

We establish the general dimensions of the frame members on the basis of the given technological dimensions, bearing in mind the necessity of providing a passageway 400 mm wide at the level of the top of the crane girders (Fig. 9-24*a*). The depth of the crane girder is taken from example 6-1, a type KP80 crane runway rail being employed. When establishing the height to the bottom of the roof truss, the possibility of deflection of the trusses and the protruding elements of the ties (within the limits of 200-250 mm) which may hinder the travelling of the cranes should not be forgotten.

For this example we select a structural scheme in the form of a frame having fixed columns and a pin-connected collar beam (Fig. 9-24*b*).

II. DETERMINATION OF LOADS ACTING ON FRAME

A. Dead Loads

The design roof load is taken as a uniformly distributed load of 340 kg/m^2 (0.34 t/m^2) and two concentrated loads originated by the skylight walls each equal to 3.1 tons (see example 7-1).

The design load per column is the support pressure of the roof truss and is equal to

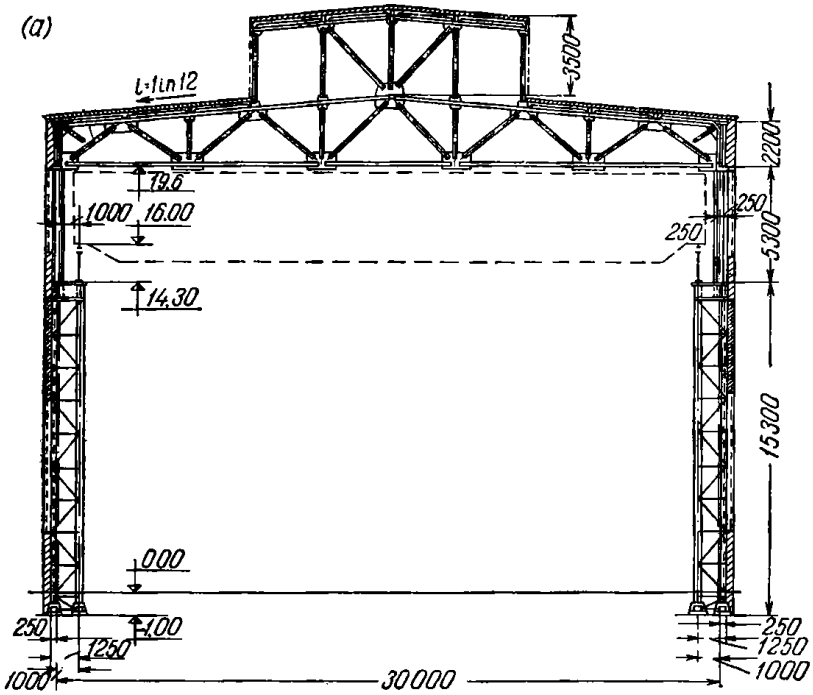
$$P_1 = 0.34 \times 12 \times \frac{30}{2} + 3.1 = 64.3 \text{ tons}$$

The own weight of the metal members is taken approximately as follows:

(a) weight of top part of column $P_2 = 1.5$ t;

(b) weight of bottom part of column $P_3 = 5$ t;

(c) weight of crane girders and bracing members $P_4 = 6$ t.


(а)
L-1 Ln 12
3500
2200
19.6
1000
16.00
250
250
5300
14.30
15300
000
250
100
250
1250
1250
1000
1000
30000

With regard to the design of the facade, we take the design load of the wall on a column in the form of two concentrated forces $P_5=6$ t and $P_6=5$ t applied respectively at the elevations 14.3 and 6.5 metres.

B Live Loads

1. *Snow Load.* When investigating the frame, the snow load can be taken as a uniformly distributed one equal to $100\times1.4=140$ kg/m^2 (according to the Building Standards and Regulations for the region of Moscow):

$$P_7=0.14\times12\times\frac{30}{2}=25.2\text{ t}$$

2. *Crane Loads.*

(a) In accordance with GOST 3332-54 the total weight of a crane with a lifting capacity of $Q=50$ tons is $G=98$ tons, the weight of the trolley is $g=18.5$ tons, the maximum working load of a crane wheel $P_{max}=50.5$ tons.

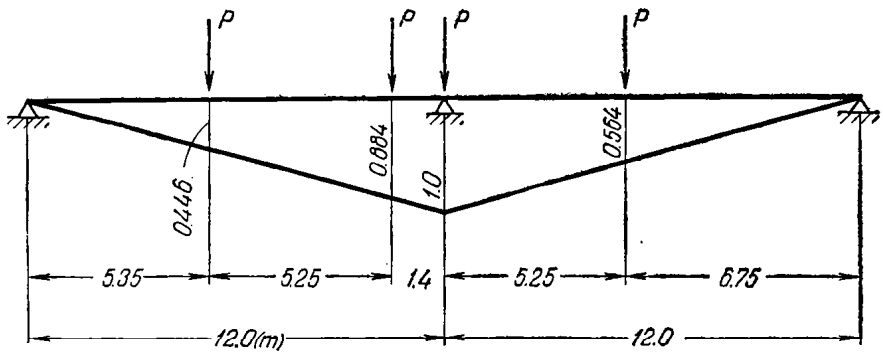


Fig. 9-25. To example 9-1. Determination of load on supports induced by crane loads

(b) The minimum working load of a crane wheel is found from equation (9-5)

$$P_{min}=\frac{Q+G}{0.5N_{cw}}-P_{max}=\frac{50+98}{0.5\times4}-50.5=23.5\text{ t}$$

where $N_{cw}=4$ is the total number of crane wheels.

(c) We find the maximum and minimum load F on a column induced by two cranes.

We plot an influence line of the support load, arrange the cranes to obtain the maximum load (Fig. 9-25) and compute the ordinates y of the influence line. The design loads F are determined (with the introduction of a load factor $n=1.2$) from equation (9-4)

$$F_{max}=P_{max}n\sum y=50.5\times1.2\,(0.446+0.884+1.0+$$
$$+0.564)=50.5\times1.2\times2.894=175.4\text{ t}$$
$$F_{min}=23.5\times1.2\times2.894=81.5\text{ t}$$

(d) We find the horizontal load on a column induced by the lateral braking forces, for which end we determine the lateral braking force for one crane wheel from equation (9-6)

$$F_{br \cdot cw} = \frac{Q+g}{20N_1} = \frac{50+18.5}{20\times 2} = 1.71 \text{ t}$$

in which N_1 is the number of wheels of a crane on one side.

The horizontal load on a column induced by the lateral braking forces of two cranes

$$F_{br} = F_{br \cdot cw} n \sum y = 1.71 \times 1.2 \times 2.894 = 5.9 \text{ t}$$

3. The *wind load* on a column is taken as a uniformly distributed one with a service pressure of 30 kg/m^2 at a height of up to 10 metres; we replace the trapezoidal wind pressure diagram for a height from 10 to 20 metres with a rectangular one having a mean ordinate value of 35 kg/m^2, and for a height above 20 metres—

TABLE 9-4 **Table of Design Loads**

Loads	Magnitude of force, t (t/m)	Elevation at which force is applied, m	Eccentricity of force relative to centre of gravity of section, m
A. Dead Loads			
Weight of roof P_1	64.3	19.5	—
Weight of upper part of column P_2	1.5	14.3	—
Weight of lower part of column P_3	5	0	—
Weight of crane girders and bracing platforms P_4	6	14.3	0.6
Weight of wall P_5	6	14.3	0.75
Weight of wall P_6	5	6.5	0.75
B. Live Loads			
Snow P_7	25.2	19.5	—
Maximum vertical crane load F_{max}	175.4	14.3	0.6
Minimum vertical crane load F_{min}	81.5	14.3	0.6
Lateral braking F_{br}	5.9	15.87	—
Wind load (uniformly distributed) q_{act_1}	0.311	Up to 10 m	—
Ditto, q_{act_2}	0.363	From 10 to 20 m	—
Ditto, q_{suc_1}	0.233	Up to 10 m	—
Ditto, q_{suc_2}	0.272	From 10 to 20 m	—
Wind load (concentrated) $P_{w \cdot act}$	3.74	19.5	—
Ditto, $P_{w \cdot suc}$	2.8	19.5	—

of 45 kg/m². The pressure of the wind on the roof and skylight is transmitted to the column in the form of a concentrated force P_w. The wind load is found from the equation

$$q_w = nq_s C_a bc$$

where q_s = service wind velocity head; b = 12 m = spacing of columns; n = 1.2 = load factor; C_a = 0.8 and 0.6 = aerodynamical coefficients of active pressure and suction; c = 0.9 = additional load combination factor.

$$q_{act_1} = 30 \times 12 \times 1.2 \times 0.8 \times 0.9 = 311 \text{ kg/m}$$
$$q_{act_2} = 35 \times 12 \times 1.2 \times 0.8 \times 0.9 = 363 \text{ kg/m}$$
$$q_{suc_1} = 30 \times 12 \times 1.2 \times 0.6 \times 0.9 = 233 \text{ kg/m}$$
$$q_{suc_2} = 35 \times 12 \times 1.2 \times 0.6 \times 0.9 = 272 \text{ kg/m}$$
$$P_{w \cdot act} = 45 \times 12 \times 1.2 \times 0.8 \times 0.9 \times 8 = 3{,}740 \text{ kg}$$
$$P_{w \cdot suc} = 45 \times 12 \times 1.2 \times 0.6 \times 0.9 \times 8 = 2{,}800 \text{ kg}$$

Here the distance from the bottom of the roof truss to the ridge of the sky light has been taken equal to 8 metres.

All the loads and the elevations at which they are applied are entered in Table 9 4.

III. STATICAL ANALYSIS OF FRAME

The system of frame employed with fixed columns and pinned connection of the collar beam is statically indeterminate with one redundant constraint.

We shall investigate the frame using the method of redundant reactions, assuming the normal force in the collar beam X_1 to be the redundant one. Therefore the main system will consist of two columns fixed at their bottoms. The canonical equation for determining the unknown force has the form

$$\delta_{11} X_1 + \Delta_{1e} = 0, \quad \text{whence } X_1 = -\frac{\Delta_{1e}}{\delta_{11}}$$

Here δ_{11} = displacement of points of application of the forces $X_1 = 1$ in their direction (conversion or diversion) produced by these same forces

Δ_{1e} = displacement of the same points in the same direction induced by external load.

The displacements δ_{11} and Δ_{1e} can be found from the equation

$$EI_0 \delta_{ik} = \sum \int_a^b M_i M_k \frac{I_0}{I_i} dx = \sum (M_{a \cdot i} M_{a \cdot k} + 4 M_{c \cdot i} M_{c \cdot k} + M_{b \cdot i} M_{b \cdot k}) \frac{L}{6} \frac{I_0}{I_i}$$

where $M_{a \cdot i}$ and $M_{a \cdot k}$ = values of ordinates of moment diagrams at end a of a column

$M_{b \cdot i}$ and $M_{b \cdot k}$ = ditto at end b

$M_{c \cdot i}$ and $M_{c \cdot k}$ = ditto at middle of column

L = length of column

$\frac{I_0}{I_i}$ = ratio of moments of inertia of column sections (generally $I_0 = 1$).

TABLE 9-5 **Table of Design Forces in Columns of Frame**

Kind of load	No. of loading	Sketch of frame and moment diagrams	Row of columns	Load combination factor c	Roof-supporting part of column				Crane runway part of column				
					Section 1-1		Section 2-2		Section 3-3		Section 4-4		
					M t-m	F t	M t-m	F t	M t-m	F t	M t-m	F t	Q t
Weight of members	1	$X_1 = -1.85$ $X_1 = -0.6$			6.7	64.3	−9.6	65.8	+21.4	77.8	−7.0	87.8	−1.85
Snow	2			1.0	−2.16	25.2	−3.12	25.2	+7.0	25.2	−2.2	25.2	−0.6
	2′			0.9	−1.95	22.7	−2.8	22.7	+6.3	22.7	−1.98	22.7	−0.54
Vertical crane loads	3	$X_1 = 4.58$ M_A M_B	A	1.0	+16.5	—	+23.8	—	−81.2	175.4	−11.0	175.4	+4.58
	3		B		+16.5	—	+23.8	—	−25.0	81.5	+45.2	81.5	+4.58
	3′		A		+14.85	—	+21.4	—	−73.0	158.0	− 9.0	158.0	+4.12
	3′		B	0.9	+14.85	—	+21.4	—	−22.5	73.3	+40.0	73.3	+4.12

Table 9-5, concluded

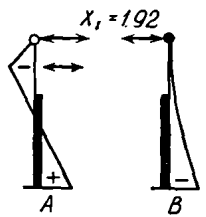
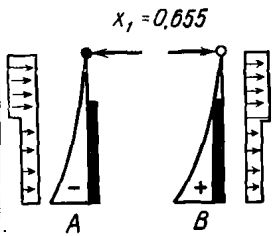
Kind of load	No. of loading	Sketch of frame and moment diagrams	Row of columns	Load combination factor c	Roof-supporting part of column				Crane runway part of column				
					Section 1-1		Section 2-2		Section 3-3		Section 4-4		
					M t-m	F t	M t-m	F t	M t-m	F t	M t-m	F t	Q t
Lateral braking	4		A	1.0	±6.93	—	±0.7	—	±0.7	—	+58.6	—	±3.98
	4		B		±6.93	—	±10.0	—	±10.0	—	−39.4	—	±1.92
	4′		A	0.9	±6.24	—	±0.63	—	±0.63	—	+52.6	—	±3.58
	4′		B		±6.24	—	±9.0	—	±9.0	—	−35.5	—	±1.73
Wind: from left	5		A	0.9	−11.7	—	−18.2	—	−18.2	—	−89.0	—	±9.0
	5		B		+ 8.04	—	+12.65	—	+12.65	—	+63.7	—	±6.5
from right	5′		A	0.9	+ 8.04	—	+12.65	—	+12.65	—	+63.7	—	+6.5
	5′		B		−11.7	—	−18.2	—	−18.2	—	−89.0	—	+9.0

TABLE 9-6 **Table of Design Load Combinations in Columns of Frame**

Design moments and forces	No. of loading	Load combination factor c	Roof-supporting part				Crane runway part				
			Section 1-1		Section 2-2		Section 3-3			Section 4-4	
			M t-m	F t	M t-m	F t	M t-m	F t	M t-m	F t	Q t
$+M_{max}$; F	1, 3, 4	1.0	+16.73	64.3	+24.2	65.8	28.4	103.0	+77.6	169.3	+6.71
	1, 3′, 4′, 5′	0.9	+22.43	64.3	+34.45	65.8	+41.1	100.5	+155.5	160.8	+10.7
$-M_{max}$; F	1, 2 or 1, 3, 4	1.0	−8.86	89.5	−12.72	91	−60.5	253.2	−88.8	288.4	
	1, 2′, 5 or 1, 3, 4, 5	0.9	−20.35	87.0	−30.6	88.5	−70.4	235.8	−191.6	268.5	+15.7
$-F_{max}$; $\pm M$	1, 2, 3, 4	1.0	+14.57	89.5	+21.8	91	−53.5	278.4	−88.8	288.4	
	1, 2′, 3′, 4′, 5′	0.9	+20.48	87	+30.65	88.5	−64.2	258.5	−191.6	268.5	15.7
F_{min}; $\pm M$	1, 5	0.9	—	—	—	—	—	—	+ 80	87.8	8.35
	1, 5′								−128	−87.8	10.85

Thus, for the two trapezoidal moment diagrams shown in Fig. 9-26 we obtain

$$EI_0\delta_{12}=\frac{L}{6}(2aa'+2bb'+ab'+a'b)$$

where a, a', b and b' are the values of the ordinates of the moment diagrams for the ends of the column.

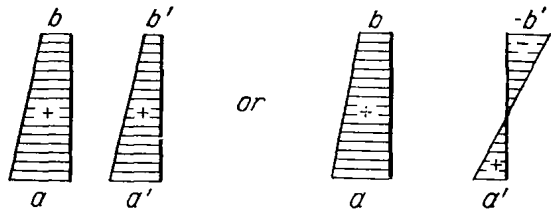


Fig. 9-26. To example 9-1. Trapezoidal moment diagrams

After determining the redundant force X_1, we find the moment M in any section of the frame

$$M=M_e+M_1X_1$$

where M_e = moment induced by external loads in main statically determinate system

M_1 = moment induced by force $X_1=1$.

When plotting the diagram of moments, we lay them off at the side of the tension fibre, considering the diagrams at the inner side of the columns to be positive. The values of the moments in the characteristic sections of the columns are summed up in Table 9-5, while the design combinations of forces are contained in Table 9-6.

9-5. DESIGN OF SKYLIGHTS AND WALL FRAMEWORK

(1) Skylights

Skylights, which are installed on the roof trusses, serve for lighting and aeration of the premises. There are distinguished skylights with *external* (Fig. 9-27*a* and *b*) and *internal* (Fig. 9-27*c*) water drainage. In the majority of cases skylights have vertical glazing.

The skylights can be arranged along the shop (longitudinal skylights) and also across the shop (lateral skylights). The former are the ones used to the greatest extent. The width of longitudinal skylights generally varies from one-half to one-third of the span.

In heated shops with a skylight with exceeding nine metres, internal drainage is usually provided, while with a smaller width of the skylight, and also in unheated shops, external water drainage from the skylights is the rule.

The height of skylights depends on the required light area filled with standard sashes having a height of 1.25, 1.5 and 1.75 metres and suspended from the framework of the skylights. The skylight glazing should be raised somewhat above the surface of the roof (at least 400 mm). The height of the skylight verticals at the glazed surface is about one metre more than the total height of the sashes.

Skylights can be designed with a number of vertical posts (Fig. 9-27*a*), or in the form of separate trusses (Fig. 9-27*b* and *c*). In the former design, rigidity of the members should be ensured by installing diagonals. Skylights of this type (consisting of separate elements)

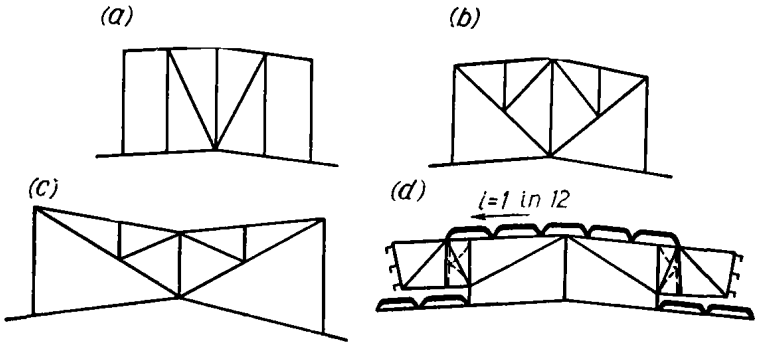


Fig. 9-27. Diagrammatic views of skylights

are lighter than the other types and are more convenient for transportation. Skylights in the form of separate trusses have fewer joints connecting them to the roof truss and are therefore more convenient for erection.

To ensure the most efficient aeration of industrial buildings, special aeration skylights are being used at present (Fig. 9-27*d*) provided with movable (opening) sashes and wind shields.

The main element of a skylight that must be investigated is the external post, which besides the vertical load also resists the wind load.

The section of the post is generally formed of two unequal leg angles or channels.

Figure 9-28 illustrates a post of a skylight with metal window sashes suspended from it. The post is connected to the projecting gusset plate of the roof truss by means of unfinished bolts or welding. A flange connection can also be used.

From roof maintenance considerations, longitudinal skylights must have interruptions at least every 80 metres, and should not

reach the ends of the building. Horizontal and vertical ties similar to those used in roof trusses are installed to ensure general rigidity of the skylights.

(2) Wall Framework

Three types of industrial building walls are distinguished:

1. Thin walls of reinforced concrete slabs, half-brick walls or walls of corrugated asbestos-cement sheets with a strengthened section.

2. Framework walls consisting of one-brick or brick-and-half blocks, or of some other kind of wall blocks.

3. Self-bearing walls of reinforced concrete heat-insulated slabs (panels) as well as of brick-and-half or two-brick blocks or of other wall blocks (when weakened by openings not more than 50-60%).

Thin walls are employed in the hot shops of metallurgical and other works. An example of the use of precast reinforced concrete slabs is shown in Fig. 9-29*a*. When employing a half-brick wall it is necessary to design a steel framework of horizontal collar beams and vertical posts, which will take the weight of the wall, as well as the horizontal wind load. Figure 9-29*b* pictures an example of such a framework that divides the masonry into separate panels.

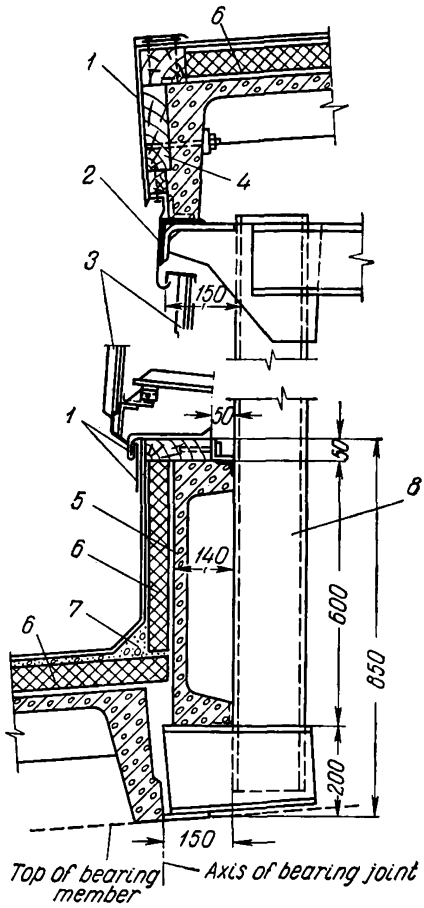


Fig. 9-28. Post of skylight:
1—roofing steel; *2*—glazing purlin; *3*—steel sash; *4*—board; *5*—type ПКЖБ reinforced concrete side slab; *6*—heat insulation; *7*—concrete; *8*—frame of skylight

The area of a half-brick wall panel is generally taken not over 9 to 12 square metres, bordering it with a frame consisting of channels or I sections No. 14. It is better policy to extend the wall panel upward than horizontally. Such a panel will have a good resistance to horizontal wind loads, transmitting them through the posts and collar beams to the columns. The beams located over openings must, be-

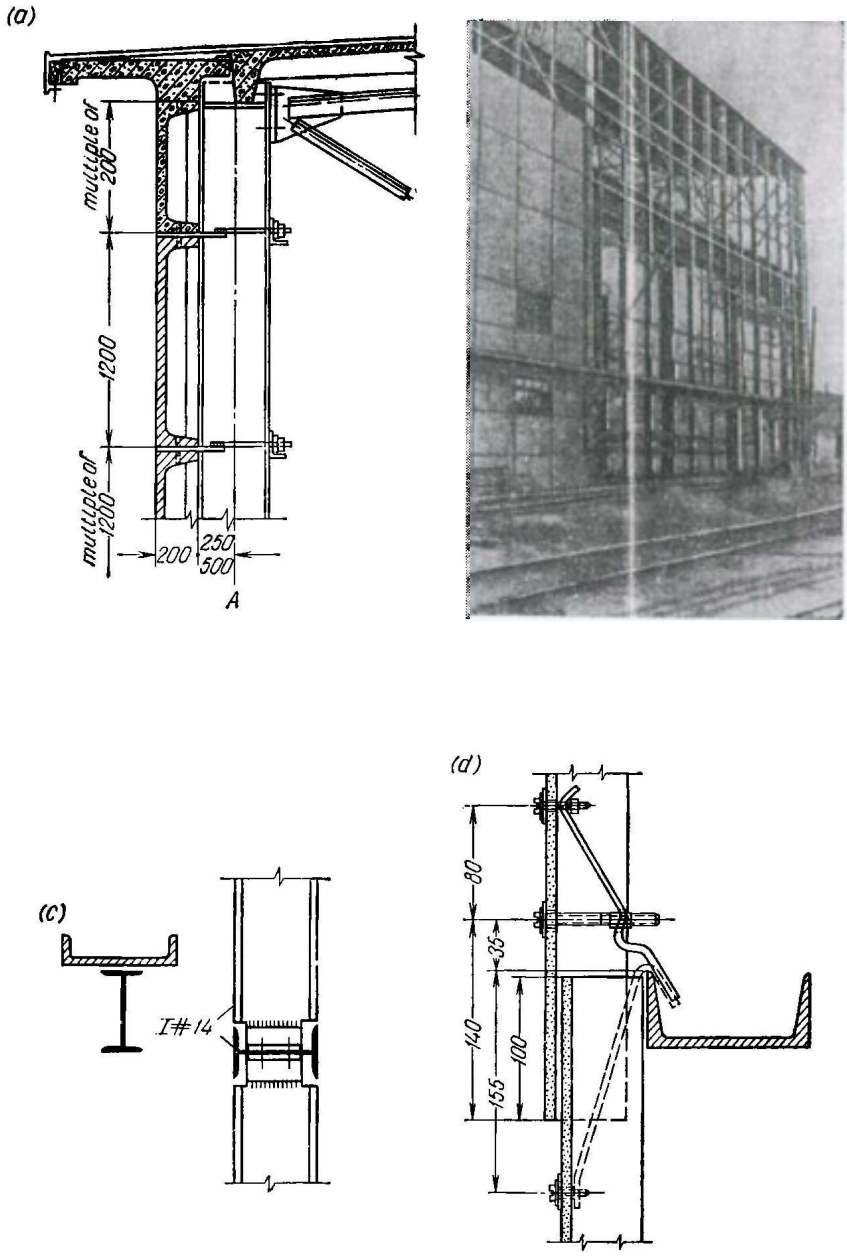


Fig. 9-29. Wall framework of thin wall

sides the horizontal loads, also withstand the vertical load caused by the weight of the wall. For this reason their section is made stiff in both directions (Fig. 9-29*c*). A wall made from asbestos-cement sheets is not so sturdy. Its connection is depicted in Fig. 9-29*d*.

Thick framework walls of one-brick or brick-and-half blocks or other materials are employed in heated industrial buildings, especially in high ones.

The collar beams (trusses) resist not the entire weight of the wall above them, but only part of it, since a self-bearing vault is formed inside the masonry. For this reason collar beams are investigated for a conventional triangular load (hatched part in Fig. 9-30). With respect to the value of the bending moment, such a load is equivalent to subjecting the beam to a uniformly distributed load of the masonry with a height of 0.6*b*. The connection of the collar beam to a column is investigated for the

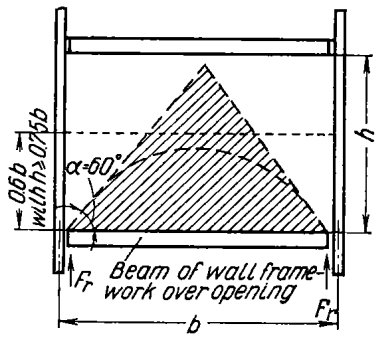


Fig. 9-30. Diagram for analysis of collar beam

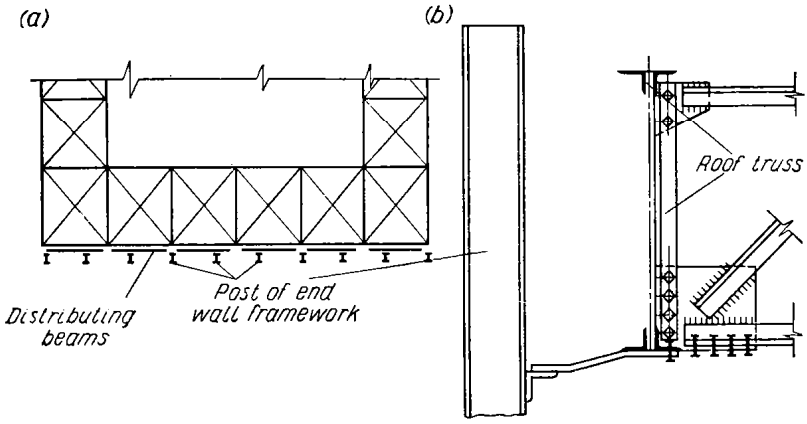


Fig. 9-31. Bearing of end framework posts

total support reaction F_r, equal to half the weight of the framed wall panel. Unfinished bolts are used, as a rule, for connecting the collar beams to the columns.

Independent bearing walls are erected on a foundation or a marginal beam and take their whole own weight, the collar beams over

openings being supported directly on the wall, which is connected to the columns with the aid of cramps embedded in the wall and movable in a vertical direction. In brick self-bearing walls the horizontal wind loads can be transmitted to the columns through horizontal collar beams embedded in the masonry. The collar beams are connected to the columns by means of flexible elements to make possible free settlement of the wall.

The posts of the end wall framework generally have an I section. They resist the vertical load of the walls and bend in a horizontal direction under the action of the wind. The horisontal wind load is transmitted by the posts at the bottom to the marginal beam or the foundation, and at the top to the horizontal wind truss included in the system of ties along the bottom chords of the roof trusses (Fig. 9-31*a*). Should the arrangement of the end framework posts not coincide with the joints of the tie truss, distributing beams (ordinarily of channels) are installed that transmit the horizontal loads from the posts to the joints of the truss. Figure 9-31*b* illustrates the connection of a post to the joint of an end roof truss with the aid of a plate hinge transmitting a horizontal load, and not preventing free deflection of the truss.

CHAPTER TEN

Special members

10-1. HEAVY TRUSSES OF LARGE-SPAN ROOFS

Heavy trusses is the name given to trusses whose elements have a double-plane section. The necessity of using such sections appears when the forces in the chords of the truss elements exceed 350 to 400 tons. Such great forces occur in trusses extending over large spans reaching 80 to 100 metres in length and subjected to heavy loads. Examples of such structures are the large-span trusses of aircraft assembly shops, hangars (Fig. 10-1), ship-building slips, from which

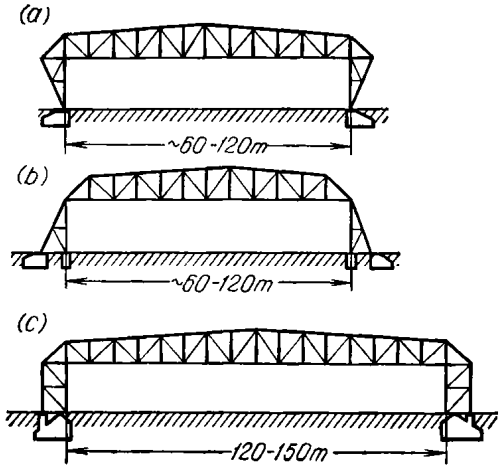


Fig. 10-1. Systems of trussed large-span frames

cranes are suspended, large-span crane trusses with heavy cranes, large-span trusses of public structures (exhibition pavilions, sports structures) and bridges.

The main trusses of large-span structures can be of a great variety both with respect to their type and their configuration. They are employed in the form of girder trusses or collars of trussed frames with parallel or polygonal chords, in the form of arch trusses, bow-tring arches, girders strengthened with a parabolic arch or struts, etc.

Figure 10-1*a* illustrates a frame with two pin connections at the supports, and Fig. 10-1*b*—at the top of the columns. The first type of frame is more advantageous, since the rigid corner of the frame facilitates relieving of the collar, reducing the stresses in it. Figure 10-1*c* pictures a frame with no pin connections, whose drawback is the necessity of developing the foundation constraining the legs. Besides, such a frame is more sensitive to changes in temperature.

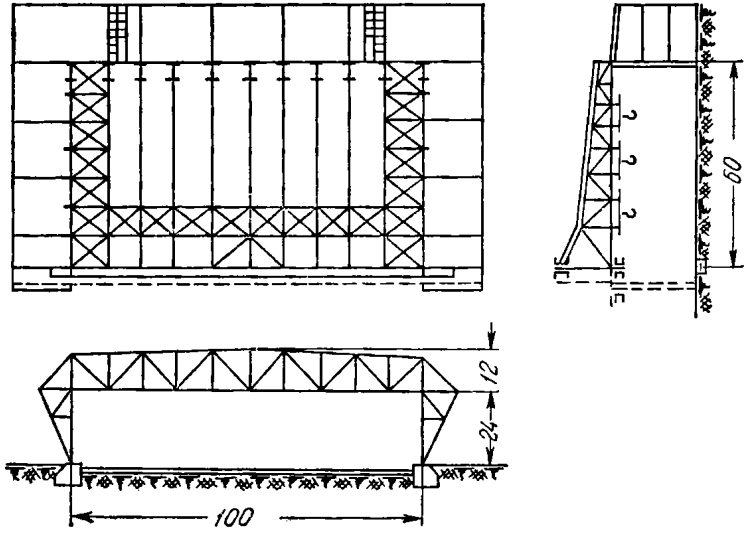


Fig. 10-2. Hangar (plan view and sections)

Figure 10-2 depicts an example of a hangar consisting of a main two-hinged frame with a span of 100 metres above the entrance gate, which supports roof trusses with a span of 60 metres and with a spacing equal to the panel of the main frame. On the other side, the roof trusses are supported on columns. Light travelling cranes (Q=3 to 5 tons) are suspended from girders connected to the roof trusses at the joints.

The entrance gates arranged along the frame consist of separate leaves mounted at the bottom on wheels. On top they rest horizontally against special guides, the latter, in turn, bearing against the ties along the bottom chords of the trusses.

The main frame has a thrust that in good soils can be resisted by the foundations, and in poor soils by a steel tie bar subjected to tension, which is laid in a special box-shaped duct under the floor.

The depth of the main truss (collar of the frame) is appointed within the limits of 1/8 to 1/10th of the span, depending on the tolerated deflection.

Frame constructions make possible the wide use of prestressing, which reduces the weight of the members. In particular, in the frame pictured in Fig. 10-2 the extreme braces at the columns can be tensioned, relieving the collar to a still greater extent.

When designing a large-span structure, special attention should be devoted to arrangement of the ties ensuring stability of the top truss chords and rigidity of the structure as a whole, and also resisting the horizontal (wind) loads.

A distinguishing feature of large-span structures consists in the own weight of the roof being the dominant load for them. Hence it follows that in designing such structures, great importance should be attached to selecting the type of roof. The use of an aluminium roof in these structures is usually justified from an economical viewpoint.

A double-pinned frame (Fig. 10-2) is investigated as a statically indeterminate system with one redundant member (the extra unknown force X_1 is the thrust) with regard to deformation of the tie bar (if employed), i.e.,

$$X_1 = -\frac{\Delta_{1e}}{\delta_{11} + \frac{EL_{tb}}{A_{tb}E_{tb}}} = -\frac{\sum \frac{F_1 F_e}{A_i} L_i}{\sum \frac{F_1^2 L_i}{A_i} + \frac{EL_{tb}}{E_{tb}A_{tb}}} \tag{10-1}$$

Here F_1 = forces in elements of main system induced by force $X_1 = 1$

F_e = forces in elements of main system induced by external loads

L_i and A_i = lengths and sectional areas of elements of truss chords and columns (the lattice may be omitted)

L_{tb} and A_{tb} = length and sectional area of tie bar.

The forces F in the elements will be

$$F = F_e + X_1 F_1 \tag{10-2}$$

Seeing that the quantity A_i is found both in the numerator and the denominator of equation (10-1), then only the ratio $\frac{A_i}{A_{ref}}$ will be of importance, where A_{ref} is the conditional or reference sectional area, for example 1 or 100 cm^2.

The investigations are carried out in tabular form.

When it is necessary to take into consideration the influence of the difference in ambient temperatures on a member, the corresponding thrust is determined from the equation

$$X_1 = \frac{E\Delta_{tt}}{\delta_{11} + \frac{EL_{tb}}{E_{tb}A_{tb}}} \tag{10-3}$$

Here $$\Delta_{tt} = \alpha t \sum F_1 L_i$$

in which α = coefficient of thermal expansion
t = difference in temperatures.

Owing to the large stresses, the elements are designed with double-plane sections, generally of the H-shaped, channel and other types (Fig. 10-3).

The sections of heavy truss elements are first selected for the heaviest compression element of the chords, then for the lightest one, thus establishing a gradation of shapes and sections. The depth of the section h (Fig. 10-3*a*) should not exceed one-tenth of the length of a panel, in accordance with the assumption that the joints in

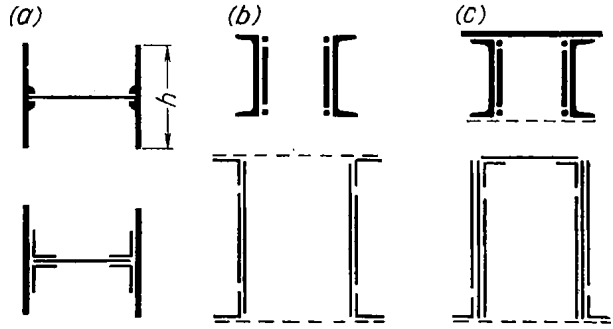


Fig. 10-3. Types of sections of bridge truss elements

the truss layout used for purposes of analysis are pinned, and not rigid ones. It is desirable that compression elements have an approximately identical slenderness ratio in both directions, and symmetrical sections. The axes of the chord elements should not be displaced with respect to each other by more than 1.5% of the section depth, otherwise account must be taken of the additional moment induced by the eccentrically applied axial forces.

The joints of truss elements having an H section are generally designed with the use of external cover gusset plates, ensuring the required transmission of the forces at the chord joint by means of additional straps. The horizontal plate of the H section, in view of the employment of flat jigs for drilling the holes, is sometimes left uncovered at the joint, transmitting the entire force field through the gusset plate. The batten plates or lattices connecting the branches of the section into a single unit are investigated similarly to axially loaded columns.

Designers should make every effort to use the minimum number of elements, and also to give the gusset plates a simple shape, without any notches and internal angles. The joint connections may be

riveted, on high-strength bolts, or welded. Figure 10-4 shows a riveted joint of a heavy bridge-type truss having H-section elements. The rivets should be arranged symmetrfcally with one pitch or a

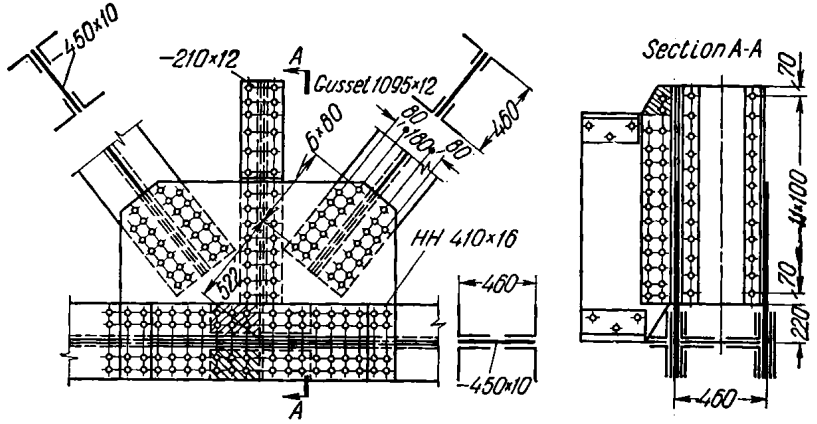


Fig. 10-4. Riveted joint of heavy truss

multiple of it, the minimum possible number of rivets being used. Figure 10-5 pictures a welded joint of a similar truss.

The welded connections of elements should be so designed as to ensure the lowest possible concentration of stresses in them. For this end it is essential to create a smooth transition of the force field,

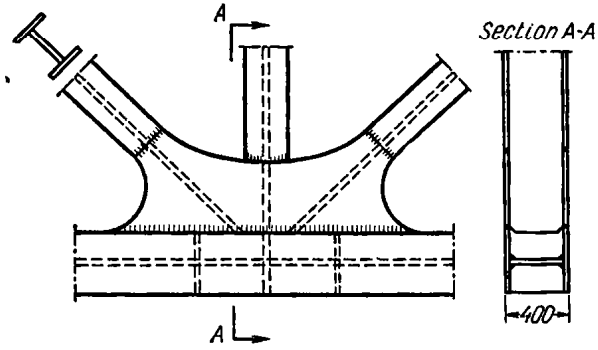


Fig. 10-5. Welded joint of heavy truss

making coves in the gusset plates and processing the welds in the required manner.

The bearing parts (supports) of heavy girder-type trusses and large-span frames, which resist a large load reaching several hundred tons, are so designed as to accurately transmit the support reactions at the designed spot. The design must ensure that these reactions

depend to a very low degree on changes in the distance between the points of support caused by deflection of the trusses, or by a change in the slope of the deflection curve. With a view to these requirements, there are distinguished fixed and expansion bearings. Beams and girders having a span up to 40 metres are designed on rocker type or tangential bearings (Fig. 10-6*a*).

With longer spans, the displacement of the bearing points caused by deflection of the trusses or by temperature changes becomes quite

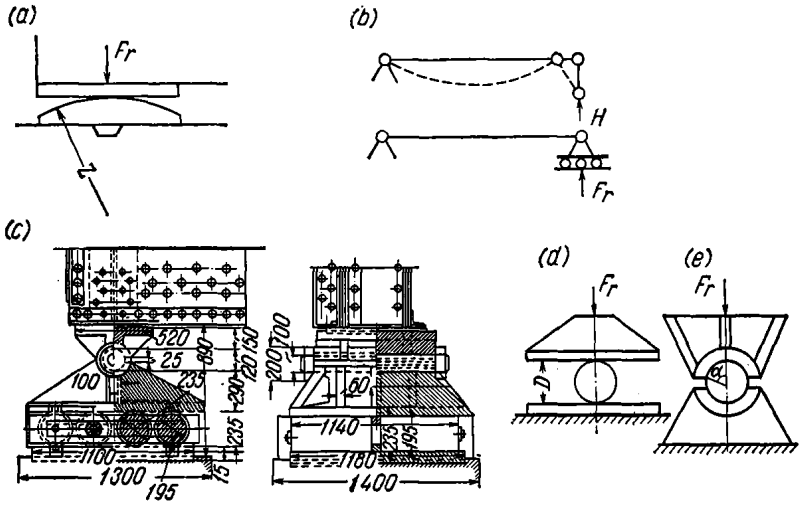


Fig. 10-6. Bearings of heavy trusses

noticeable (Fig. 10-6*b*). For this reason the expansion pinned bearings of large-span members are designed on rollers (Fig. 10-6*c*) Fixed supports are designed in the same way, but without rollers. Steel castings are used as the material for the bearings, while the rollers or the cylindrical bushes of the pinned connection are made from grade BCт. 5 steel.

Bearing rollers (Fig. 10-6*d*) are investigated for conventional diametrical compression by means of the expression

$$\sigma = \frac{F_r}{NDL} \leqslant R_{c \cdot r} \qquad (10\text{-}4)$$

where F_r = support reaction
N = number of rollers
D = diameter of roller
L = length of roller
$R_{c \cdot r}$ = design strength of rollers in diametrical compression with free contact (see Table 2-9 and 2-10).

A cylindrical bush or journal with tight contact (a central angle of contact $\alpha \geqslant \frac{\pi}{4}$, Fig. 10-6*e*) is investigated for local crushing by means of the formula

$$\sigma = \frac{F_r}{1.25rL} \leqslant R_{cr.t} \tag{10-5}$$

in which r is the radius of the pin or journal.

The values of the design crushing strengths for tight contact $R_{cr.t}$ are contained in Tables 2-9 and 2-10.

Should the pinned bearings be designed with a spherical head, the crushing stress with tight contact is determined from the expression

$$\sigma = \frac{6F_r}{\pi D^2} \frac{1}{1-\cos^3\varphi} \leqslant R_{cr.t} \tag{10-6}$$

where D = diameter of spherical surface
φ = angle of contact of the head with the spherical equalizer.

10-2. ARCHED, DOME-SHAPED AND SUSPENDED ROOFS

(1) Arched Roofs

It is rational to employ arched roofs for large spans exceeding 60 to 80 metres. Their principal merit is the low weight of the members, which is explained by the arches being a thrust system, with the section mainly withstanding compression induced by a normal force, and with insignificant bending. This is why they are sufficiently economical. In comparison with a frame member, an arch occupies more space. The hatched area in Fig. 10-7 shows the "dead" space that cannot always be utilized. An arched member is more liable to deformation than a frame, since the linear stiffness of an arch i_a is less than that of a frame collar beam i_{cb} (Fig. 10-7*a*)

$$i_a = \frac{I_a}{S} < \frac{I_{cb}}{L_{cb}} = i_{cb}$$

This explains the use of arched members in large-span roofs where there are no dynamic or large horizontal loads, in buildings designed for use as pavilions, markets, warehouses, etc.

Arches are divided with respect to design into *fixed*, *two-hinged* and *three-hinged* ones. The expediency of employing one of these systems is determined by the bearings. Figure 10-7*b* gives a diagram of the moments induced by a uniformly distributed load for each of these types of arches. The greatest moment is obtained in a three-hinged arch at one fourth of the span, therefore such arches are the heaviest ones. Being statically determinate, however, they are not sensitive to settlement of the supports and changes in temperature,

and for this reason they are used in the appropriate circumstances. In accordance with the diagram of moments, the configuration of three-hinged arches is usually designed in the form of two crescent-shaped half-arches (Fig. 10-8*a*).

Two-hinged arches, which the moments are distributed rather uniformly in, are the main type employed. The uniform distribution of the moments often permits them to be designed with parallel chords (Fig. 10-9*a*), which is the most rational design from the fabrication viewpoint, and sometimes with a crescent-shaped configuration (Fig. 10-8*b*). Fixed arches can be designed with the lightest weight, since the moment in the middle half of the span is minimal, and though it is high at the supports, there is a relatively small increase in weight of the section, since in the other types of arches it becomes necessary to develop the section at the supports from considerations of design. It must be noted, however, that fixed arches require very good and firm (rocky) soil capable of resisting high support moments, as otherwise such large foundations must be employed that the structure as a whole becomes costly. This is why fixed arches are used comparatively seldom. In general, when selecting the type of arch to be used in a structure, the question of the foundations must always be solved simultaneously.

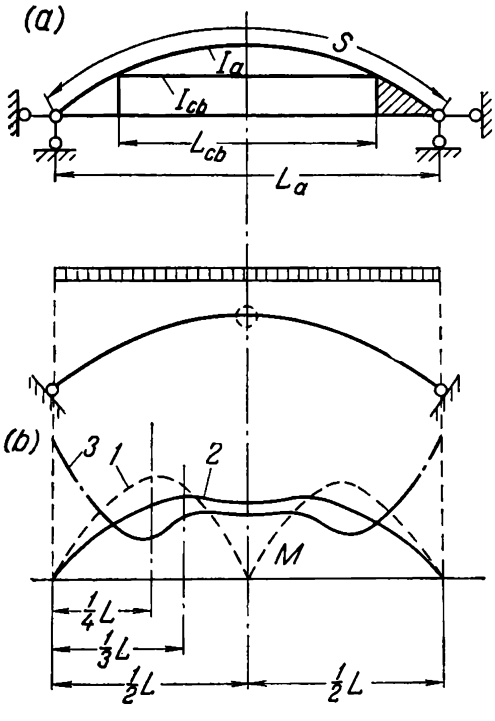


Fig. 10-7. Schematic view of arches and diagrams of moments induced by uniformly distributed load:
1—three-hinged arch; *2*—two-hinged arch; *3*—fixed arch

Thus, the most widespread type is the two-hinged arch, and under conditions of weak soils—the three-hinged one.

The general dimensions of an arch are its span L and its rise r_a. The rise is appointed on the basis of technological or economical considerations, while for exhibition pavilions and other public structures account is also taken of architectural considerations. The

most advantageous $\frac{r_a}{L}$ ratios are approximately $\frac{1}{6}$ to $\frac{1}{5}$. In public structures arches are designed with an $\frac{r_a}{L}$ ratio of about $\frac{1}{2}$ to $\frac{1}{5}$. An increase in the rise r_a leads to a reduction of the normal force and an increase in the moment, and vice versa.

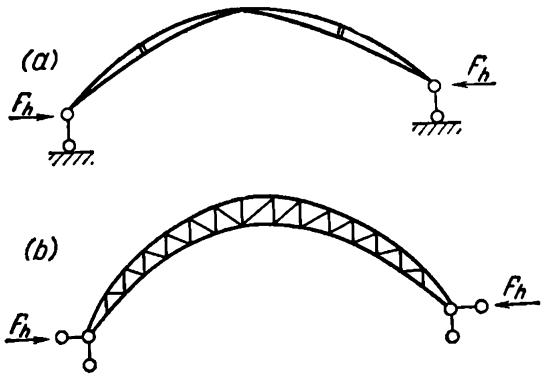


Fig. 10-8. Configuration of arches

In a direction along a building, arches are generally located with a spacing of 12-24 metres. Latticed truss purlins with a spacing of

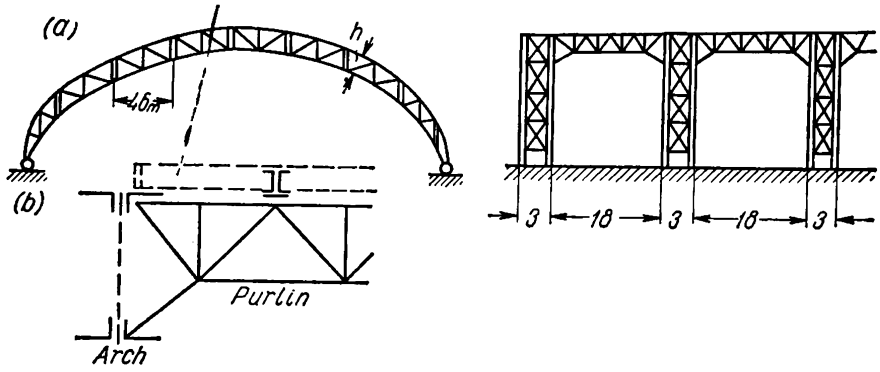


Fig. 10-9. Design of arched roof

4-6 metres are installed in the intervals between them (Fig. 10-9*a* and *b*). For convenience of erection, twin arches are sometimes manufactured with a distance of about three metres between them (Fig. 10-9*a*). These arches can stand independently in the process of erection without losing their general stability. At the same time this three-metre distance between the two members of the twin arch

makes possible the complete shop fabrication of parts of the arch twin member 4-6 metres long (a unit consisting of 2-3 panels).

The depth of an arch section is selected for small spans within the limits of $\frac{h}{L} \cong \frac{1}{30}$ to $\frac{1}{40}$, and for large ones $\frac{h}{L} \cong \frac{1}{40}$ to $\frac{1}{60}$.

The section of an arch may be solid (for small spans) or latticed (for large ones) with a diagonal or, less frequently, with a triangular lattice.

Investigation of arches is commenced with determination of the loads, which include the weight of the roof, snow (over the whole span and over half of its length), wind loads and the weight of the members. All the loads are determined as prescribed by the Building Standards and Regulations.

The weight of the arch can be found from the equation

$$g = \frac{G}{S} \text{ kg/m}$$

in which g = weight of one linear metre of arch
S = length of arch
G = total weight of arch, equal to

$$G = \frac{2}{3} r_a LBC_g \text{ kg} \tag{10-7}$$

where B = spacing of arches, m
C_g = weight factor equal to 2-4.

The forces acting along the axis of an arch are found from the formulas

$$M_x = M_b - F_h \quad yF_x = Q_b \sin \alpha + F_h \cos \alpha$$

$$Q_x = Q_b \cos \alpha - F_h \sin \alpha$$

Here F_h = thrust
y = ordinate of arch axis ($y_{max} = r_a$)
α = angle between tangent to arch axis and the horizontal
M_b and Q_b = beam moment and shear force obtained when considering the arch as a beam with a span of L (Fig. 10-10*a*).

A two-hinged arch is a statically indeterminate system with one redundant quantity. The thrust F_h is taken as the unknown force X_1, which is found from the equation

$$F_h = X_1 = -\frac{\Delta_{1e}}{\delta_{11}} \tag{10-8}$$

In constant-dimension arches with a gentle slope and with a uniformly distributed load $F_h = \frac{qL^2}{8r_a}$.

The forces in open-web arches with parallel chords can be determined in the same way with the following resolution of the forces along the elements of the section (Fig. 10-10*b*):

the force in the top chord

$$F_{ch.t} = -\frac{M_x}{h} - \frac{F_x}{2} \quad (10\text{-}9a)$$

the force in the bottom chord

$$F_{ch.b} = +\frac{M_x}{h} - \frac{F_x}{2} \quad (10\text{-}9b)$$

the force in the diagonals

$$F_d = Q_x \frac{\cos\alpha}{\cos\beta} \quad (10\text{-}9c)$$

the force in the verticals

$$F_v = Q_x \cos\alpha \quad (10\text{-}9d)$$

When investigating open-web arches with a relatively large section, equations (10-10) should be employed for calculating the displacements used in determining the thrust, namely

$$E\delta_{11} = \sum_{i=1}^{n} \frac{F_1^2 L_i}{A_i} \quad (10\text{-}10a)$$

$$E\Delta_{1e} = \sum_{i=1}^{n} \frac{F_1 F_e}{A_i} L_i \quad (10\text{-}10b)$$

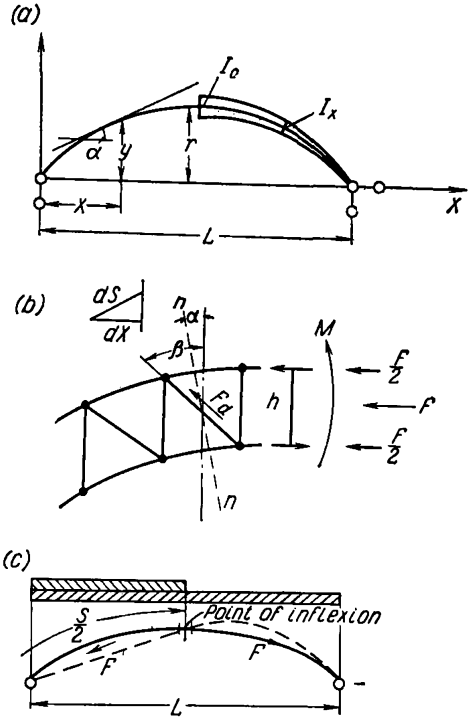


Fig. 10-10. To analysis of two-hinged arch

Here it will be sufficient to sum the forces only along the chords, neglecting the forces in the lattice elements owing to their slight influence on the magnitude of the thrust.

The sections of the elements of arch chords and lattices are generally taken of two angles and are designed as in ordinary trusses. With large forces in the chords, the same as in heavy trusses, a change-over to double-plane, mainly H, sections becomes necessary.

To ensure stability of the compression elements of the chords in a direction perpendicular to the plane of the arch, it is necessary to install horizontal ties, and also purlins or braces between the arches, similar to the arrangement of ties in ordinary trusses. It is desirable that the distance between the purlins (braces) should not exceed 16 to 20 chord widths. When checking a compression element of an

arch chord for buckling its effective length in a vertical plane (in the plane of the arch) is taken equal to the length of the panel and in a plane perpendicular to that of the arch to the distance between the braced points.

It is also necessary to check the general stability of the arch in a vertical plane. Since the most probable form of the loss of general stability of an arch in the vertical plane is S-shaped buckling with an inflexion point of the axis near the middle of the arch length (Fig. 10-10*c*), the critical force can be approximately determined from the formula of Euler-Yasinski. The effective length is taken equal to half the length of the arch S multipled by a length coefficient k taken from Table 10-1, i.e.,

$$F_{cr} = \frac{\pi^2 E I_x}{\left(k \frac{S}{2}\right)^2} \tag{10-11}$$

TABLE 10-1 **Length Coefficient k for Arches**

Type of arch	r_a/L			
	1/20	1/5	1/3	1/2.5
Three-hinged .	1.2	1.2	1.2	1.3
Two-hinged	1	1.1	1.2	1.3
Fixed . .	0.7	0.75	0.8	0.85

Here the moment of inertia of the arch section I_x is taken at one-fourth of the span length. The following relation must be observed

$$\frac{F_{cr}}{F} > 1.2 \text{ to } 1.3$$

where F is the force induced by the design loads.

The depth of an arch section is sometimes determined by checking its general stability. To make possible the arrangement of arch-type members on walls or columns of various structures, bowstring arches are employed, the tie bar taking the thrust. Figure 10-11 depicts the bowstring arch used in the roof of the Palace of Sports in Luzhniki, Moscow.

The thrust (force in the tie bar) is determined with account of the expansion of the tie bar.

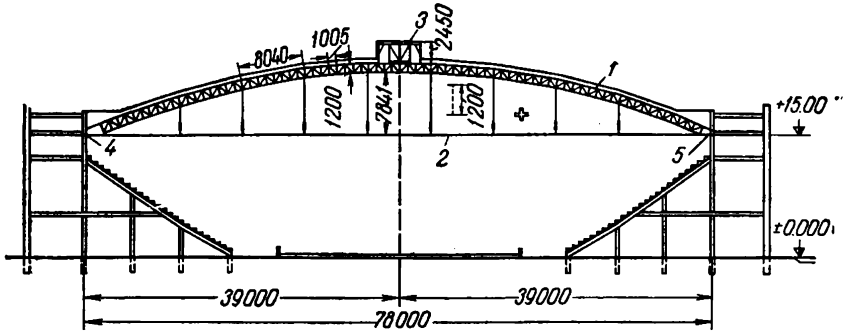


Fig. 10-11. Bowstring arch used in roof of Palace of Sports in Luzhniki (Moscow):
1—arch; *2*—tie bar; *3*—aeration skylight; *4*—fixed support; *5*—pinned support

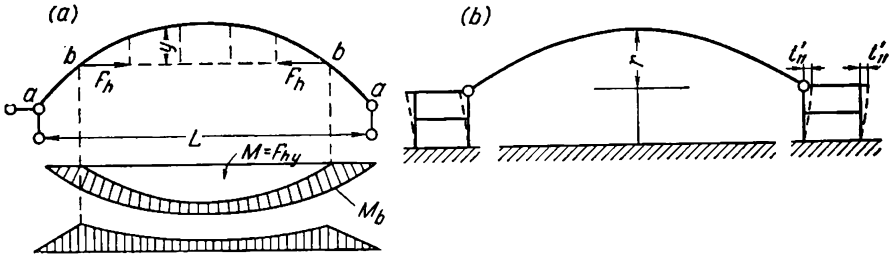


Fig. 10-12. Methods of increasing the inside clearance in arched structures

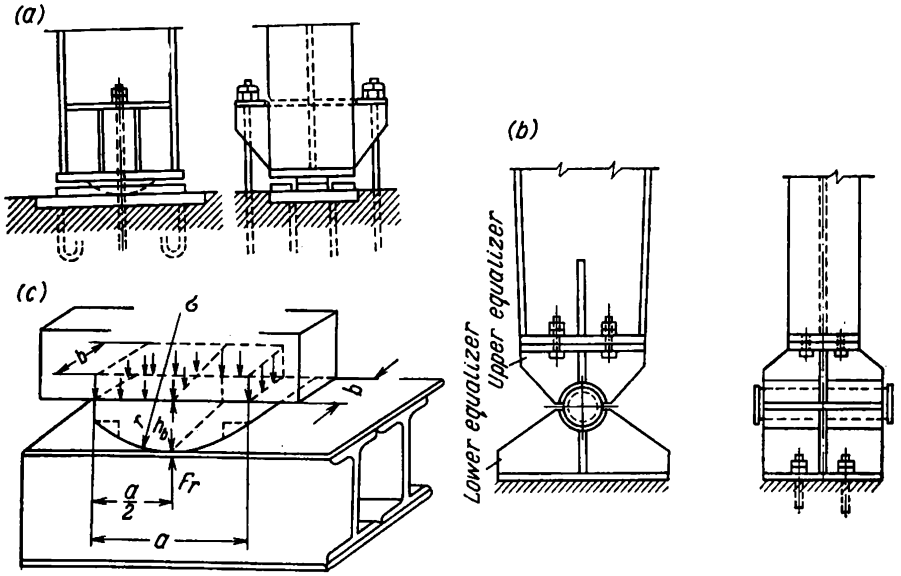


Fig. 10-13. Bearings of arched members

The height of the tie bar can be increased, arranging it above the supports and thus increasing the useful height of the premises. In this instance on parts *a-b* (Fig. 10-12*a*) the arch functions in the same way as a simple beam subjected to bending, while the relieving influence of the force F_h in the tie covers only the part of the arch above the tie bar (with the ordinates y).

To increase the inside clearance of a structure and to employ an economical r_a/L ratio of about 1/5 to 1/6, arches can be located on auxiliary side structures (Fig. 10-12*b*) or pilasters. Here the thrust is distributed according to equation (10-8), in whose denominator the displacement of the bearing member δ'_{11} induced by a unit horizontal force should be added to the quantity δ_{11}.

The bearings of arched members with large spans subjected to longitudinal forces of about 800 to 1,200 tons are generally made of steel castings with a cylindrical bush similar to bridge bearings (Fig. 10-13*b* and 10-6*c*). With lower forces, rocker type or tangential bearings are used (Fig. 10-13*a* and 10-6*a*). Such bearings can be investigated by means of equation (10-4), taking $D=2r$ (r is the radius of the bearer) and $N=1$. The required thickness of the bearer h_b is determined on the assumption of its bending along the section of contact with the bearing plate of the grillage and uniform distribution of the support reaction over the whole bearer (Fig. 10-13*c*)

$$h_b \geqslant \sqrt{\frac{3F_r a}{2R_b b}} \qquad (10\text{-}12)$$

where R_b = design bending strength of the bearer material
a and b = dimensions of bearer (see Fig. 10-13*c*).

The bearings should be anchored in the foundations.

(2) Dome-Shaped Roofs

Dome-shaped roofs are very rational for use mainly in round buildings. Three main types of domes are distinguished, namely, *ribbed*, *ribbed and ringed*, and *polygonal domes*.

1. *Ribbed domes* are in essence a system of two- or three-hinged bracing arches with bearings arranged along a circle (Fig. 10-14*a*). Such systems can be considered as bar ones. In the intervals between the arches, purlins are installed for supporting the roof and the ties. The thrust of the arches F_h can be withstood either by the structure which the dome is supported on, or by a bearing ring. In this instance the latter will serve as a sort of conditional tie bar for each arch.

Should the sectional area of a conditional tie bar A_{tb} be so selected that its elastic deformations will equal those of a ring with axial-symmetrical loading induced by all the ribs, then an element of

the dome can be analyzed as an arch located along the diameter with a conditional tie bar sectional area A_{tb} and with the tributary area hatched in Fig. 10-14*a*. If the bearing ring is designed in the form of a polygon (Fig. 10-14*b*), then

$$A_{tb} = \frac{2LA_r}{L_r} \sin^2 \frac{\varphi}{2}$$

in which L_r = length of section of ring between ribs

φ = angle between ribs.

If the ring is a round one (Fig. 10-14*c*), then

$$A_{tb} = \frac{2\pi A_r}{N}$$

where N is the number of ribs (half-arches) in the dome.

The sectional area of the ring is found from the expression

$$A_r \geqslant \frac{F_r}{R}$$

in which F_r is the force in the bearing ring. In a polygonal ring

$$F_r = \frac{F_h}{2 \sin \frac{\varphi}{2}} \qquad (10\text{-}13a)$$

and in a circular one

$$F_r = \frac{F_h r}{L_r} \qquad (10\text{-}13b)$$

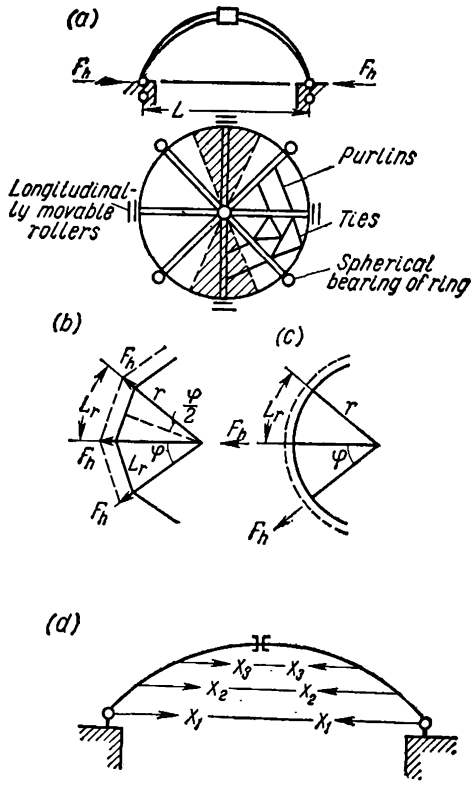


Fig. 10-14. Diagrammatic views of domes

The bearing ring is laid either on flat plates or slabs, or on tangential or roller bearings. Four longitudinally movable rollers are installed, the remaining bearings being spherical ones (Fig. 10-14*a*).

The central ring at the apex of a dome formed by three-hinged arches resists mainly compression induced by the thrust forces F_h and should be checked for stability. It is essential that the stresses in it should not exceed the critical ones

$$\sigma \leqslant \sigma_{cr}$$

where

$$\sigma = \frac{F_h r}{A_r L_r} \qquad (10\text{-}14a)$$

$$\sigma_{cr} = \frac{3EI_r}{A_r r^2} \qquad (10\text{-}14b)$$

in which r = radius of central ring at apex
A_r = sectional area of this ring
L_r = distance between thrust forces applied to the ring at the apex of the dome
I_r = moment of inertia of ring section with respect to vertical axis.

With ribs in the form of two-hinged arches, the section of the central ring is subjected not only to compression, but also to bending, and therefore must be investigated for resistance to the total moment equal to the moment at the place of connection of the rib to the ring.

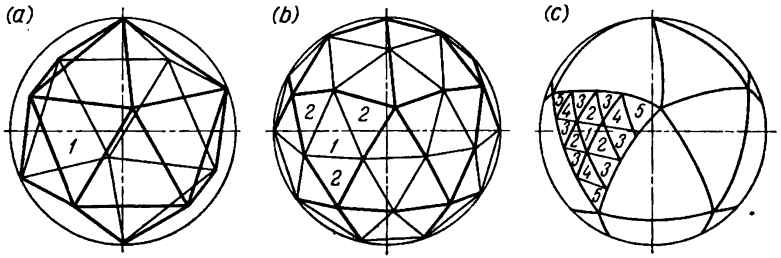


Fig. 10-15. Icosahedron and its development

The section of the ring is designed either of the box type consisting of two channels or I shapes, or as an H shape made up of rolled elements.

2. *Ribbed and Ringed Domes.* If in a ribbed dome, which is in essence a bar system of arches connected only by means of a bearing ring, all the ring purlins are made to take the load, then we obtain a three-dimensional or space member with a number of rings serving as conditional tie bars for the separate arches. Such a dome is referred to as a ribbed and ringed one. It is more rational and lighter than a ribbed dome, since almost all the elements are used to resist the loads. Figure 10-14*d* depicts a ribbed and ringed dome with three rows of rings and, accordingly, with three redundant quantities X_1, X_2 and X_3. Each ring is selected with its own radius r and length L_r. Otherwise the investigations are similar to those used for a ribbed dome and are carried on as for an arched system with several conditional tie bars.

3. *Polygonal Domes.* A polygonal dome may be designed in a number of ways. Thus, for example, a sphere may be divided by meridionally directed and annular ribs, and a diagonal installed in each of the rectangles obtained in this way. In such a design the lengths of the elements in each tier and the angles between them will be different, and this will result in large labour requirements for

erecting the dome. To eliminate this great difference in element types, good practice dictates using the geometry of polyhedrons inscribed in a sphere. As is known from solid geometry, there are altogether five such polyhedrons, among which the one having the greatest number of identical faces (20) is the icosahedron (Fig. 10-15*a*).

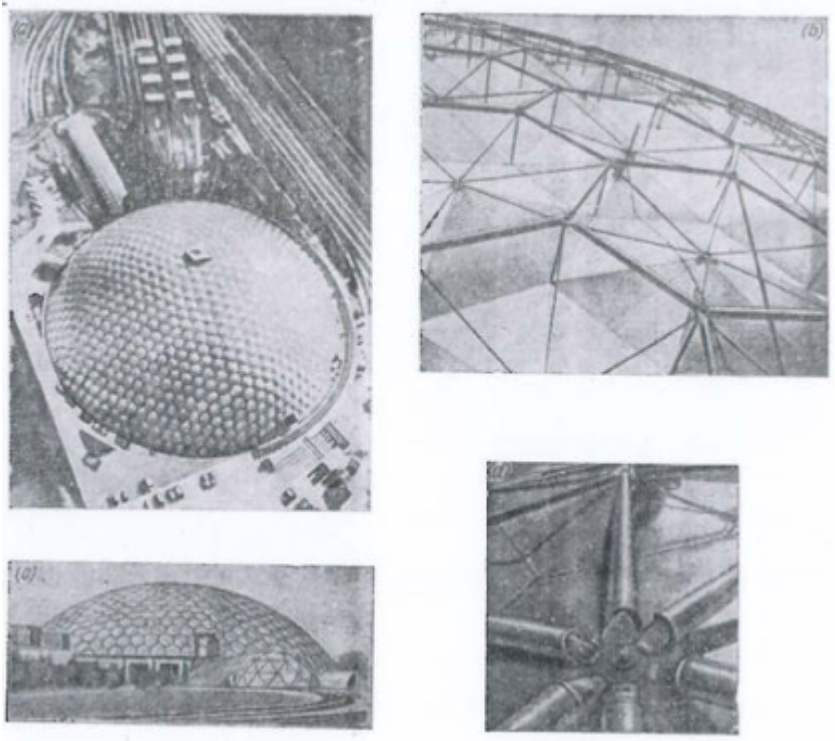


Fig. 10-16. Polygonal domes with a diameter of 118 metres

When two different types of faces are used, an 80-face polyhedron is obtained (Fig. 10-15*b*), and with five different types of faces—a 320-face one (Fig. 10-15*c*), etc. This principle of construction has served as the basis for a large number of such geodetical domes.

Figure 10-16*a* and *b* pictures a dome built in the town of Baton Rouge (USA) and having a diameter of 118 metres and a height of 36.5 metres. Each cell of the dome consists of a convex hexagonal panel field-welded from steel plates with a thickness of 3.2 millimetres, and a hexagonal tubular framework connected to the plate panel

by means of round bars. The consumption of metal in this dome was 55.5 kilograms per square metre of floor.

Certain difficulties encountered in the erection of this dome, which required a high precision of work, led to changes in design in the construction of a similar dome at Wood River (USA) and to an original method of erection. The tubular framework forming the triangular lacing was welded directly to the apices of the convex plate panels with the aid of round gusset plates (Fig. 10-16*c* and *d*). The central part of the dome with a height of 10 metres (up to a diameter of 61 m) was assembled on metal falsework. Next the shell was jacked up, the falsework was dismantled and sheets of nylon fabric were so attached to the shell from underneath as to form a closed space under it, which air was delivered into under pressure. On this air bed, the lower part of the dome was erected up to the bearing ring.

Fig. 10-17. Polygonal aluminium dome (Sokolniki, Moscow)

Figure 10-17 illustrates the design of an aluminium dome of rhombic panels, each of which is a bent plate with braces.

A polygonal dome can be analyzed in the same way as a shell, by means of the membrane theory. The shell is assumed to be solid and axially symmetrical. The load of a dome generally consists of its weight (dead load), snow and wind loads.

In considering the equilibrium of a shell under a uniformly distributed load p kg/m^2 directed toward the centre, we shall obtain the basic equation for a spherical membrane shell (see page 456)[1].

$$T_1 + T_2 = pR \qquad (10\text{-}15)$$

where T_1 = meridional force in shell, kg/m
T_2 = annular force, kg/m
R = radius of sphere.

1. *Forces in Dome Induced by Dead Load of g kg/m^2.* Let us denote by G the total weight of the part of the dome from its apex to the level y (or cut off by the radius R at an angle of φ) as shown in Fig. 10-18*a*, i.e.,

$$G = -g2\pi R(R - y)$$

[1] On page 456 the forces T_1 and T_2 are taken in the form of resultant ones measured in kilograms.

This total weight will be balanced by the vertical reaction

$$T_1 2\pi r \sin\varphi = G \tag{10-16}$$

Hence the meridional compressive force T_1 will be

$$T_1 = \frac{G}{2\pi r \sin\varphi} = -g\frac{R^2}{R+y} = -g\frac{R}{1+\cos\varphi} \tag{10-17}$$

The annular force T_2 is found from equation (10-15), substituting for the pressure perpendicular to the surface of the shell the load induced by the weight of the members $p = -g\cos\varphi$.

$$-g\frac{R}{1+\cos\varphi} + T_2 = -g\cos\varphi R,$$

whence

$$T_2 = -Rg\left(\cos\varphi - \frac{1}{1+\cos\varphi}\right)$$

or

$$T_2 = -Rg\left(\frac{y}{R} - \frac{1}{1+\frac{y}{R}}\right) = -g\frac{y^2+yR-R^2}{y+R} \tag{10-18}$$

Upon solving the equation $y^2+yR-R^2=0$ we shall find the boundary line where the annular force is equal to zero and a transition from compression to tension takes place, namely,

$$y = 0.618R \text{ and } \varphi = 51 \text{ deg } 49 \text{ min}$$

At the apex of the dome, with $y=R$, we have

$$T_1 = T_2 = -g\frac{R}{2}$$

and for $y = 0$

$$T_1 = T_2 = gR \tag{10-19}$$

The corresponding force diagrams are shown in Fig. 10-18*a*.

2. *Force in Dome Induced by Uniformly Distributed Load q kg/m²* (Fig. 10-18*a*).

Similarly we obtain

$$G = -q\pi r^2$$

and

$$T_1 = \frac{G}{2\pi r \sin\varphi} = -\frac{\pi r^2 q}{2\pi r\frac{r}{R}} = -\frac{qR}{2} \tag{10-20}$$

By substituting in shell equation (10-15) $p = -q\cos^2\varphi = -\frac{qy^2}{R^2}$ we find

$$T_2 = -\frac{qy^2}{R^2}R + \frac{qR}{2} = -\frac{q}{2R}(2y^2 - R^2) = -\frac{qR}{2}\cos 2\varphi \tag{10-21}$$

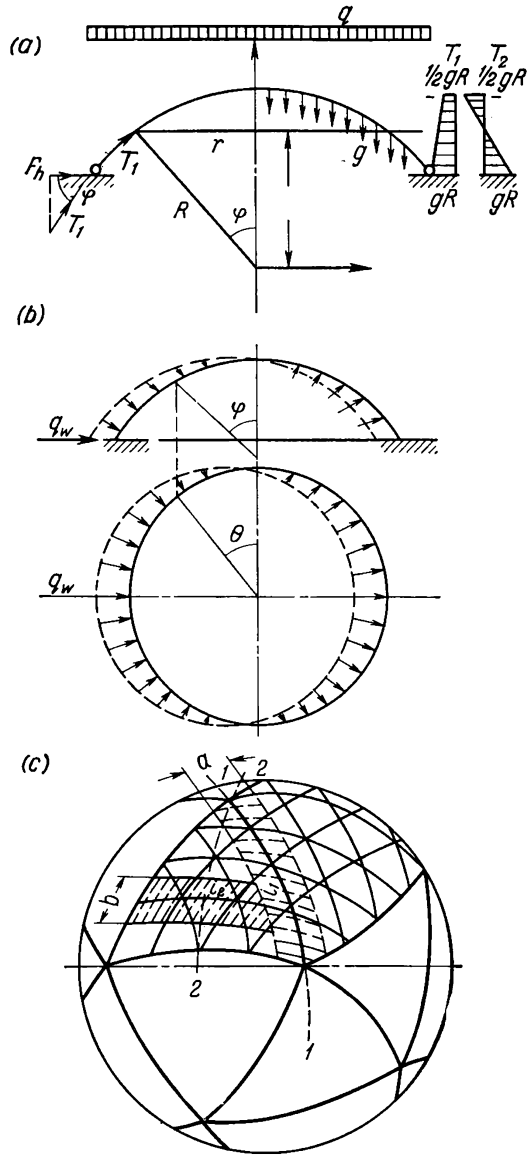


Fig. 10-18. To analysis of polygonal dome

Upon assuming that $2y^2-R^2=0$, we find the value of $y=0.707$ ($\varphi=45$ deg) at which the annular force vanishes.

At the apex of the dome with $y=R$ we have

$$T_1=T_2=-\frac{qR}{2}$$

and for $y=0$

$$T_1=-\frac{qR}{2} \text{ and } T_2=+\frac{qR}{2} \tag{10-22}$$

3. *Forces in Dome Induced by Wind Load.* The forces induced by the wind load are determined on the assumption that the pressure of the wind (normal to the surface) is equal to (Fig. 10-18*b*)

$$q_w=q_{w\cdot v}\sin\varphi\sin\theta$$

in which $q_{w\cdot v}$ is the design wind load on a vertical plane.

For a dome in the form of the surface of a hemisphere, the meridional forces will be

$$T_1=q_{w\cdot v}R\frac{\cos\varphi}{\sin^3\varphi}\left(\frac{2}{3}-\cos\varphi+\frac{1}{3}\cos^3\varphi\right)\sin\theta \tag{10-23}$$

and the annular forces will be

$$T_2=q_{w\cdot v}R\left[\sin\varphi-\frac{\cos\varphi}{\sin^3\varphi}\left(\frac{2}{3}-\cos\varphi+\frac{1}{3}\cos^3\varphi\right)\right]\sin\theta \tag{10-24}$$

For determining the forces in the bars of a dome, it is always possible to select a bar following a meridian which collects from a definite "force" area the force T_1. Thus, for instance, in Fig. 10-18*c* the bar i_1 in section *1-1* is directed along a meridian. This bar collects the force from the hatched area having a width of a, i.e., the force in it is

$$F_1=T_1a \text{ kg} \tag{10-25}$$

In the same way the force in bar i_2, directed along a ring (Fig. 10-18*c*), is determined. For this purpose we shall consider section *2-2* with a width of the "force" area equal to b. The force in the bar is

$$F_2=T_2b \text{ kg} \tag{10-26}$$

Since the forces T_1 and T_2, expressed in kg/m, can be determined at any point of the sphere, the forces in all the bars of the dome can be found.

Besides the axial forces, the bars, depending on the design of the roof, may be subjected to bending induced by a local load, which must be taken into account when selecting their sections. It is also essential to ensure the required stiffness of the bars in a vertical

plane (to avoid the loss of stability) by satisfying the condition that

$$I_d \geqslant \frac{T_1 Ra}{0.5E} \sqrt{\frac{T_1 R}{0.5E}}$$

The elements of a dome are ordinarily made either with a tubular section, or of relatively small angle, channel or T shapes.

(3) Suspension (Guy) Systems

In suspension (guy) systems the main bearing elements of the members are guys (cables, wire ropes).

Owing to the high strength of rope wire (from 120 to 210 kg/mm^2) and the complete utilization of the sectional area of a rope in tension, suspension members have a light and economical design. It is advantageous to employ such structures where large spans are involved. The merits of such structures also include rapid erection of them.

The main shortcomings that lead to certain difficulties in solving the structural members are the high deformability of suspension

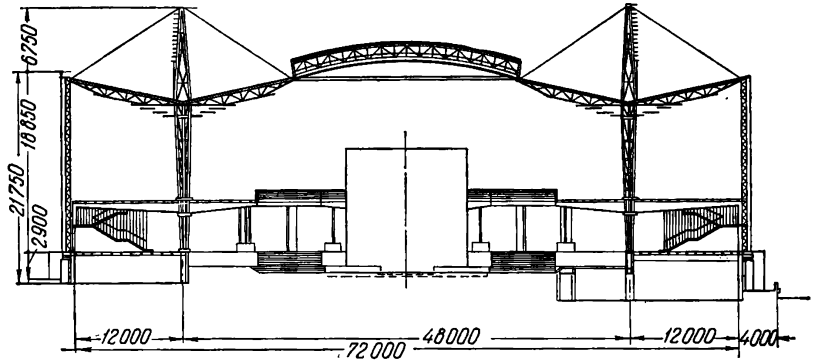


Fig. 10-19. USSR pavilion at World Fair at Brussels, 1958

systems under the action of live loads and, in roofs, the complications involved in draining away water.

Suspension systems are thrust ones. The members taking the thrust form a considerable part of the structure as regards the consumption of material.

Guy systems may be divided into two types, namely, plane (guy-bar) and three-dimensional or space (in the form of membranes or polygonal members) systems.

The building of the USSR pavilion at the World Fair at Brussels (1958) can serve as an example of the cantilever guy system; the guys

are connected at one side to a cantilever and through the posts of the external framework to the foundations, while at the other side they are loaded with the weight of the middle part of the roof member (Fig. 10-19). A number of bridges of the guy-rope system have been erected.

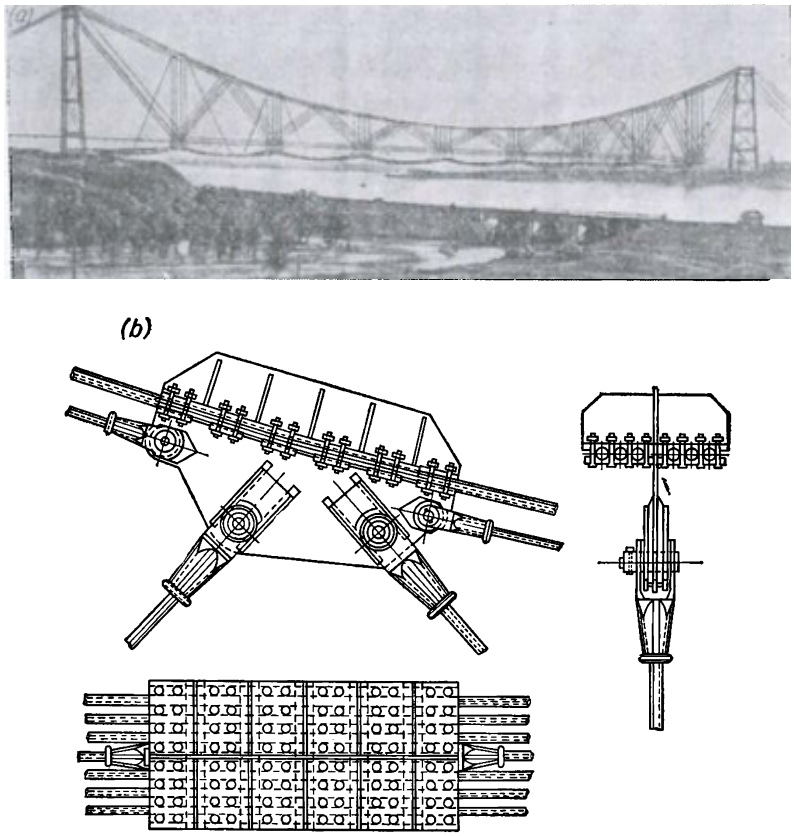


Fig. 10-20. Crossing of Volga River at Volgograd:
a—general view; *b*—top chord joint

The guy bridge trusses with a span of 874 metres used for the crossing of the Volga River at Volgograd (Fig. 10-20), designed of wire ropes according to a system proposed by engineers V. Vakhurkin and G. Popov in 1955, proved to be very rational. As the result of

prestressing of the bottom chord, also made from ropes, all the elements of the truss were in tension. The roadway members were suspended at the joints. The compressive forces appearing in the elements of the truss under the action of the service loads do not exceed the prestressing forces, and thus, notwithstanding the presence of flexible elements, the truss behaves as a rigid system. Figure 10-20*b* pictures the design of a top truss chord joint.

Such members can be successfully employed for large-span roofs.

Space polygonal suspension members, depending on the external contour of the structure, may be of the barrel type (over a rectangular contour), spherical or in the form of other shells with a positive Gaussian curvature (covering an ellipsoidal contour, etc.), a negative Gaussian curvature, a double curvature (saddle-shaped), etc.

The design of barrel and round suspension shells can be considered from the viewpoint of analysis as a set of plane flexible strings. However, such strings are too deformable, and under the influence of uneven loading or gusts of wind and suction great displacements of the roof in different directions may appear. To eliminate these undesirable displacements, the practice is followed of laying a flooring of reinforced concrete slabs on the ropes or cables, an additional load being applied during erection, after which all the joints are concreted in situ. After removal of the additional load and elastic rebounding of the ropes, the roof begins to function in the reverse direction as a shell and becomes sufficiently rigid. Figure 10-21*a* shows diagrammatically such a roof with a span of 94 metres (stadium at Montevideo). Here the ropes are anchored in the external reinforced concrete compressed ring, while at the centre of the structure they are connected to a metal tensioned ring. Water is drained off along suspended pipes.

When light aluminium panels are employed, guy ropes must be installed (Fig. 10-21*b*).

Convex suspension roofs can be designed in the form of a two-chord system of ropes or cables. Several designs are possible, for example, in the form of a "bicycle wheel" with tensioned strings and a central drum (the USA pavilion at the Brussels World Fair, 1958, Fig. 10-22*a*). Similar systems are possible without a central drum, with the installation between two ropes of compression posts (Fig. 10-22*b*) or tension posts (Fig. 10-22*c*). Semi-transparent plastics laid over the purlins can be used as the roof sheathing in such roofs.

Polygonal roofs with a double curvature (Fig. 10-23) are a rational design of roofs. The bearing cables are arranged along a concave surface, while the tensioning ropes directed at right angles to them have a convexity facing upward. As a result, a saddle-shaped relatively rigid surface is obtained. The cables and ropes are anchored in inclined arches whose weight facilitates their tensioning. The weight

in which r = radius of central ring at apex
A_r = sectional area of this ring
L_r = distance between thrust forces applied to the ring at the apex of the dome
I_r = moment of inertia of ring section with respect to vertical axis.

With ribs in the form of two-hinged arches, the section of the central ring is subjected not only to compression, but also to bending, and therefore must be investigated for resistance to the total moment equal to the moment at the place of connection of the rib to the ring.

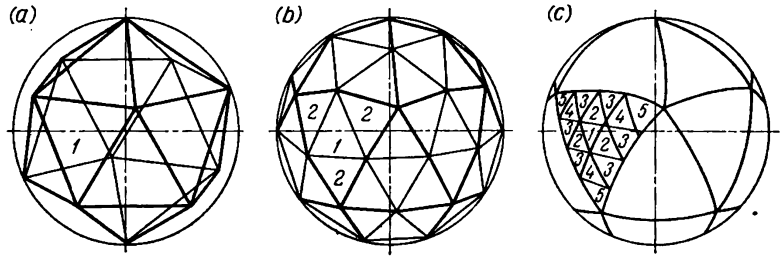


Fig. 10-15. Icosahedron and its development

The section of the ring is designed either of the box type consisting of two channels or I shapes, or as an H shape made up of rolled elements.

2. *Ribbed and Ringed Domes.* If in a ribbed dome, which is in essence a bar system of arches connected only by means of a bearing ring, all the ring purlins are made to take the load, then we obtain a three-dimensional or space member with a number of rings serving as conditional tie bars for the separate arches. Such a dome is referred to as a ribbed and ringed one. It is more rational and lighter than a ribbed dome, since almost all the elements are used to resist the loads. Figure 10-14*d* depicts a ribbed and ringed dome with three rows of rings and, accordingly, with three redundant quantities X_1, X_2 and X_3. Each ring is selected with its own radius r and length L_r. Otherwise the investigations are similar to those used for a ribbed dome and are carried on as for an arched system with several conditional tie bars.

3. *Polygonal Domes.* A polygonal dome may be designed in a number of ways. Thus, for example, a sphere may be divided by meridionally directed and annular ribs, and a diagonal installed in each of the rectangles obtained in this way. In such a design the lengths of the elements in each tier and the angles between them will be different, and this will result in large labour requirements for

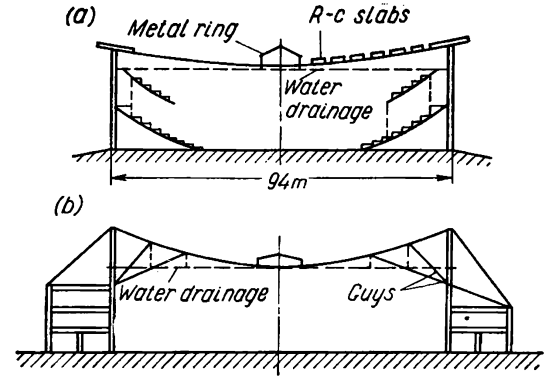


Fig. 10-21. Suspension roofs:
a—roof of stadium at Montevideo; *b*—roof with guy ropes

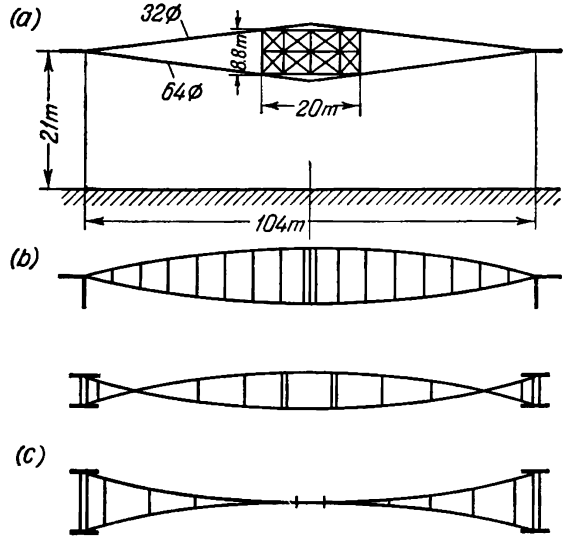


Fig. 10-22. Convex suspension roofs and roofs with tie rods

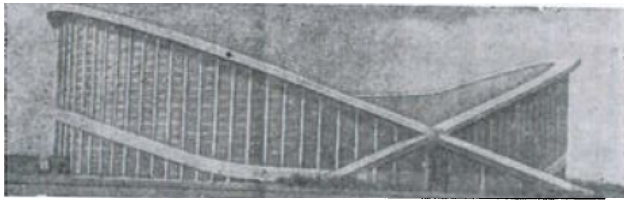


Fig. 10-23. Polygonal roof with double curvature

of the arches, however, is not sufficient, and the vertical external columns simultaneously serve as tie rods anchored in the foundation.

It must be remembered that before installation the cables and ropes should be preliminarily tensioned with a force exceeding the design one by 20-25%.

An approximate method can be used for investigating the strength of the ropes of roofs with a negative curvature supported on an elliptical contour if it is assumed that the loaded structure retains its parabolic configuration.

Let F_{h_1} and F_{h_2} denote the forces in the bearing and the tensioning elements, b_1 and b_2—the major and minor diameters of the ellipse, δ_1—the deflection of the bearing rope, δ_2—the deflection or rise of the tensioning rope, q—the intensity of the external uniformly distributed load (dead and live) in t/m, $q_1 = q_u b_{br}$ in t/m—the intensity of the live load, equal to the product of the load per square metre q_u and the distance between the bearing ropes b_{br}, q_{ps}—the intensity of prestressing transmitted from the tensioning ropes to the bearing ones.

The magnitude of q_{ps} is found on the assumption that the bearing ring will not take bending in a horizontal plane and that each tensioning rope transmits an identical load to the bearing rope, i.e., both ropes are under a uniformly distributed load and should therefore retain a parabolic configuration

$$q_{ps} = \frac{q}{\frac{\delta_1}{\delta_2} - 1} \text{ t/m} \qquad (10\text{-}27)$$

The forces in the bearing rope and, accordingly, in the tensioning rope will be

$$F_{h_1} = \frac{(q + q_{ps}) L_1^2}{8\delta_1} \quad \text{and} \quad F_{h_2} = \frac{q_{ps} L_2^2}{8\delta_2} \qquad (10\text{-}27a)$$

in which L_1 = length of bearing rope
L_2 = length of tensioning rope.

After the roof has been loaded with a live load q_1, the following force will remain in the tensioning rope

$$F'_{h_2} = \frac{(q_{ps} - q_1) L_2^2}{8\delta_2}$$

In these investigations the value of the modulus of elasticity of the rope (with a stiff core) is taken equal to $E \cong 16 \times 10^6$ t/m^2.

Ropes and cables should be well protected against the weather, for which purpose galvanized or plastic-coated ropes are used. The ropes are anchored by embedding them in steel sleeves with grade ЦАМ9-1.5, ЦАМ10-5 alloys, babbitt metal B-95, etc. The

ropes are unbraided in the sleeve and the ends of the wires are bent down. It is also possible to wedge the ropes in a special casing.

Below are considered the methods of analyzing flexible cables (ropes, guys).

Analysis of Flexible Non-Extensible Cable. Let us assume that we have a cable freely suspended in span L and loaded with a uniformly distributed load q (Fig. 10—24*a*). Let us further assume that the length S of the cable has such a value that under the action of

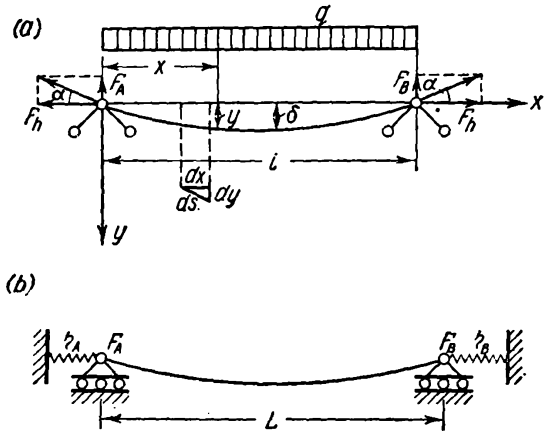


Fig. 10-24. To analysis of flexible cables

the load the deflections y (or their maximum value δ) are relatively small, within the limits of $\frac{\delta}{L} = \frac{1}{6}$ to $\frac{1}{20}$. Then it can be assumed that the load is uniformly distributed over a horizontal line, while the equilibrium curve of the elastic line is a quadratic parabola. The distinctive feature of flexible cable investigation consists in a different form of cable equilibrium corresponding to each new loading. Seeing that in any section of the cable the moment is equal to zero, this condition can be written down for the section x

$$M_x - F_h y = 0$$

whence

$$y = \frac{M_x}{F_h} \tag{10-28}$$

Here M_x = sum of moments of all the vertical forces acting on the left-hand part of the cable, which is equivalent to a bending moment in the section of a simple beam.

Upon differentiating equation (10-28), we get

$$\frac{dy}{dx}=\frac{1}{F_h}\frac{dM_x}{dx}=\frac{Q_x}{F_h} \tag{10-29}$$

where Q_x = shear force found in the same way as for a simple beam.

When Q and F_h are known, we can find the longitudinal force in a flexible cable

$$F_l=\sqrt{Q^2+F_h^2} \tag{10-30}$$

Thus, to determine the force in a cable, rope or guy it is necessary to know the thrust F_h or the ordinate of the equilibrium curve y (or its maximum value—the deflection δ).

With a uniformly distributed load q we shall have

$$M_x=\frac{qL^2}{8}\quad F_h=\frac{M}{\delta}=\frac{qL^2}{8\delta}\quad \tan\alpha=\frac{Q}{F_h}=\frac{4\delta}{L} \tag{10-31}$$

When determining the force in the cable, the length of the cable S, which is directly related to the deflection δ (Fig. 10-24*a*), may be selected instead of the latter.

The length of the cable is determined from equation (10-32), in which relation (10-29) has been used and the radical has been replaced by expansion in a series

$$S=\int_0^L ds=\int_0^L \sqrt{1+\left(\frac{dy}{dx}\right)^2}dx\cong\int_0^L\left[1+\frac{1}{2}\left(\frac{Q_x}{F_h}\right)^2-\frac{1}{8}\left(\frac{Q_x}{F_h}\right)^4+\ldots\right]\times$$

$$\times dx\cong L+\frac{1}{2F_h^2}\int_0^L Q_x^2dx \tag{10-32}$$

The quantity $\int_0^L Q_x^2dx=D$ is a characteristic of the load.

Thus the magnitude of the thrust F_h may be expressed through the initial length of the cable (the pre-cut length) S

$$F_h=\sqrt{\frac{D}{2(S-L)}}$$

A more precise value of the thrust can be obtained from one of the following formulas

$$F_h=\sqrt{\frac{LD}{S^2-L^2}}\text{ or }F_h=\frac{\sqrt{3LD}}{4\delta} \tag{10-33}$$

in which

$$D=\int_0^L Q_x\,dx\text{ or }D=\int_0^L M_x q_x\,dx \tag{10-34}$$

From formulas (10-33) it follows that

$$\sqrt{\frac{LD}{S^2-L^2}}=\sqrt{\frac{3LD}{4\delta}}$$

whence we find the length of an arc for gentle curves (P. Chebyshev's formula)

$$S=\sqrt{L^2+\frac{16}{3}\delta^2}=L\sqrt{1+\frac{16}{3}\left(\frac{\delta}{L}\right)^2}\cong L\left(1+\frac{8}{3}\frac{\delta^2}{L^2}\right) \quad (10\text{-}35)$$

Table 10-2 contains certain values of the quantity D calculated for different loads.

Analysis of Flexible Elastic Cable with Fixed Supports. If the deflection of the cable is less than $\frac{\delta}{L}=\frac{1}{r_0}<\frac{1}{20}$, then it should be investigated with account taken of elastic strains. The thrust F_h is found from the equation

$$F_h^3+\frac{8EA}{3r_0^2r_s}H^2=\frac{DEA}{2Lr_s^3} \quad (10\text{-}36)$$

in which E = modulus of elasticity of the cable
A = cross-sectional area of the cable
$r_0=\frac{L}{\delta}$ and $r_s=\frac{S}{L}$ = quantities determined without account taken of elastic strains.

Analysis of Flexible Elastic Cable with Yielding Supports. Let us assume that the supports have a yielding of η_A and η_B (measured in metres), which is characterized by their displacement under the action of a force equal to 1 ton (Fig. 10-24*b*).

The thrust F_{h_1} in this instance is determined from the equation

$$F_{h_1}^4\left(\frac{4Lr_s^3}{EAF_h}+\frac{D}{F_h^4}\right)-F_{h_1}^3\left(\frac{4Lr_s^3}{EA}+4\eta\right)+D=0 \quad (10\text{-}37)$$

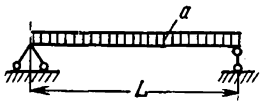
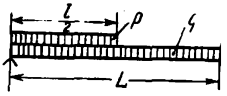
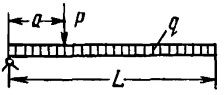
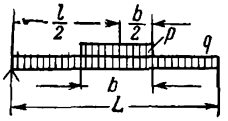
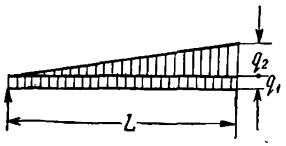
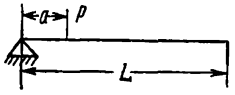
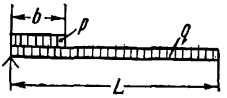
where F_h = thrust found from equation (10-36)
$\eta=\eta_A+\eta_B$.

Analysis of Flexible Elastic Cable Prestressed with Force F_{ps}. Let us assume that we have a cable whose length is somewhat less than the span L ($S<L$), the cable being loaded with a vertical load and subjected to prestressing with a force F_{ps}.

If we denote by F_h the total thrust appearing in the cable under the simultaneous action of the prestressing caused by the force F_{ps} and the vertical load (i. e., the force F_{ps} will form part of the support reaction), then F_h will be found from the equation

$$F_h^3-F_{ps}F_h^2=\frac{DEA}{2L} \quad (10\text{-}38)$$

TABLE 10-2 Quantity D Characterizing Loads

Item No.	Loading diagram	Value of D (in kg²-m)	Notation
1		$\frac{q^2L^3}{12}$	—
2		$\frac{q^2L^3}{12}\left(1+\gamma+\frac{5}{16}\gamma^2\right)$	$\gamma=\frac{p}{q}$
3		$\frac{q^2L^3}{12}\left[1+(4-3\beta)\beta^3\gamma^2+\right.$ $\left.+(6-4\beta)\beta^2\gamma\right]$	$\beta=\frac{b}{L}$
4		$\frac{q^2L^3}{12}\left[1+(3-2\beta)\beta^2\gamma^2+\right.$ $\left.+(3-\beta^2)\beta\gamma\right]$	—
5		$\frac{q^2L^3}{12}+\frac{q_1q_2L^3}{12}+\frac{q_2^2L^3}{45}$	—
6		$\frac{P^2(L-a)a}{L}$	—
7		$\frac{q^2L^3}{12}\left[12\alpha\gamma_1(1-\alpha)\right.$ $\left.(1+\gamma_1)+1\right]$	$\gamma_1=\frac{P}{qL}$ $\alpha=\frac{a}{L}$

In a particular case, when the length of the cable is equal to that of the span and there is no prestressing, i. e., $S=L$, $r_s=1$ and $r_0=\infty$ (analysis of a string), we have

$$F_h=\sqrt[3]{\frac{DEA}{2L}}=\sqrt[3]{\frac{q^2L^2EA}{24}} \tag{10-39}$$

Example 10-1. A steel cable (rope) crossing a span $L=100$ m has a weight $g=10$ kg/m, a cross-sectional area $A=12$ cm^2 and a modulus of elasticity $E=16\times\times10^6$ t/m^2. The cable is subjected to a load $P=1$ t applied at the middle of the span.

Find the force in the cable and its deflection if the initial deflection is $\delta=5$ m.

Solution. We determine the characteristics of the cable

$$EA=16\times10^6\times12\times10^{-4}=19{,}200 \text{ t}$$

$$D=\frac{10^2\times10^{-6}\times100^3}{12}[12\times0.5\times1(1-0.5)(1+1)+1]=58.3 \text{ t}^2\text{-m}$$

(see line 7 of Table 10-2, where $\alpha=0.5$ and $\gamma_1=1$);

$$r_0=\frac{L}{\delta}=\frac{100}{5}=20$$

Next we find from equation (10-35) the initial length of the cable (the precut length)

$$S=\sqrt{100^2+\frac{16}{3}\times5^2}=100.665 \text{ m}$$

whence

$$r_s=\frac{S}{L}=1.00665 \quad r_s^3=1.02$$

Upon substitution of the values obtained in equation (10-36), we get

$$F_h^3+125.49F_h^2=5{,}487, \text{ whence } F_h=6.45 \text{ t}$$

The maximum force in the cable is found from equation (10-30)

$$F_{l.\,max}=\sqrt{1^2+6.45^2}=6.53 \text{ t}$$

The maximum moment at the middle of the span

$$M_{max}=\frac{0.01\times100^2}{8}+\frac{1\times100}{4}=37.5 \text{ t-m}$$

The ordinate of the cable equilibrium line at the middle of the span (the deflection) will be

$$y_{max}=\delta=\frac{M_{max}}{F_h}=\frac{37.5}{6.45}=5.81 \text{ m}$$

Example 10-2. Find the force required for prestressing the cable considered in example 10-1 to ensure its deflection at the middle of the span being 1 metre.

Solution.

$$F_h=\frac{M_{max}}{y_{max}}=\frac{37.5}{1}=37.5 \text{ t}$$

From equation (10-38) we find the prestressing force

$$F_{ps}=37.5-\frac{58.3\times19{,}200}{2\times100\times37.5^2}=33.52 \text{ t}$$

10-3. TOWERS AND MASTS

(1) General

High structures with a relatively small cross section and with a large ratio between the height and the maximum width are known as towers or masts.

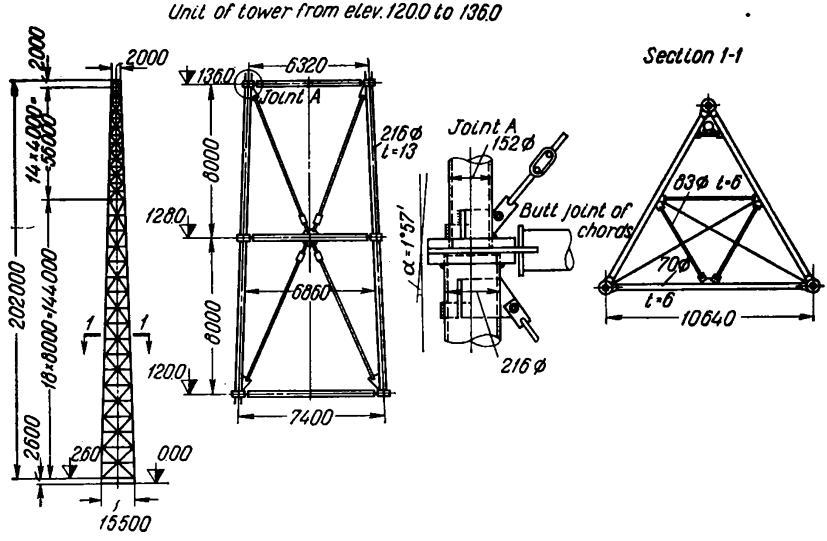


Fig. 10-25 Triangular radio tower

A *tower*, also known as a *pylon*, is a single-cantilever freely standing self-supporting structure fixed at its base. A *mast* is a structure pin-connected to its foundation and braced with guys or other elements.

Water towers, radio and television towers, the towers of power transmission lines and radio-relay systems, etc., are all examples of structures belonging to the tower family. Figure 10-25 pictures a triangular radio tower made of tubes and having a height of 202 metres. Examples of structures of the mast type are radio masts (Fig. 10-27), television masts (Fig. 10-28), etc.

Owing to the great height of tower- and mast-type structures, the main load which determines to a certain degree the constructive form of the structure is the wind load (USSR Building Standards and Regulations).

Vertical loads play a part only in such structures as water towers or in the members of mast-type structures which are subjected to the vertical component of the guy tensioning forces.

The design uniformly distributed wind load is found from the equation

$$q = QC_aC_wA \qquad (10\text{-}40)$$

where $Q = q_s n$ = service velocity head of wind in kg/m² at given height multiplied by load factor $n = 1.3$

C_a = aerodynamical coefficient, equal to 1.4 for angle sections and 0.45-1.2 for cylindrical bars

C_w = coefficient increasing the velocity head of the wind in order to allow for its dynamic and gusty nature (usually this coefficient ranges from 1.1 to 2)

A = net area in square metres of the projection of the structure or its part being investigated onto a plane perpendicular to the direction of the wind.

(2) Towers

High radio towers most frequently have a triangular or rectangular cross section in a horizontal plane and, as a rule, a pyramidal configuration in height.

The width of the base is taken equal to 1/8-1/15th of the height, with an inclination or pitch of the sides of 1/16 to 1/40. Seeing that tubular sections, in comparison with other ones, have good streamlining and a considerably lower aerodynamical coefficient C_a, the elements of tower chords are generally made of tubes with flanges welded to their ends which form a bolted joint convenient for erection (see Fig. 10-25). The lattice or trellis of a tower consists of cross diagonals made from round steel with a tensioning device, and rigid braces also having a tubular section.

The weight of radio towers 100 to 200 metres high is respectively about 0.2-0.3 t/m for triangular and 0.4-0.5 t/m for rectangular towers designed of tubes.

Water towers are designed in the form of posts located in plan view at the corners either of a square or a polygon inscribed in a circle. The sections of the posts consist of I shapes or channels, but may also be built up ones.

Towers are investigated as a trussed squared beam fixed at its base. The vertical load is uniformly distributed between the posts (chords).

The total force induced by the vertical and horizontal loads in a tower post will be

$$F = F_v \pm F_h \qquad (10\text{-}41)$$

where $F_v = \frac{P}{N \cos \alpha}$ = force in post induced by vertical loads uniformly distributed between N posts of the tower, forming with the vertical axis an angle α (if inclined)

F_h = force in post induced by horizontal forces and originated by tilting moment.

In a quadrangular tower, the maximum force in a chord will be obtained when the horizontal (wind) load is directed toward

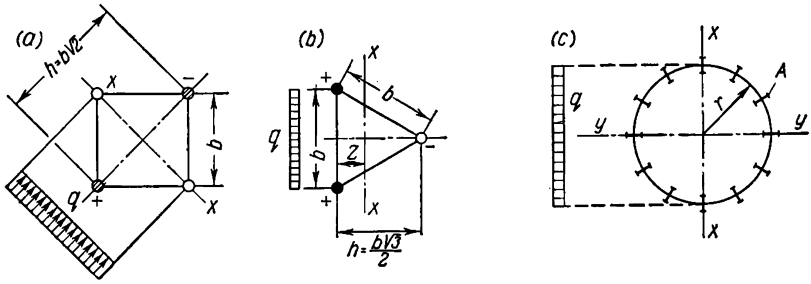


Fig. 10-26. To analysis of towers

a rib of the tower and only two chords are subjected to bending (Fig. 10-26*a*)

$$F_q = \frac{M}{b\sqrt{2}} \tag{10-42}$$

In a triangular tower the maximum force in a chord induced by the horizontal load q will be (Fig. 10-26*b*)

$$F_q = \frac{2M}{b\sqrt{3}} \tag{10-43}$$

In a multipost tower, the force F_h in a post with a section of A is determined in accordance with the tilting moment with respect to axis x-x (Fig. 10-26*c*) from the expression

$$F_h = \sigma A = \frac{M}{W_x} A = \frac{M_r A}{I_x} = \frac{2M}{Nr} \tag{10-44}$$

where the equatorial moment of inertia I_x in symmetrical sections is equal to half of the polar moment of inertia J

$$I_x = \frac{J}{2} = \frac{Nr^2 A}{2} \tag{10-45}$$

The plane of the maximum radius of gyration of the post sections, as a rule, is arranged along the radii of the tower. When investigating the posts for stability in compression, their effective length in the plane of a tower side is taken equal to the distance between the horizontal rings and the joints of the lattice. In towers with four and six posts, the effective length of a post in buckling in a radial direction is taken equal to the distance between joints of the lattice, whereas with eight or more posts this length is taken equal to the total length (height of the tower), if in a horizontal plane there are no rigidly secured membranes possessing adequate stiffness in a vertical plane.

The lattice of a tower is subjected to a shear force and is analyzed on the assumption that this force is resisted by two plane vertical trusses.

(3) Masts

A mast with guys consists of a shaft or pole of a triangular, square or round cross section. The chords of a triangular or quadrangular shaft are made of tubes connected by means of flanges in the same way as tower members. It is possible to use angle sections, although this will be less rational. A round shaft section is formed of a rolled and welded plate.

The guys are made of steel ropes with a steel core. They are embedded in concrete anchor slabs in the ground. The guys are arranged along the height of the mast either parallel to each other and at an angle of 45 deg to the mast, each guy having its own foundation (Fig. 10-27), or a group of guys are directed to several tiers of the mast from one foundation and are braced with yards (Fig. 10-28). In the latter case the maximum angle of inclination of a top tier guy to the horizontal is 60 deg. In a plan view the guys are arranged in a radial direction in relation to the axis of the mast shaft, three being used with a triangular or round section of the shaft and four with a quadrangular section. The ends of guy ropes are embedded in steel sleeves with the aid of zinc or other alloys.

Radio masts are lighter and cheaper than radio towers, but require a greater area for arrangement of the guys.

Radio masts are investigated mainly for the wind load and the vertical component of the guy tensioning forces with various combinations of temperatures and magnitudes of the wind velocity head (with a view to the appropriate standards and regulations). Should there be wire antennae between masts, the possibility of their complete or partial rupture should be taken into consideration. Allowance must also be made for the formation of an ice crust on all the elements of the members.

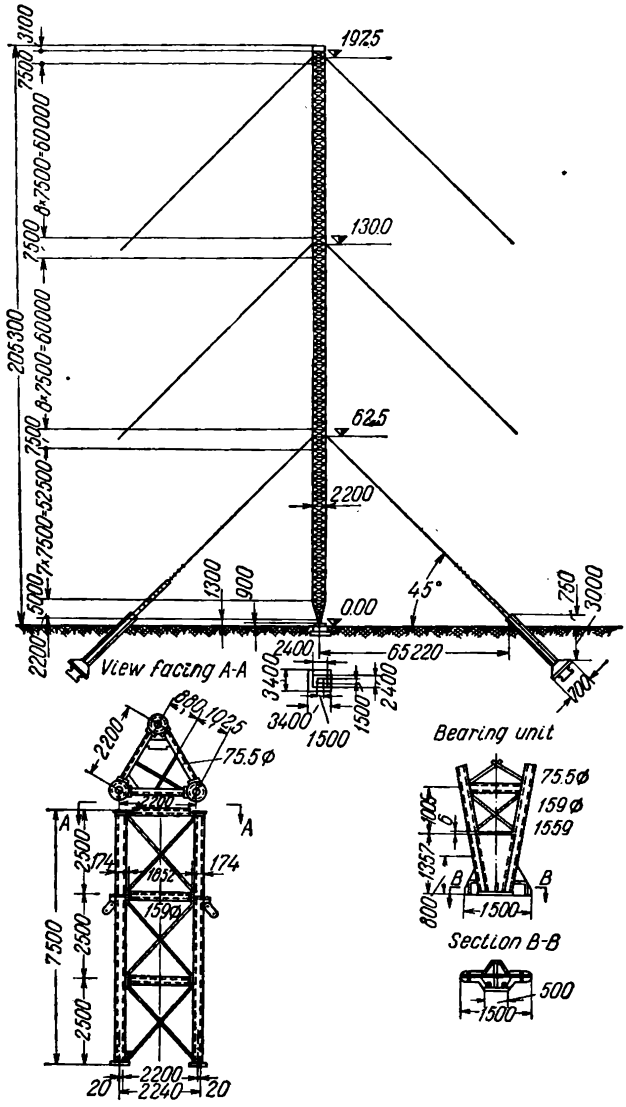


Fig. 10-27. Triangular radio mast

The maximum bending deflection of the mast shaft at the places of guy connection should not exceed 1/100th of the height of these connections from the foundation. The top of the mast should also deflect by not more than 1/100th of the length of the cantilever.

A mast is investigated in two stages. In the first stage the shaft is considered as a bar in compression and bending on stiff supports (guys). The longitudinal force is determined as the sum of the weight of the members and the equipment located on the mast, and of the vertical components of the guy reactive forces. These forces are used to select the sections of the mast shaft and guys, after which the second, more precise stage of investigations is carried out. In this stage the guy is analyzed as a tensioned flexible cable with an initial tension (or a predetermined sag) and the possible horizontal displacements of the guy connections are determined. Next the mast shaft is investigated as a bar in compression and bending on flexible yielding bearings.

The shaft of a mast rests on a central bearing. In insulated masts the bearing is made from barrel-shaped porcelain insulators carrying a cast steel equalizing bearing.

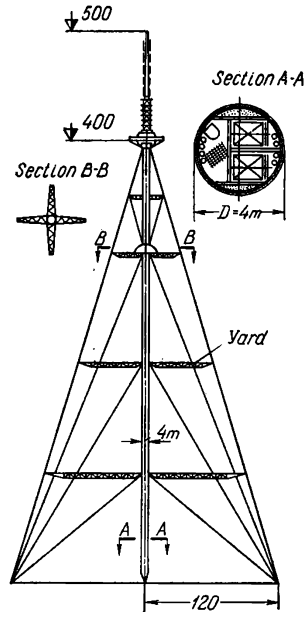


Fig. 10-28. Television mast with yards used to brace the guys

(4) Towers of Power Transmission Lines

Metal power transmission line towers are mainly used for high-voltage lines (220-330-500 kV). There are distinguished *suspension*, *strain* or *anchor*, *angle*, and *dead-end* or *terminal* towers.

Suspension towers (Figs. 10-29 and 10-30*a*) as their name implies, are designed mainly for supporting the tensioned wires. They are spaced, depending on the relief of the terrain, from 200 to 400 metres apart for lines with a voltage of 220-330 kV and from 400 to 600 metres apart for lines with a higher voltage. The height of the wires is so selected that there will be a distance of at least 6-10 metres from the surface of the ground to the spot with the greatest sag. This requires a tower height ranging from 20 to 40 metres. The weight of a suspension tower for 220-500-kV lines is 4-8 tons.

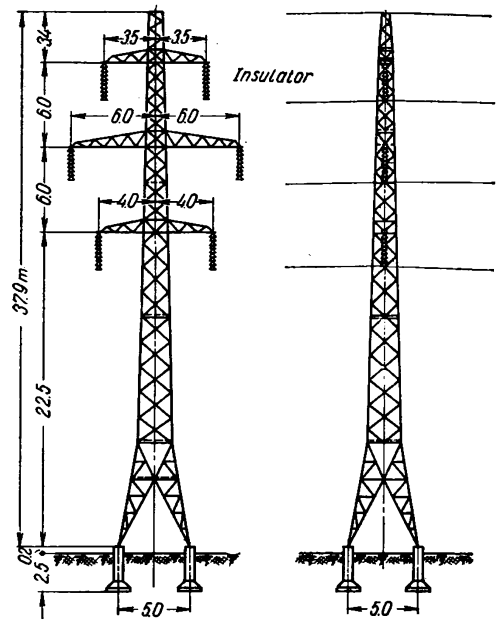


Fig. 10-29. Single-shaft suspension tower of 220-kV double-circuit power transmission line

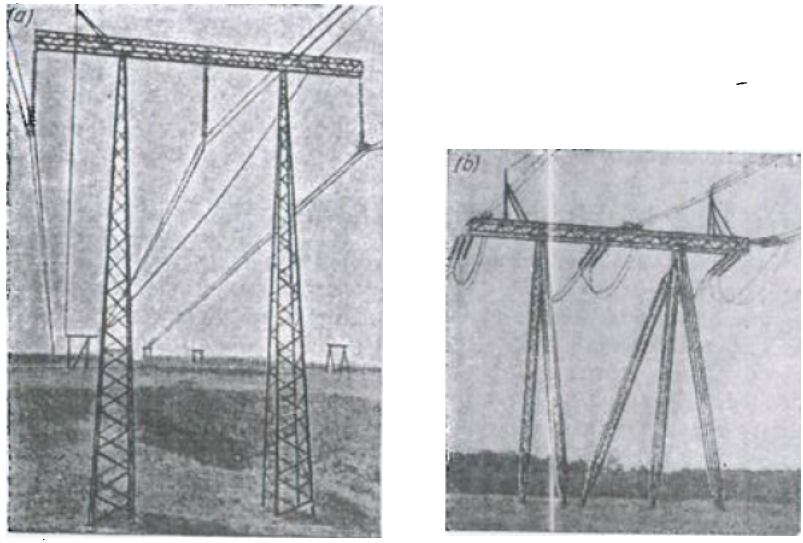


Fig. 10-30. Portal towers of power transmission lines:
a—suspension tower; *b*—strain and angle tower

Strain or *anchor towers* are installed approximately after 5-7 suspension towers. Their main designation is to completely resist the tension of the wires in case of their rupture on one side. Strain towers are about one-and-a-half times heavier than suspension ones.

Angle towers are located at spots where the direction of the line changes, and serve for resisting the component induced by the change in direction of the tension. It is good practice to combine strain and angle towers (Fig. 10-30*b*). Such towers are from 2 to 2.5 times heavier than suspension ones.

A great variety of tower designs are used. Among them are *single-shaft*, *single-circuit*, *double-circuit* (Fig. 10-29) and *portal* (Fig. 10-30) towers. The use of masts with guys will be very economical for power transmission lines.

It is often necessary to install power transmission line towers and masts in places difficult of access. It is therefore very important that the members of the towers be light in weight and convenient for transportation and erection. In accordance with these requirements, together with the widespread employment of steel 3 in towers, use is also made of low-alloy steel and, of course, aluminium alloys will be widely introduced for this purpose.

Owing to their use in large numbers, the members of towers should be unified and standardized. This has been partly done in the USSR for 220- and 330-kV lines, the design illustrated in Fig. 10-29 being used for standard towers.

A single-shaft tower, as a rule, is designed of four chord angles and lattices that form four plane trusses. The design loads of the towers consist of vertical and horizontal ones. The vertical loads consist of the weight of the members, the weight of the wires with the insulator strings, the lightning-arrestor cable and the ice crust on the wires and the cable. The vertical loads are distributed between the chords of the tower. The horizontal loads are made up of the wind load and that induced by the possible rupture of the wires (partial or complete) on one side of the tower.

The different load combinations are established by special standards for designing power transmission lines.

The tower is investigated for the action of horizontal loads as a squared cantilever beam fixed in its base. In this instance the horizontal load can be transferred to the axis of symmetry of the beam, with the addition of the moment M_t that acts in a horizontal plane and causes twisting of the tower. The horizontal force applied along the axis of the beam induces its bending, which is resisted by two plane trusses. The stresses in the chords and lattice of these trusses are determined in the usual ways.

The twisting moment M_t induces shear forces which are resisted by the lattices of the plane trusses.

In each cross section of the rectangular tower the twisting moment induces two pairs of forces (Fig. 10-31*a*)

$$Q_1 b + Q_2 a = M_t \qquad (10\text{-}46)$$

If a tower with a rectangular cross section were a solid thin-walled closed element (in the form of a rectangular tube), then the twisting moment would induce shearing stresses over the whole contour.

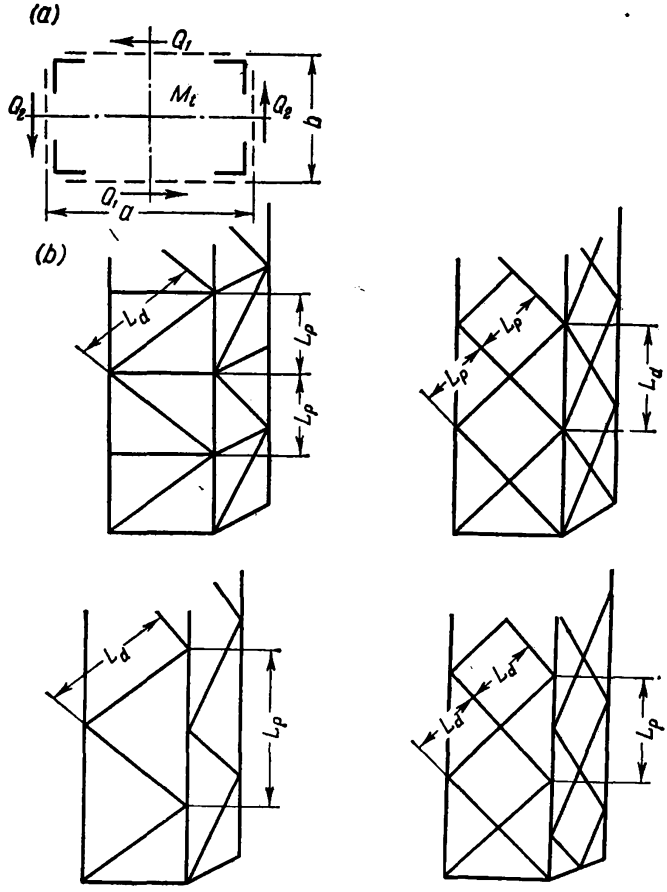


Fig. 10-31. To analysis of power transmission line towers

It is known that in a closed thin-walled section the shearing stress multiplied by the thickness of the section is the shearing force per

unit of length $f_{sh} = \tau t$. It is constant over the whole section and equals the twisting moment divided by twice the area limited by the closed contour

$$f_{sh} = \tau t \text{ kg/cm} = \text{const} \quad f_{sh} = \frac{M_t}{2ab} \tag{10-47}$$

It may be considered that the shear force is the sum of the flow of shearing stresses on each side of the rectangle and, therefore,

$$Q_1 = f_{sh} = \frac{M_t}{2b} \text{ and } Q_2 = f_{sh} = \frac{M_t}{2a} \tag{10-48}$$

The shear forces caused by bending of the tower should be added to these forces.

Having determined the shear forces, it is easy to find the forces in the elements of the lattice (see Chapter 8).

The lattices of the tower sides can be triangular, triangular with braces, and of the cross type without braces (except for the extreme ones that ensure rigidity of the member). The points of intersection of the diagonals on adjacent sides may coincide on the chord (Fig. 10-31*b*) or be displaced by half the panel length (herringbone arrangement, Fig. 10-31*c*). The design of the lattice affects the determination of the effective length of a chord, consisting of one angle.

For towers with the lattice joints of adjacent sides coinciding (Fig. 10-31*b*), the effective length of the chord when determining its slenderness ratio is taken equal to the length of the panel L_p, the radius of gyration of the angle being taken at its minimum value (with respect to an axis inclined to the legs of the angle).

For towers in which the joints of adjacent sides do not coincide (Fig. 10-31*c*), the effective length of the chord is taken equal to the length of the panel L_p multiplied by a coefficient k_p, i. e., $L_{p.e} = k_p L_p$.

The coefficient k_p is determined in accordance with the USSR Building Standards and Regulations, depending on the ratio between the linear stiffnesses of the chord and of the diagonal, and varies from $k_p = 1$ to 1.13. The diagonals are welded to the chords either directly or through gusset plates. All field connections may also be made by means of unfinished bolts, and one bolt may be used for each connection. The kind of connection is taken into account by using the respective service condition factors, as well as the proper length coefficients for the diagonals.

CHAPTER ELEVEN

Thin-shell structures

11-1. GENERAL

(1) Nomenclature of Thin-Shell Structures

Thin-shell structures is the name given to structures in which steel plates and sheets are the main part, forming steel shells. They are designed for storing or processing gases, liquids or bulk (loose) materials. The following kinds of shell structures are distinguished:

1. *Gas holders*—for the storage and distribution of gases.
2. *Tanks* and *reservoirs* for the storage of water, oil products and other liquids.
3. *Bunkers* for the storage of bulk materials (ore, coal, cement and so on).
4. *Special structures* for iron and steel, chemical and other branches of industry (blast furnaces, air heaters, autoclaves, various large chemical apparatus, etc.).
5. *Large-diameter tubes*, *pipes* and *pipelines* employed at iron and steel, chemical and other plants, hydroelectric power stations, as well as in the capacity of trunk oil and gas pipelines, etc.

(2) Features of Steel Shells

Steel shells are employed not only as members resisting various loads, but also as vessels, owing to the density of steel and its gas- and watertightness. For this reason the connections (splices) of sheets and plates in shells must be not only strong, but also tight.

Shell structures in the majority of cases are shells of revolution (cylindrical, spherical, conical shells, etc.), i.e., they have the most advantageous shape for withstanding the loads induced by gases and liquids.

The dimensions of shells are frequently greater than those tolerated by railway clearance gauges, and therefore the work at the shops is limited to the fabrication of semifinished articles (the preparation of sheets and plates, details, etc.), the remaining jobs being performed in the field. This increases the total labour required for the fabrication and erection of shell structures. Besides, the necessity of rolling the plates to form spherical and other surfaces having a curvature in two directions is the cause of difficulties in production which raise

the cost of fabricating the members. A feature characteristic of shell structures, which are almost exclusively of a welded design, is also the great total length of the welds, which is the result of the relatively small width of rolled sheets of the standard sizes (1,400 to 1,500 mm). This, in turn, leads to a considerable expenditure of labour for machining the edges of the plates for welding.

A method of fabricating and erecting large tanks and reservoirs has been recently developed that considerably increases industrialization and mechanization of construction work by transferring the greater part of the work to the shop—the so-called *roll method* (see Fig. 11-14). The essence of this method consists in the walls and bottom of a tank being welded at the shop into single large sheets by means of automatic welding. The sheets are then made up into rolls with a diameter of about 2.5-3 metres that can be shipped by railway. At the construction site these rolls are unrolled and erected in place. Thus the number of field welds is brought down to a minimum.

11-2. THEORY UNDERLYING ANALYSIS OF THIN SHELLS OF REVOLUTION

The surface of a shell of revolution has an axis of symmetry and two radii of curvature perpendicular to the surface, namely, R_1 = = a meridional radius forming the curve of revolution, and R_2 = a circular radius of revolution having its origin on the axis of symmetry (Fig. 11-1*a*). The angles φ (latitude) and α (longitudinal) characterize respectively the location of the radii.

A spherical surface is characterized by the relation $R_1 = R_2$; a cylinder—by the relations $R_1 = \infty$, $R_2 = r$ and $\varphi = \frac{\pi}{2}$; a cone—by the relations $R_1 = \infty$, $R_2 \sin \varphi = r$ and $\varphi = \text{const}$ (a constant angle). Here r is the radius of the middle of the shell surface, perpendicular to the axis of symmetry.

Let us consider an element of a shell cut out remote from the ends, with a thickness t and with sides dS_1 and dS_2 (Fig. 11-1*b*), the area of which is subjected to a uniformly distributed load p. It is known that in thin shells or membranes, which are characterized by a small ratio between the thickness of the shell and its radius $\left(\frac{t}{R} < \frac{1}{30}\right)$, the conditions of equilibrium can be observed when only axial forces are present, viz., meridional ones T_1 and annular ones (hoop forces) T_2, directed along a tangent to the middle surface of the shell. These forces are the resultants of the normal stresses applied to the sides of the element

$$T_1 = \sigma_1 dS_2 t \text{ and } T_2 = \sigma_2 dS_1 t$$

Let us take the sum of the projections of all the forces on the direction of the corresponding radius of curvature. In accordance with the condition of equilibrium, this sum should equal zero (Fig. 11-1*b*)

$$2T_1 \sin\frac{d\varphi}{2} + 2T_2 \sin\frac{d\alpha}{2} - pdA = 0$$

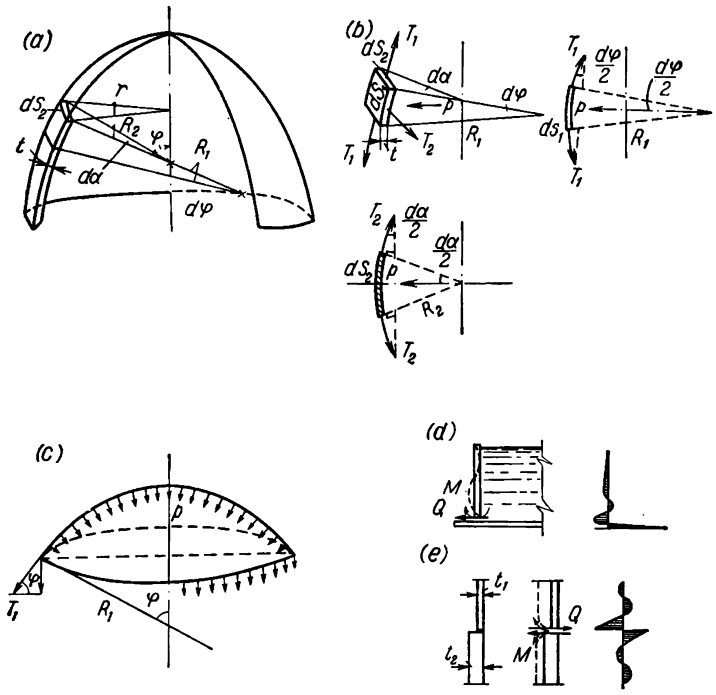


Fig. 11-1. To analysis of shells

Seeing that with small angles

$$\sin\frac{d\varphi}{2} = \frac{d\varphi}{2} \qquad \sin\frac{d\alpha}{2} = \frac{d\alpha}{2} \qquad d\varphi = \frac{dS}{R_1}$$

$$d\alpha = \frac{dS_2}{R_2} \quad \text{and} \quad dA = dS_1 dS_2$$

then, upon dividing both parts of the equation obtained by $dS_1 dS_2$, we get

$$\frac{T_1}{R_1 dS_2} + \frac{T_2}{R_2 dS_1} = p$$

Upon expressing T_1 and T_2 through the stress, we get the basic equation for thin flexible shells

$$\frac{\sigma_1}{R_1} + \frac{\sigma_2}{R_2} = \frac{p}{t} \tag{11-1}$$

where σ_1 = stress along the generatrix (meridian)
σ_2 = annular (hoop) stress.

For a cylindrical shell in which $R_1 = \infty$, we obtain the annular stresses

$$\sigma_2 = \frac{pR_2}{t} \tag{11-2}$$

For a spherical shell whose radius in all directions is the same ($R_1 = R_2 = R$), the stress conditions of each element are also the same in all directions and, therefore,

$$\sigma_1 = \sigma_2 = \frac{pR}{2t} \tag{11-3}$$

Thus, with the same radius, the stress in a spherical shell will be half that in a cylindrical one.

The general equation (11-1) contains two unknowns σ_1 and σ_2, and therefore for solving the problem it is essential to have a second equation. This equation can be obtained by considering a section of the shell along a parallel circle and equating to zero the sum of the projections of all the forces on the axis of symmetry (Fig. 11-1*c*)

$$\frac{T_1}{dS_2} \sin\varphi 2\pi r - p\pi r^2 = 0$$

Seeing that $R_2 \sin\varphi = r$ (Fig. 11-1*a*) and $T_1 = \sigma_1 dS_2 t$ then

$$\sigma_1 = \frac{pR_2}{2t} \tag{11-4}$$

Substitution of equation (11-4) in equation (11-1) yields the relation between the annular and the meridional stresses

$$\sigma_2 = \sigma_1 \left(2 - \frac{R_2}{R_1}\right) \tag{11-5}$$

The equations for thin shells obtained above, deduced from the conditions of equilibrium when only axial forces (meridional and annular ones) are present, assume that the shell is absolutely flexible, i.e., that its stiffness with respect to bending and twisting is equal to zero. The stresses in such a momentless shell are uniformly distributed over the section. There is also freedom for axial strains. Such assumptions for behaviour of a shell hold for parts at some distance from the bearing connections or places of shell bending, i.e., from places where the position of the centre of the radius of curvature

R_1 changes by a step, or the thickness of the shell changes; in other words, at some distance from all those places where the conditions for axial strains change. At these places there appear thrust forces and "end" bending moments which cause bending of the shell owing to restriction of the strains in the conditions of continuity of the section (Fig. 11-1*d* and *e*). The bending moments spread over a comparatively narrow zone of the shell, and are rapidly damped (the "end effect") owing to the strains of the shell having to overcome the elastic resistance of the adjacent parts (similarly to a beam on an elastic foundation). The determination of these moments and shear forces from the condition of continuity of the section of connected shells is a problem with two statically indeterminate quantities.

The sharper the violation of the smooth surface of the shell, the greater will be the additional bending moments and the shear forces. For this reason sharp bends at places of shell connection should be avoided in designing. When such connections are essential from considerations of design, they should be checked and strengthened if necessary. Generally such strengthening consists in thickening the wall at the place of bending or in installing a thrust ring.

Should a cylindrical or spherical shell be subjected to compression on all sides (for example, when there is a vacuum) the stability of the shell must be checked. This check consists in ascertaining that the design stresses in the shell (with account of the load factor) do not exceed the critical stresses multiplied by the service condition factor

$$\sigma \leqslant \sigma_{cr} k_s \tag{11-6}$$

With the action of an external uniform pressure p normal to the surface, the critical stress in a spherical shell is determined from the equation (for shells of steel 3 with $\frac{r}{t} = 200$ to 500)

$$\sigma_{cr} = 0.1 \frac{Et}{r} \tag{11-7}$$

The critical normal stresses in a cylindrical shell with uniform (along the circular surface) compression parallel to the generatrices can be found from the equation (with $\frac{r}{t} > \frac{525}{R}$)

$$\sigma_{cr} = \frac{C_{sh} Et}{r} \tag{11-8}$$

in which E = modulus of elasticity
t = thickness of shell
r = radius of middle surface of shell
C_{sh} = coefficient taken from Table 11-1
R = design strength of steel, t/cm^2.

TABLE 11-1 Values of Coefficient C_{sh}

r/t	$\leqslant 500$	750	1,000	1,500
C_{sh}	0.12	0.10	0.08	0.07

When $\frac{r}{t} \leqslant \frac{525}{R}$, the critical stress is determined from the equation

$$\sigma_{cr} = \varphi_{sh} R \tag{11-9}$$

where φ_{sh} is taken from Table 11-2 (for steel 3).

TABLE 11-2 Values of Factor φ_{sh}

r/t	50	100	150	200	250
φ_{sh}	0.85	0.71	0.6	0.53	0.47

With eccentrical compression of a cylindrical shell in a direction parallel to the generatrices, or pure bending of the shell, the critical normal stresses computed from equations (11-8) and (11-9) are increased $(1+0.1\alpha)$ times.

Here $\alpha = 1 - \frac{\sigma'}{\sigma}$, in which σ is the maximum design compressive stress (it is assumed that $\sigma > 0$) and σ' is the corresponding stress at the opposite end of the generatrix diameter.

For a cylindrical shell (with a vacuum inside) subjected to an external uniform pressure p, the critical stresses are determined when $0.5 \leqslant \frac{L}{r} \leqslant 10$ by means of the equation

$$\sigma_{2_{cr}} = 0.55E\left(\frac{r}{L}\right)\left(\frac{t}{r}\right)^{3/2} \tag{11-10a}$$

when $\frac{L}{r} \geqslant 20$ from the equation

$$\sigma_{2_{cr}} = 0.17E\left(\frac{t}{r}\right)^2 \tag{11-10b}$$

Here L is the length of the cylindrical shell between the bearing or stiffening rings.

11-3. GAS HOLDERS

Gas holders are designed for the storage of gas and to control its pressure during consumption.

With respect to internal pressure and design, gas holders are divided into two groups:

1. *Constant-volume* gas holders functioning at high gas pressures, from 2 to 20 atm gauge (1 atm gauge corresponds to a pressure of 1 kg/cm^2).

2. *Variable-volume* gas holders of the wet and dry type, functioning at a low pressure not exceeding 500 mm w. g. (0.05 atm gauge).

(1) Constant-Volume (High Pressure) Gas Holders

High-capacity gas holder plants ordinarily consist of several tens or hundreds of identical constant-volume gas holders. The volume of a gas holder may vary within wide limits (from 100 to 5,000 cubic metres). Constant volume gas holders may be *spherical* and *cylindrical*. The consumption of metal for the construction of spherical gas holders is about 20% less than that required for their cylindrical counterparts. However, spherical gas holders are used to a smaller extent owing to the difficulty of manufacturing plates with a double curvature, and also because of their large size, which exceeds railway clearance gauges.

Cylindrical gas holders with respect to their dimensions may be normal-size (within the limits of railway clearance gauges) or over-size (exceeding these limits). It is known that the cost of operation of a gas plant lowers with an increase in the volume of the gas holders and a reduction in their number. This is why it is good practice to have over-size cylindrical gas holders with a large diameter (Fig. 11-2), arranging them on two supports. In this case, however, the greater part of the work involved in fabricating the gas holder must be performed in the field, which raises the initial cost of the installation.

Normal-size gas holders, which can be shipped by railway in a completely fabricated state (Fig. 11-3), are made with a diameter of 3.25 metres (corresponding to over-size railway consignments of the first degree, which extend a small amount beyond the standard size established in accordance with the normal railway clearance gauge for 1,524-mm railways). This makes it possible to fabricate them entirely in shop conditions with the use of automatic welding.

Normal-size gas holders can be arranged at the gas holder plant vertically (Fig. 11-4) or horizontally.

A cylindrical gas holder consists of a cylindrical casing and two hemispheres. The latter, as a rule, are butt-welded to the cylindrical part, which can be done owing to the presence of a common tan-

Fig. 11-2. Horizontal over-size gas holders

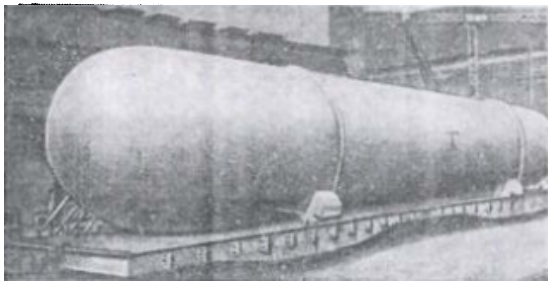


Fig. 11-3. Normal-size gas holder on flat car

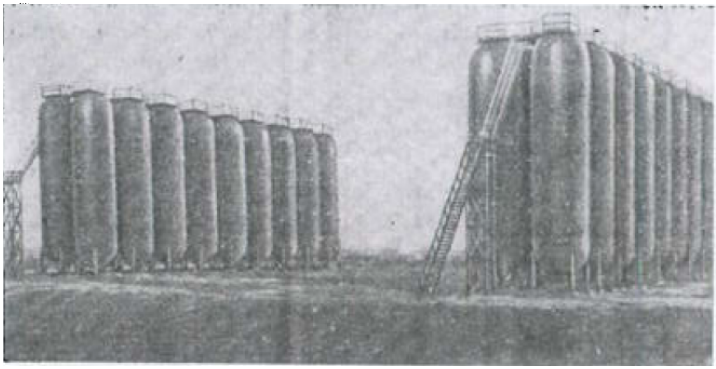


Fig. 11-4. Battery of vertical gas holders

gent at the place of connection (Fig. 11-5*a*). Because of the different thickness of the spherical and cylindrical shells, however, additional stresses develop at the joint as a result of the end effect. For this reason the welding must be of a high quality with a sealing bead used without fail on the opposite side. When using a lap joint to connect the hemisphere with the cylinder, the plates are made to overlap over a length of about 5-6t (Fig. 11-5*b*).

The plates or sheets of the cylindrical part of a gas holder are either butt-welded (Fig. 11-5*c*) or lap-welded (Fig. 11-5*d*). A butt weld is better and more economical.

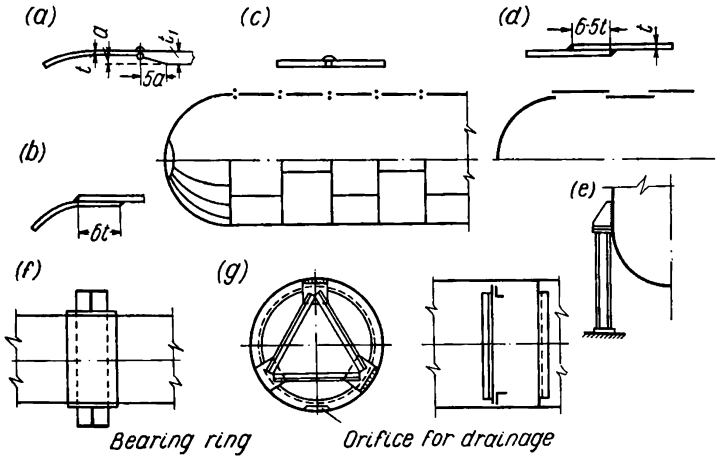


Fig. 11-5. Details of design of cylindrical constant-volume gas holders

Vertical gas holders are installed on supports, as pictured in Fig. 11-5*e*. An external bearing ring (Fig. 11-5*f*) or an internal stiffening triangle (Fig. 11-5*g*) is installed on the supports of horizontal gas holders.

To increase the stiffness of the shell in an annular direction (in view of the action of annular forces), stiffening ribs are sometimes installed. In this case the stability of a ring is checked whose section A consists of a stiffening angle and part of the wall of the casing equal to the spacing of the rings (Fig. 11-5*g*). The critical stress for such a ring is determined from the equation

$$\sigma_{cr} = \frac{3EI_r}{Ar^2} \qquad (11\text{-}11)$$

where I_r is the total moment of inertia of part of the shell and the stiffening angle (rib).

When stiffening ribs are used, they are generally installed one per casing unit (a ring with a width equal to that of one plate, i.e., about 1,500 mm).

(2) Variable-Volume (Low Pressure) Gas Holders

With respect to their design, variable-volume gas holders may be divided into two groups, namely, *wet* and *dry* ones. Wet gas holders are mainly of a telescopic design. The upper part of such a gas holder—its bell, can rise under the influence of the gas pressure, carrying

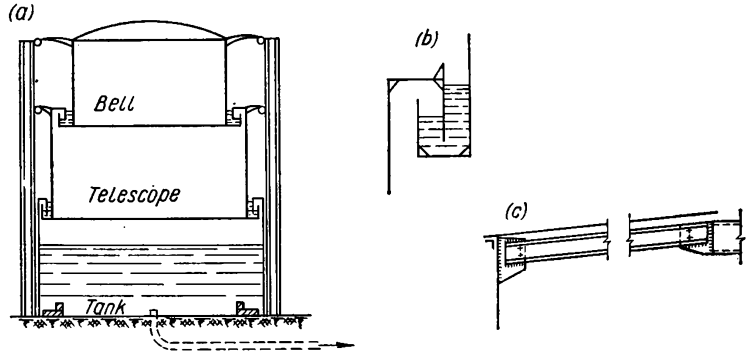


Fig. 11-6. Schematic view of wet gas holder

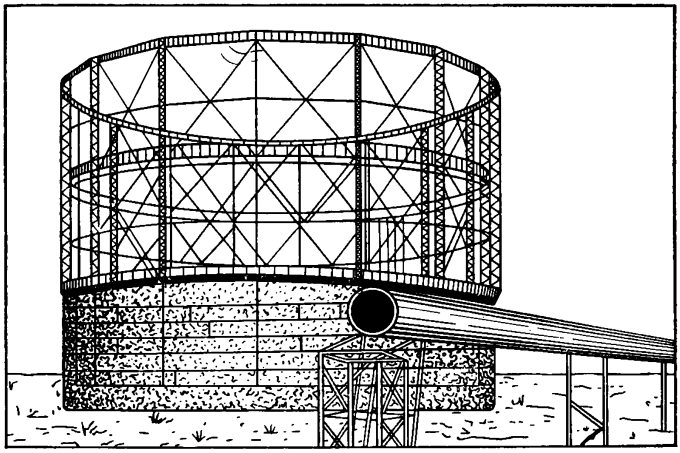


Fig. 11-7. Wet gas holder with vertical guides

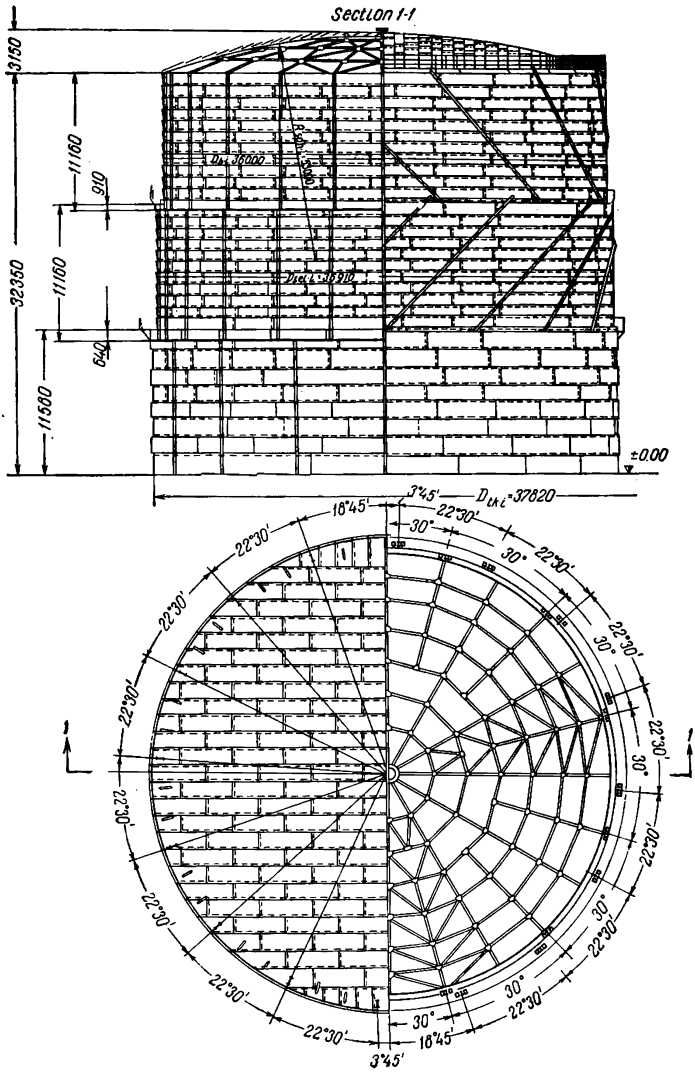


Fig. 11-8. Wet gas holder with helical guides

along the intermediate annular units—the telescope (Fig. 11-6*a*). Gas holders are sometimes designed with several telescopes.

The lower part of a wet gas holder—its tank, is filled with water. The gas is fed in from below and fills the whole holder. The bell and the telescope as they rise draw water with their trough-shaped pockets located along the edges, the water in the pocket serving as a hydraulic seal that does not let the gas escape to the atmosphere (Fig. 11-6*b*).

In order to ensure smooth movement of the bell and the telescope, and to avoid misalignment, the gas holder is encircled with vertical guides comprising I-section posts. The guides are connected by means of cross braces into a rigid space member (Fig. 11-7). Rollers mounted on arms fixed to the rising parts of the gas holder move along the posts.

Upon lowering to the bottom, the moving parts rest on special seats. The tank is computed for the pressure of the water, therefore the thickness of its walls is somewhat greater than that of the telescope walls. The minimum wall thickness is taken equal to 4 mm, while that of the bell roof is 2.5-3 mm. The bell roof is made from sheets placed on inclined rafters consisting of channels (Fig. 11-6*c*). The rafters are connected at the centre to a channel ring. The rafters are also connected to each other by means of annular angles and a lattice arranged in one plane (to facilitate the laying out of the sheets). Wet gas holders are designed with a capacity varying from 100 to 30,000 cubic metres.

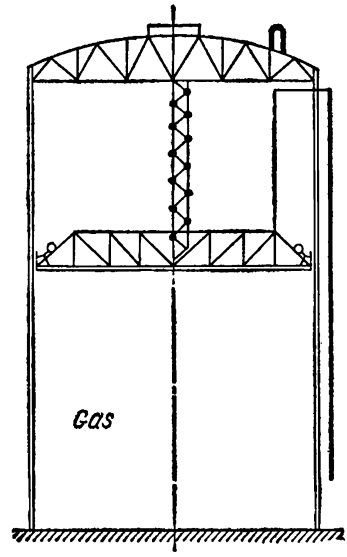


Fig. 11-9. Schematic view of dry gas holder

In recent years there has been developed a design of gas holders with helical guides (Fig. 11-8). In these holders a rail arranged along a helical line at an angle of 45 deg is secured to the outer surface of the movable telescopes and the bell. Double-flange rollers are secured on the upper ring of the tank, and also on the upper end of each telescope. Owing to the absence of a special space framework, the weight of helical gas holders is reduced by 10-15% in comparison with that of a gas holder with vertical guides having the same capacity.

A dry gas holder comprises an immovable casing with a bottom and a roof inside which a piston moves (Fig. 11-9). Dry gas holders are

designed with a capacity ranging from 10,000 to 100,000 cubic metres. When the gas holder is filled from below, the gas after reaching a certain pressure overcomes the weight of the piston and lifts it. When part of the gas has been used up, the piston lowers and forces out the gas with its weight.

To increase the reliability of operation of a dry gas holder, the latest tendency is to replace the movable piston with a section of rubberized fabric.

11-4. TANKS FOR LIQUIDS

(1) Surface Tanks

Surface vertical cylindrical tanks with a flat bottom placed on a filled-in sand bed are widely used for the storage of oil products.

During the storage of oil products, their evaporation takes place (as a result gases accumulate under the roof). The extent of this

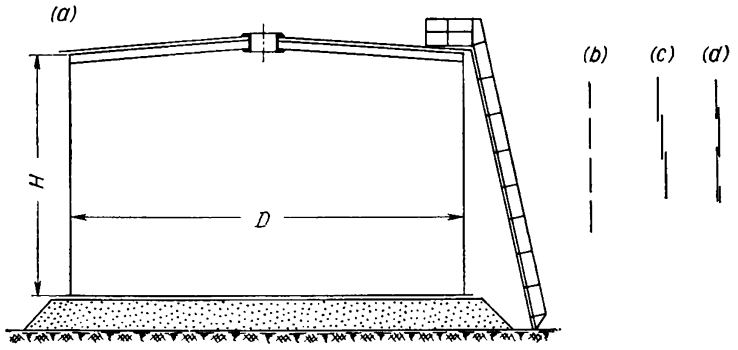


Fig. 11-10. Surface low-pressure tank

evaporation varies both with a change in temperature ("low breathing") and upon discharging and charging ("high breathing") and leads to great losses. To reduce the losses of the oil products, tanks of various types are employed.

For the storage of light oil products with a low vapour pressure (kerosene, diesel fuel, etc.), as well as of dark refined oil products and oils, use is made of low-pressure tanks with an internal pressure up to 200 mm w. g. (0.02 kg/cm^2) that allow a vacuum of 25 mm w. g. (Fig. 11-10).

The storage of oil products with a high vapour pressure (various grades of petrol, oil, etc.) requires the employment of cylindrical

tanks with an increased pressure (0.2-0.3 kg/cm^2), as well as tanks with pontoons or floating roofs.

Use can also be made of low-pressure tanks, but with the connection of several tanks into one gas-equalization system with a gas collector.

Low-Pressure Tanks. At present it is recommended to design low-pressure tanks with a panel roof, and having a nominal capacity varying from 100 to 5,000 cubic metres. Tanks have also been designed with a capacity of 10, 15 and 20 thousand cubic metres.

The main constructive elements of low-pressure tanks are the bottom, wall (casing) and roof. The bottom, which rests on a sand bed, is subjected only to compression induced by the pressure of the liquid. From considerations of design its thickness is taken equal to $t = 4$ mm with a tank diameter D less than 18 metres, $t = 5$ mm for a diameter of 18-25 metres and $t = 6$ mm with a diameter D exceeding 25 metres. The border sheets of the bottom are designed with a thickness of 8 mm. The thickness of a tank wall is taken in accordance with the results of analysis equal to 4-5 mm and above. The arrangement of the plates or sheets along the height of the tank may be end to end (Fig. 11-10*b*), telescopic (Fig. 11-10*c*) and stepped (Fig. 11-10*d*). End-to-end connection (butt joints) should be considered the most rational kind of connection for plates with a thickness of 6 mm and above. Such an arrangement of the plates saves metal and also makes it possible to employ a modern industrial method of tank manufacture—the roll method.

At present the roof of a tank is made in the form of panels consisting of thin sheets (t=2.5 to 3 mm) laid on a framework (Fig. 11—11). The panels are supported on the central post and the casing of the tank. The employment of shop-made panels greatly simplifies the erection of the roof in comparison with the previously used design in the form of separate sheets laid on rafters and purlins.

Two combinations of loads are used in investigating the members of the roof:

1. Loads directed toward the inside of the tank—snow, the own weight of the members and the vacuum that may be formed as the result of rapid cooling of the vapours. This vacuum is taken equal to 25 mm w.g., i. e., equal to 25 kg/m^2.

2. A load directed outwards—the vapour pressure of the evaporating liquid (gauge pressure) taken equal to 200 mm w.g. (or 200 kg/m^2).

It should be borne in mind that an excessive pressure may be created in the tank when it is being charged with the oil products, or as a result of evaporation with a low amount of liquid. For this reason the large pressure on the roof, directed upward, may remain unbalanced by the own weight not only of the roof, but also of the casing.

To counterbalance these negative forces, the central column, if it is made of a tube, is filled with concrete (or sand), while if latticed columns are used their base is concreted. The wall of the tank is anchored to concrete slabs laid in the ground around the tank.

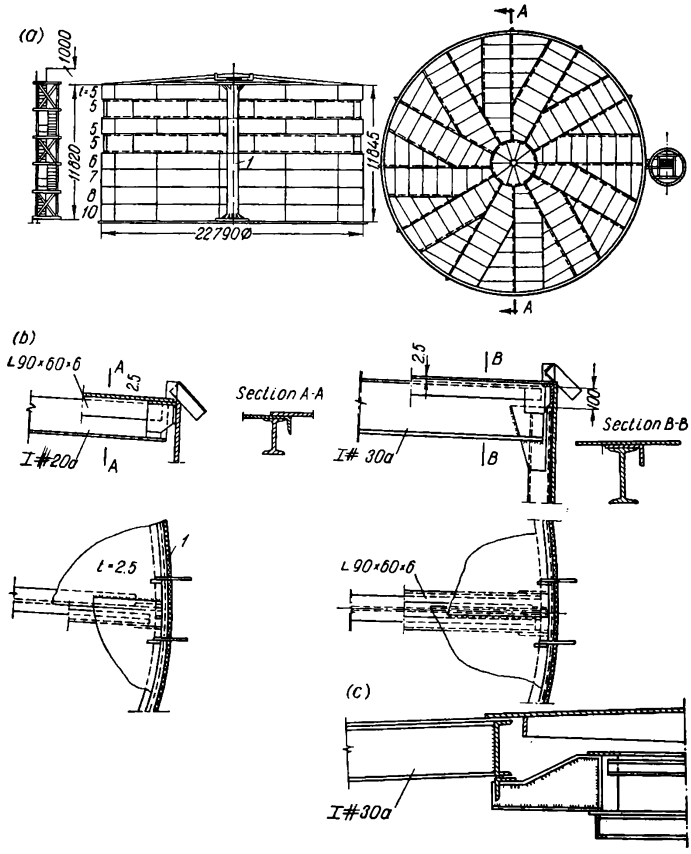


Fig. 11-11. Low-pressure 5,000 cubic-metre tank with panel roof: *a*—vertical section and plan view of panel roof; *b* and *c*—details of panel construction

Academician V. Shukhov devoted a lot of time to determining the most advantageous relations between the general dimensions of a tank—its height H and diameter D—for a given capacity. He established two rules:

1. A tank with a variable wall thickness has the lowest possible weight on condition that the amount of steel in the bottom and the roof is equal to the amount of steel in the wall.

2. A tank with a constant wall thickness has the lowest possible weight on condition that the amount of steel in the bottom and the roof is one half of the amount of steel in the wall.

These rules make it possible to design tanks of an optimal shape, which are obtained with d/H ratios of 1/1 to 1/4 for capacities from 100 to 600 cubic metres, and of 2/1 to 5/1 for capacities up to 10,000 cubic metres.

Use is made in the USSR of standards and standard designs of tanks for the storage of oil products. These standards stipulate that tanks with a capacity of from 3 to 75 cubic metres are to be designed of the horizontal type (surface and underground), and with a capacity from 100 to 10,000 cubic metres—of the vertical type (surface). All the tanks are welded.

The height of a tank is selected equal to a multiple of the width of the plates or sheets (1,400 or 1,500 mm). With a deviation from the optimal height of up to 20%, the weight grows by only 2%; this is why it will be rational to use a height that is a multiple of the plate width. The greatest optimal height of large tanks (up to 10,000 cubic metres), from the viewpoint of metal consumption, is about 12 metres (8 annular sections) regardless of any further increase in their diameter.

Figure 11-11*a* illustrates the design of a standard tank with a capacity of 5,000 cubic metres.

The consumption of steel for low-pressure tanks with a panel roof is characterized by the figures contained in Table 11-3.

TABLE 11-3 **Consumption of Steel for Tanks**

Capacity, m³	200	400	700	1,000	2,000	3,000 and 5,000
Consumption of steel in kg per cubic metre of capacity	38	33	25	24	21	20

The wall of a tank is investigated as a cylindrical shell subjected to tension by the action of the hydrostatic pressure p. At a depth x from the surface of the liquid (Fig. 11-12*a*), the design pressure on the wall will be

$$p_x = \gamma x n_1 + p_2 n_2$$

where γ = specific weight of liquid
$n_1 = 1.1$ = load factor for hydrostatic pressure of liquid
$n_2 = 1.2$ = load factor for excessive pressure of gases p_2.
The stress in the wall will be

$$\sigma_2 = \frac{p_x r}{t} = \frac{(\gamma x n_1 + p_2 n_2) r}{t} \tag{11-12}$$

The minimum thickness of the wall is determined when investigating its behaviour in a vertical (design) joint from the equation

$$t = \frac{(\gamma x n_1 + p_2 n_2) r}{k_s R C_{st}} \tag{11-13}$$

in which C_{st} = weld strength coefficient equal to the ratio between the design strength of a weld and that of the base metal
k_s = service condition factor taken equal to 0.72-0.8 for the casing of tanks.

The minimum thickness of the wall is taken equal to $t = 5$ mm. Either butt or lap joints (Fig. 11-12*b*) are used for connecting the separate wall plates or sheets both horizontally and vertically.

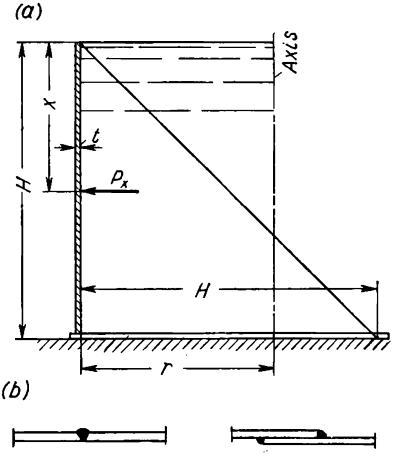


Fig. 11-12. To analysis of tank wall

The flat bottom of an aboveground tank is made from plates that are welded into strips, the latter being connected together either by means of lap (Fig. 11-13*a*) or butt joints.

A tee or corner joint (Fig. 11-13*b*) is employed for connecting the wall to the bottom, which necessitates the provision of a smooth surface at the edge of the bottom along the whole perimeter. For this purpose parts of the lap welds between the border plates are cut away over a distance of 250-300 mm and the edge of the upper border plate is set with a hammer until it coincides with the edge of the lower one, while a strap is used on the under side (Fig. 11-13*a*, joint *A*); here a gap of 4 mm should be provided to take care of shrinkage of the weld. These spots must be checked with special attention after welding.

When fabricating tanks by the roll method, the plates 1.5 metres wide are placed in a row and are butt-welded along their longitudinal

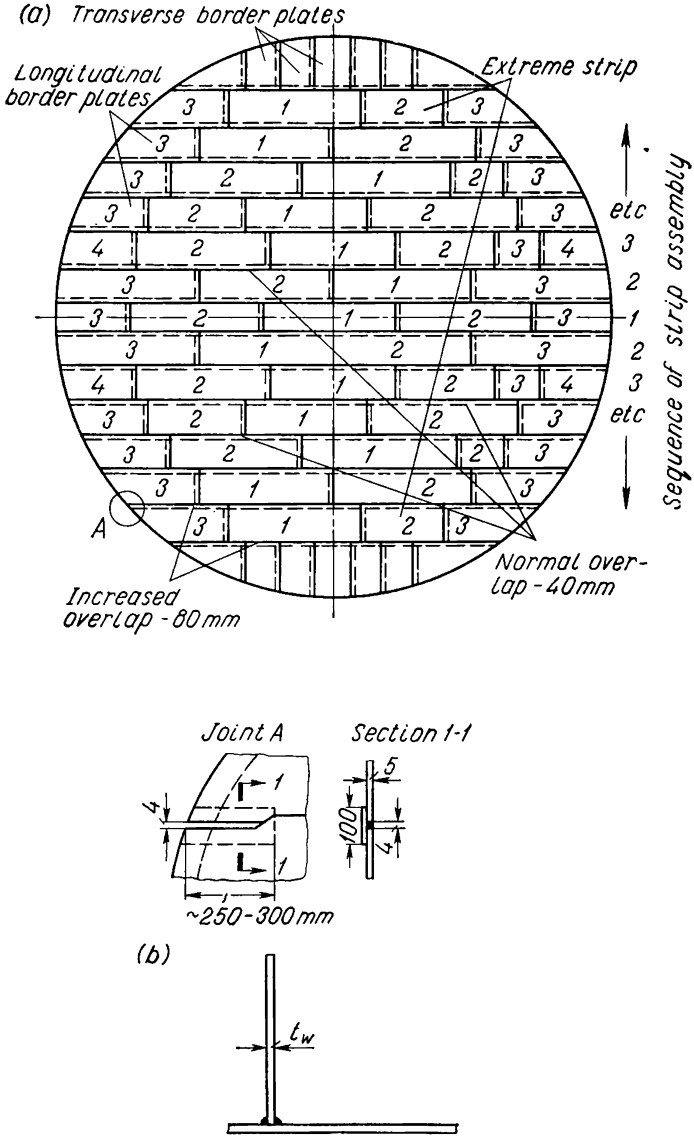


Fig. 11-13. Bottom of tank

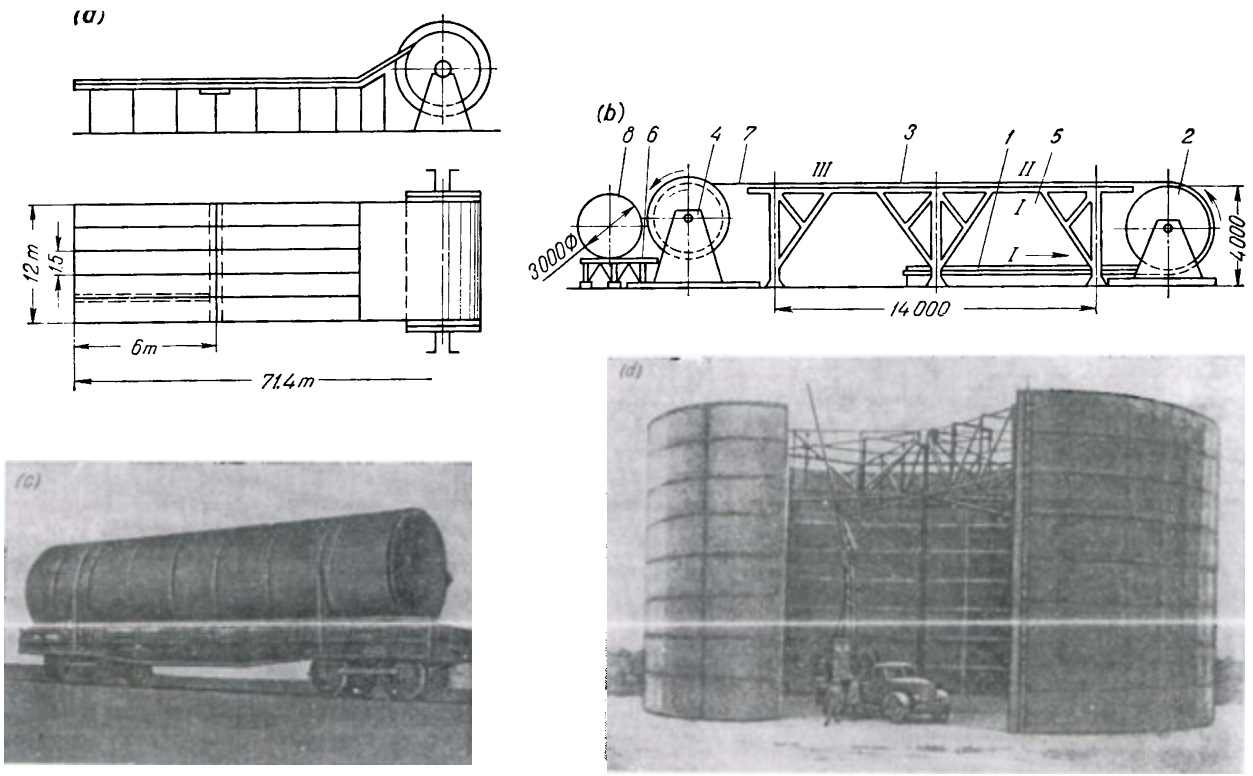


Fig. 11-14. Fabrication and erection of tanks using roll method:

a—single-tier installation with magnetic stand; *b*-double-tier stand; *1*—racks of first tier; *2*—idle drum; *3*—racks of second tier; *4*—roll-forming device; *5*—monorail; *6*—rack for finished roll; *7*—sheet of casing; *8*—finished roll; *I*, *II* and *III*—workplaces for assembly, welding and testing of sheets; *c*—roll on flat car; *d*—erection of tank casing

edge into sheet bands (Fig. 11-14*a*). Next such sections are welded across their entire width, using a lateral weld, and are rolled up into a roll as the whole sheet is built up. This is a sort of substitution for rolling of the plates into shape. The roll method is used to fabricate the walls and bottoms of tanks. At present tanks are fabricated on double-tier stands with welding on both sides, which makes possible the employment of lap joints for thin plates ($t \leqslant 6$ mm)

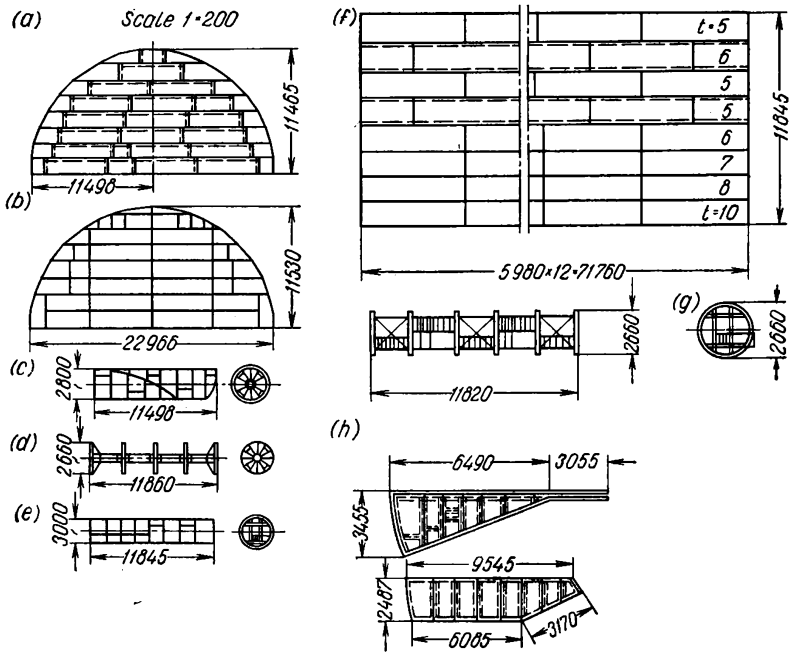


Fig. 11-15. Shipping elements of tank with a capacity of 5,000 cubic metres:
a—bottom with straight border plates; *b*—alternative of bottom for welding on a magnetic stand; *c*—bottom roll; *d*—central post; *e*—casing roll; *f*—casing of tank; *g*—shaft ladder; *h*—roofing panels

and the use of underhand welding on both sides (Fig. 11-14*b*). Figure 11-15 pictures shop-fabricated shipping elements of a standard tank with a capacity of 5,000 cubic metres.

With plate-by-plate assembly (the old method) all the vertical joints of the casing plates are staggered.

The roll method can be employed with a thickness of the metal from 4 to 12 mm (with a thickness of 14 mm some difficulties are encountered in unrolling the rolls). A sheet up to 12 metres wide

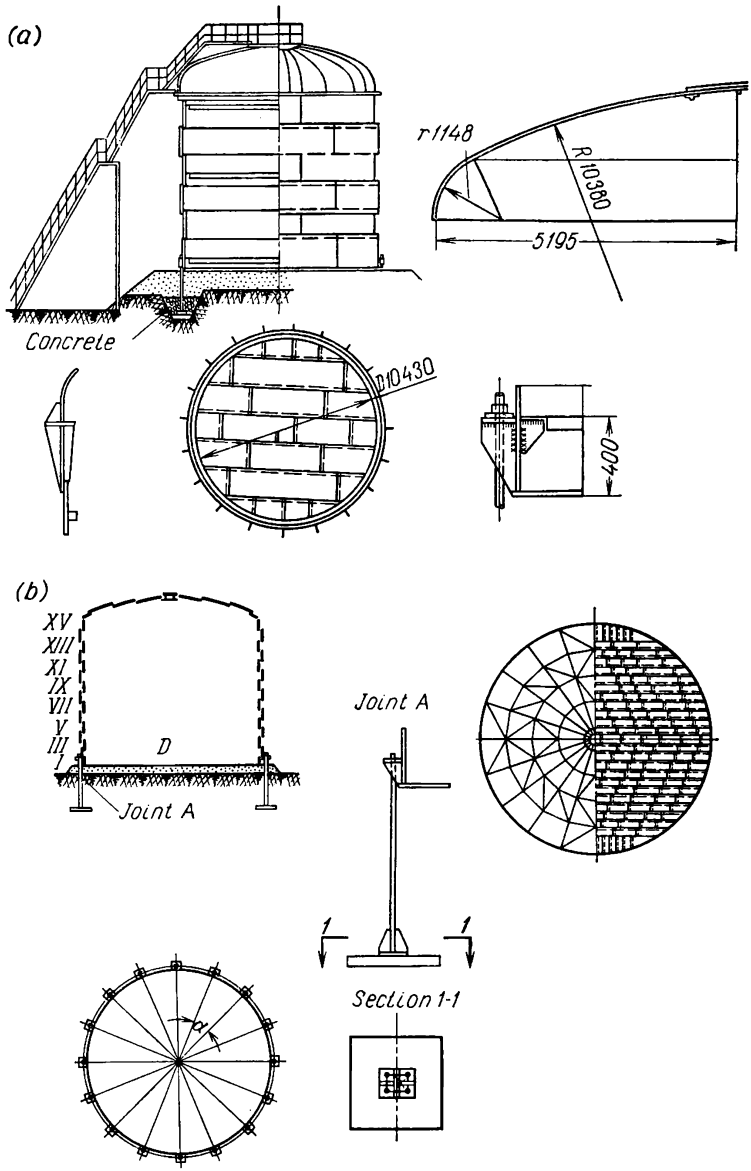


Fig. 11-16. Vertical cylindrical increased-pressure tanks:
a—with spherocylindrical roof; *b*—with spherical roof

(the width of the stand) is rolled up into a roll having a diameter of 2.3-3.2 metres (Fig. 11-14*c*).

At the erection site, the casing roll is placed in a vertical position and unrolled (Fig. 11-14*d*). The closing joint of the casing should be a lap one. After unrolling two half-bottom rolls they are connected

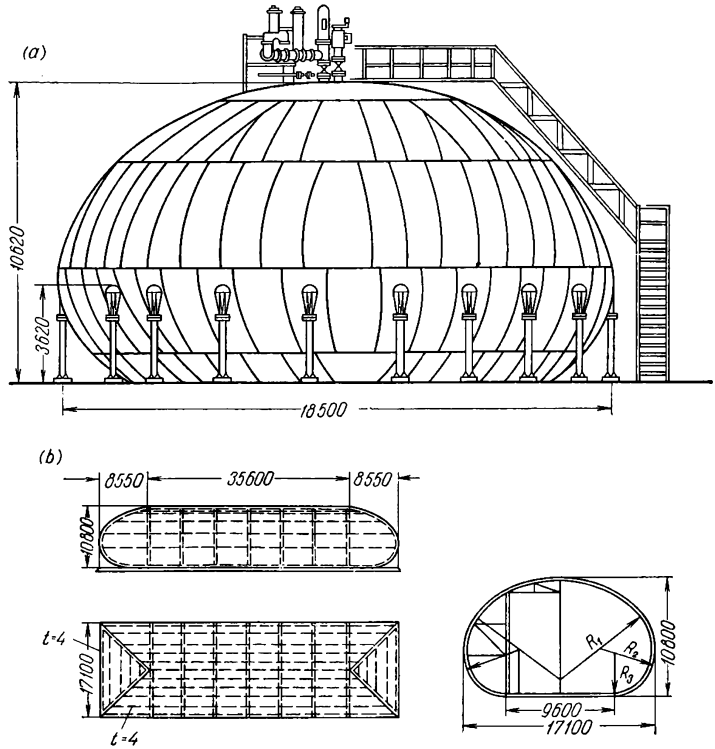


Fig. 11-17. Bulb-shaped tanks:
a—with equatorial supporting; *b*—cylindrical

by lap welding with the welds made only on the outer side, without a sealing bead underneath. This weld is subjected to special control. To protect the bottom against corrosion, the sand bed is coated with a layer of bitumen.

For low-pressure tanks, use can be made of a suspended roof consisting of plates 2.5-3 mm thick. The roof is suspended from columns whose ends stand out above the casing by 1.5-2 metres.

Increased-Pressure Tanks. As has already been indicated, tanks with an increased gauge pressure are employed for the storage of volatile oil products. Figure 11-16*a* shows the design of a vertical tank with a spherocylindrical roof. The use of separate cylindrical elements 4-6 mm thick having one radius of curvature simplifies rolling into shape, and at the same time makes it possible to obtain a roof in the form of a surface of revolution. The roof is connected to the casing through a horizontal plate. These tanks are designed for an internal gauge (excessive) pressure of 1,600-2,000 mm w.g. and a vacuum of 50 mm w.g. Illustrated in Fig. 11-16*b* is the design of a vertical tank with a spherical roof, whose flat roofing is laid in separate pieces onto the roofing framework, which forms a spherical dome made up of bars. To facilitate erection and exclude the use of overhead welds, the roof is connected to the casing through partially spread angles. These tanks are designed for an internal pressure of 1,000-1,500-2,000 mm w.g. and a vacuum of 1,000 mm w.g. Stability of the walls with a vacuum is ensured in both systems of tanks by using stiffening rings spaced 2 metres apart.

It is good policy to design large tanks with a gauge pressure of about 0.4 kg/cm^2 in the form of bulb-shaped tanks, which are uniform strength shells (Fig. 11-17).

(2) Tower Tanks

Tanks located on towers are chiefly designed for water supply purposes and have a capacity ranging from 100 to 3,000 cubic metres. A distinguishing feature in these tanks is the design of their bottom, which, as a rule, is a suspended one in the form of one shell of revolution, or of a combination of such shells. It will not be good practice to design a flat bottom in tower tanks, seeing that such a bottom will require a heavy beam grillage to withstand bending.

11-5. BUNKERS

There are distinguished flat-walled pyramidal, flexible and round bunkers.

Figure 11-18*a* shows diagrammatically a double pyramidal bunker. The structure comprises a framework formed by vertical beams and corner ribs which carry stiffeners made of angles, and a wall. Bunker walls are made from plates at least 8 mm thick. The wall resists bending in the span between the stiffeners induced by the load of the loose material contained in the bunker. The design vertical load acting on the plate of the wall at a depth of x from the surface of the loose material will be

$$q_x = \gamma x n \qquad (11\text{-}14)$$

where γ = volume or unit weight of material as filled into the bunker
$n \geqslant 1.2$ = load factor.

The horizontal load at the same depth will be

$$q_y = \gamma x n \tan^2\left(45° - \frac{\varphi}{2}\right) \qquad (11\text{-}15)$$

in which φ = angle of natural slope of a loose material, taken from reference books.

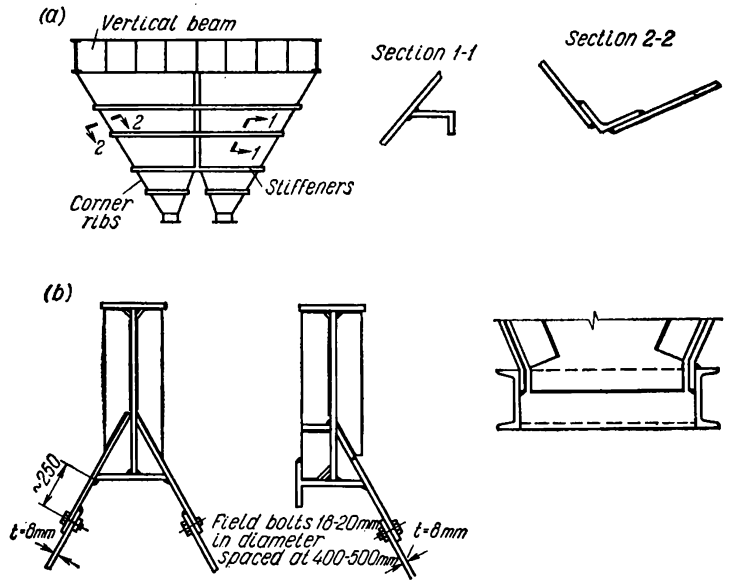


Fig. 11-18. Double pyramidal bunker and its details

The design of the details of the wall-to-beam connections and of the neck is depicted in Fig. 11-18*b*. A gate is connected to the channel (or angle) of the neck.

Flexible bunkers are intended for the storage of large quantities of loose materials. A flexible bunker is a suspended parabolic shell subjected mainly to tension. Figure 11-19 pictures a general view of a flexible bunker with a capacity of 10,000 tons for the storage of ore, having a height and a width of about 17 metres and a length of approximately 60 metres.

The equation of the bunker configuration curve is

$$y = \frac{\delta}{2L^2}\left(3x^2 - \frac{x^3}{L}\right) \qquad (11\text{-}16)$$

The cross-sectional area of the bunker (Fig. 11-20) will be

$$A = \frac{5}{4} L\delta \qquad (11\text{-}17)$$

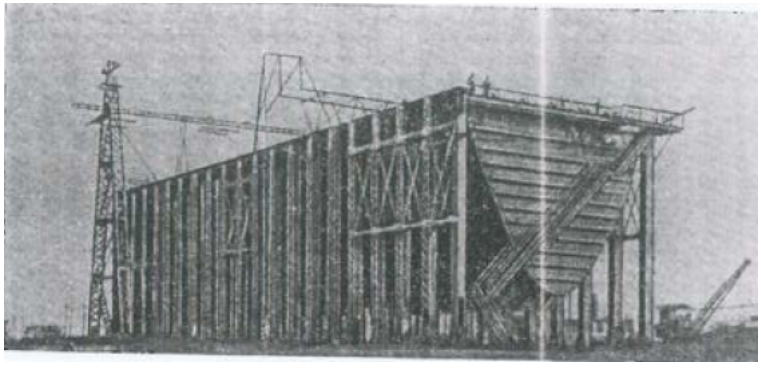


Fig. 11-19. General view of flexible bunker

The maximum value of the load ordinate along the axis of symmetry is

$$P_{max} = \frac{5}{4} \delta\gamma n \qquad (11\text{-}18)$$

where n is a load factor.

The components of the reaction at the points of suspension are

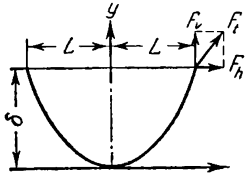


Fig. 11-20. To analysis of flexible bunker

$$F_h = \frac{P_{max} L^2}{3\delta} \quad \text{and} \quad F_v = \frac{P_{max} L}{2} \qquad (11\text{-}19)$$

The total tension force per unit of bunker length at the place where it is secured to the beams will be

$$F_t = \sqrt{F_h^2 + F_v^2} = A\gamma n \sqrt{\frac{L^2}{9\delta^2} + \frac{1}{4}} \qquad (11\text{-}20)$$

This force is used to compute the thickness of the bunker wall. All the plates of the bunker are butt-welded. When a bunker has a large length it is necessary to install intermediate transverse rigid membranes between which the horizontal components of the reaction F_h are resisted by a horizontal beam or truss.

APPENDIX I

TABLE 1 **Service and Design Loads and Load Factors**

Item No.	Kind of load	Service load, kg/m	Load factor n	Design load, kg/m²
1	Own weight of members	According to project	1.1 (0.9)	—
2	Heat and sound insulation articles (panels of light and porous materials, fillings, roof binding courses, plaster, etc.)	Ditto	1.2 (0.9)	—
	Floor Loads			
3	Halls of railway stations, theatres and cinemas with corridors and staircases, grandstands, trade and exhibition halls, museums, not under	400	1.3	520
4	Libraries, archives, stages, not under	500	1.2	600
5	Attic premises — additional to weight of equipment	75	1.4	105
6	Production premises:			
	(a) equipment loads (at least 400 kg/m² for investigating slabs and 300 kg/m² for investigating purlins and collar beams)	According to project	1.1-1.3	—
	(b) load induced by weight of employees, details, repair materials (at least 200 kg/m²) . .	Ditto	1.2-1.4	—

Table 1, concluded

Item No.	Kind of load	Service load, kg/m²	Load factor n	Design load, kg/m²
	Miscellaneous Loads			
7	Own weight of equipment	According to project	1.2	—
8	Weight of liquid	Ditto	1.1	—
9	Weight of loose materials, sludge and other bodies filling equipment (bunkers, tanks)	Ditto	1.2	—
10	Load induced by fork trucks, electric cars, etc	Ditto	1.3	—
11	Loads induced by overhead travelling cranes with a lifting capacity under five tons	Ditto	1.3	—
12	Ditto, other overhead cranes	Ditto	1.2	—

When investigating main beams and collar beams (with a distance between them of at least five metres) the standards allow reducing of the live load by the introduction of a factor of 0.9.

It is also permitted to reduce the live load on columns in multistorey buildings by applying factors varying from 0.9 to 0.5 depending upon the number of floors (from 1 to 9 and above).

TABLE 2 **Service Condition Factors k_s for Elements of Steel Members**

Item No.	Elements of members	k_s
1	Solid beams and compression elements of floor trusses under halls of theatres, clubs, cinemas, under grandstands, under the premises of shops, stores, libraries, archives, etc., with a weight of the flooring equal to or greater than the live load	0.9
2	Main compression elements (except for support ones) of floor and roof truss lattices with a slenderness ratio of $\lambda \geqslant 60$	0.8
3	Compression diagonals of space latticed members made from single angles welded along one leg:	
	(a) with herringbone and double-intersection lattice with joints not coinciding on adjacent sides .	0.8
	(b) with double-intersection lattice with joints coinciding on adjacent sides	0.9

Table 2, concluded

Item No.	Elements of members	k_s
	Ditto, with bolted connections	0.75
4	Crane girders under cranes with a lifting capacity of 5 tons and more with heavy, very heavy and very heavy continuous service duties	0.9
5	Columns of civil buildings and supports of water towers	0.9
6	Single compression angles connected on one side (by one leg) except those indicated in item 3	0.75

NOTES. 1. The service condition factors established for items 2 and 3 do not relate to the fastenings of the corresponding elements of members at joints.

2. The service condition factors for angles connected on one side are established for equal leg angles, as well as for unequal leg ones connected along their short leg.

TABLE 3 **Maximum Deflections of Bent Elements**

Item No.	Elements of members	Deflection in parts of span $\frac{1}{r_0} = \frac{\delta}{L}$
1	Crane girders and trusses:	
	(a) with manually operated cranes	1/500
	(b) with electric cranes having a lifting capacity up to 50 t	1/600
	(c) Ditto, 50 t and above	1/750
2	Monorail tracks	1/400
3	Beams of working areas of industrial buildings:	
	(a) in the absence of railway tracks:	
	main beams and girders	1/400
	other beams and girders	1/250
	(b) having broad gauge railway tracks	1/600
4	Floor beams between storeys:	
	(a) main beams	1/400
	(b) other beams	1/250

Table 3, concluded

Item No	Elements of members	Deflection in parts of span $\frac{1}{r_0} = \frac{\delta}{L}$
5	Roof and roofing beams:	
	(a) main beams	1/250
	(b) purlins	1/200
6	Elements of frameworks:	
	(a) posts, collar beams	1/300
	(b) glazing purlins (in vertical and horizontal planes)	1/200

NOTES. 1. The deflections are determined without taking account of the dynamical factor.
2. When plaster is used, the deflection of floor (ceiling) beams and girders induced only by the live load should not exceed 1/350th of the span.

TABLE 4 **Maximum Horizontal Deformations of Building and Open Trestle Members to be Allowed under the Action of Cranes**

Kind of deformation	Magnitude of unit deformation
1. Displacement of columns in buildings with heavy service conditions:	
(a) in lateral direction:	
with plane structural scheme	1/2500 H
with three-dimensional structural scheme	1/4000 H
(b) in longitudinal direction	1/4000 H
2. Displacement of columns of open crane trestles in lateral and longitudinal directions	1/4000 H
3. Horizontal deflection of bracing members (beams or trusses) with simple ones used in design . . .	1/2000 L
Symbols: H = height of column from bottom of footing to top of crane rail; L = span of bracing member (column spacing).	

NOTE. The displacements of columns (at the level of the top flange of the crane girders), as well as the deflection of the bracing members, are determined for the lateral braking forces computed for the crane with the maximum lifting capacity of all the cranes installed in the building or on the trestle or bridge. Here the magnitude of the displacement of the columns is determined from the condition of equal displacements of two opposite columns.

Appendix II

1. *Data for Analysis of Axially Loaded Elements*

TABLE 1 **Buckling Factors φ for Axially Loaded Steel Elements**

Slenderness ratio λ	Values of factor φ for steel 3 and 4									
	0	1	2	3	4	5	6	7	8	9
0	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
10	0.990	0.988	0.986	0.984	0.982	0.980	0.978	0.976	0.974	0.972
20	0.970	0.968	0.966	0.964	0.962	0.960	0.958	0.956	0.954	0.952
30	0.950	0.947	0.944	0.941	0.938	0.935	0.932	0.929	0.926	0.923
40	0.920	0.917	0.914	0.911	0.908	0.905	0.902	0.899	0.896	0 893
50	0.890	0.887	0.884	0.881	0.878	0.875	0.872	0.869	0.866	0.863
60	0.860	0.855	0.850	0.845	0.840	0.835	0.830	0.825	0.820	0.815
70	0.810	0.804	0.798	0.792	0.786	0.780	0.774	0.768	0.762	0.756
80	0.750	0.744	0.738	0.732	0.726	0.720	0.714	0.708	0.702	0.696
90	0.690	0.681	0.672	0.663	0.654	0.645	0.636	0.627	0.618	0.609
100	0.600	0.592	0.584	0.576	0.568	0.560	0.552	0.544	0.536	0.528
110	0.520	0.513	0.506	0.499	0.492	0.485	0.478	0.471	0.464	0.457
120	0.450	0.445	0.440	0.435	0.430	0.425	0.420	0.415	0.410	0.405
130	0.400	0.396	0.392	0.388	0.384	0.380	0.376	0.372	0.368	0.364
140	0.360	0.356	0.352	0.348	0.344	0.340	0.336	0.332	0.328	0.324
150	0.320	0.317	0.314	0.311	0.308	0.305	0.302	0.299	0.296	0.293
160	0.290	0.287	0.284	0.281	0.278	0.275	0.272	0.269	0.266	0.263
170	0.260	0.257	0.254	0.251	0.248	0.245	0.242	0.239	0.236	0.233
180	0.230	0.228	0.226	0.224	0.222	0.220	0.218	0.216	0.214	0.212
190	0.210	0.208	0.206	0.204	0.202	0.200	0.198	0.196	0.194	0.192
200	0.190	—	—	—	—	—	—	—	—	—
210	0.170	—	—	—	—	—	—	—	—	—
220	0.160	—	—	—	—	—	—	—	—	—

Slenderness ratio λ	Values of factor φ for low-alloy steels of grades	
	14Г2, 15ГС, 10Г2С, 10Г2СД, 15ХСНД	10ХСНД
0	1.0	1.0
10	0.98	0.98
20	0.95	0.95
30	0.92	0.92
40	0.89	0.88
50	0.84	0.82

Table 1, concluded

Slenderness ratio λ	Values of factor φ for low-alloy steels of grades	
	14Г2, 15ГС, 10Г2С, 10Г2СД, 15ХСНД	10ХСНД
60	0.78	0.75
70	0.71	0.68
80	0.63	0.59
90	0.54	0.50
100	0.46	0.43
110	0.39	0.36
120	0.33	0.31
130	0.29	0.27
140	0.25	0.23
150	0.23	0.20
160	0.21	0.18
170	0.19	0.16
180	0.17	0.14
190	0.15	0.12
200	0.13	0.11
210	0.12	0.10
220	0.11	0.09

TABLE 2 **Buckling Factors φ for Axially Loaded Elements of Aluminium Alloys**

Slenderness ratio λ	Values of factor φ for alloy of grade							
	АМц-М	АМг-М and АД31-Т	АВ-Т, АД31-Т1 and АМц-П	АМг6-М and АМг-П	АД33-Т	АВ-Т1, АМг61 and В92 (sheet)	Д1-Т	Д16-Т and В92 (shape)
0	1	1	1	1	1	1	1	1
10	0.974	0.973	0.999	0.998	0.998	0.998	0.994	0.990
20	0.947	0.945	0.998	0.997	0.996	0.996	0.988	0.980
30	0.921	0.917	0.980	0.943	0.917	0.900	0.889	0.835
40	0.895	0.870	0.880	0.830	0.800	0.780	0.766	0.700
50	0.815	0.770	0.780	0.785	0.686	0.660	0.644	0.568
60	0.730	0.685	0.690	0.628	0.587	0.557	0.539	0.455
70	0.655	0.603	0.600	0.538	0.493	0.463	0.444	0.352
80	0.585	0.580	0.525	0.460	0.416	0.387	0.361	0.268
90	0.521	0.465	0.457	0.388	0.342	0.312	0.286	0.210
100	0.463	0.415	0.395	0.332	0.280	0.252	0.231	0.171
110	0.415	0.365	0.335	0.273	0.230	0.208	0.190	0.141
120	0.375	0.327	0.283	0.230	0.194	0.175	0.160	0.118
130	0.336	0.296	0.241	0.196	0.165	0.150	0.136	0.101
140	0.300	0.265	0.208	0.169	0.143	0.129	0.118	0.087
150	0.220	0.235	0.181	0.147	0.124	0.113	0.103	0.076

2. *Data for Checking Stability of Beams*

STEEL BEAMS

TABLE 3 **Factors ψ for I Beams of Steel 3 and 4**

α	For beams without constraints in span				For beams with intermediate constraints of top flange, regardless of place of load application
	with concentrated load applied to		with uniformly distributed load applied to		
	top flange	bottom flange	top flange	bottom flange	
1	2	3	4	5	6
0.1	1.73	5.00	1.57	3.81	2.17
0.4	1.77	5.03	1.60	3.85	2.20
1.0	1.85	5.11	1.67	3.90	2.27
4	2.21	5.47	1.98	4.23	2.56
8	2.63	5.91	2.35	4.59	2.90
16	3.37	6.65	2.99	5.24	3.50
24	4.03	7.31	3.55	5.79	4.00
32	4.59	7.92	4.04	6.25	4.45
48	5.60	8.88	4.90	7.13	5.23
64	6.52	9.80	5.65	7.92	5.91
80	7.31	10.59	6.30	8.58	6.51
96	8.05	11.29	6.93	9.21	7.07
128	9.40	12.67	8.05	10.29	8.07
160	10.59	13.83	9.04	11.30	8.95
240	13.21	16.36	11.21	13.48	10.86
320	15.31	18.55	13.04	15.29	12.48
400	17.24	20.48	14.57	16.80	13.91

NOTE. With one constraint in a span and the load applied to the bottom flange the values of ψ are determined from column 2.

TABLE 4 **Factors ψ for I-Section Cantilevers of Steel 3 and 4 (with Concentrated Load Applied to Centre of Gravity of Free End of Cantilever or Below It)**

α	0.1	1	2	3	4	6	8	10	12	14	16	24	32	40
ψ	3.1	3.4	3.8	4.1	4.3	4.6	5.0	5.3	5.5	5.7	5.9	6.6	7.3	7.8

NOTE to Tables 3 and 4.
For other grades of steel the tabulated values of ψ are multiplied by:
0.83 for steel 5
0.71 for grade 14Г2, 15ГС, 10Г2С, 10Г2СД and 15ХСНД steel
0.6 for grade 10ХСНД steel.

BEAMS OF ALUMINIUM ALLOYS

The values of the factors ψ for beams of grade Д16-Т aluminium alloy are determined from Table 5. For alloys of other grades the values of ψ are determined by multiplying the values obtained in Table 5 by the factor η_0 taken from Table 6 depending upon the grade of the alloy.

TABLE 5 **Factors ψ for Beams of Aluminium Alloy of Grade Д16-Т**

α	For beams without constraints in span				For beams with intermediate constraints, regardless of place of load application
	with concentrated load applied to		with uniformly distributed load applied to		
	top flange	bottom flange	top flange	bottom flange	
1	2	3	4	5	6
0.1	0.45	1.29	0.41	0.98	0.56
0.4	0.46	1.30	0.42	0.99	0.57
1	0.48	1.32	0.43	1.01	0.59
4	0.57	1.41	0.51	1.09	0.66
8	0.68	1.52	0.61	1.18	0.75
16	0.87	1.72	0.77	1.35	0.89
24	1.04	1.89	0.91	1.49	1.03
32	1.18	2.04	1.04	1.62	1.25
48	1.45	2.30	1.26	1.84	1.35

Table 5, concluded

α	For beams without constraints in span				For beams with intermediate constraints, regardless of place of load application
	with concentrated load applied to		with uniformly distributed load applied to		
	top flange	bottom flange	top flange	bottom flange	
1	2	3	4	5	6
64	1.68	2.53	1.46	2.05	1.53
80	1.89	2.74	1.52	2.22	1.68
96	2.08	2.92	1.79	2.38	1.82
128	2.43	3.27	2.07	2.66	2.08
160	2.74	3.57	2.33	2.92	2.31
240	3.42	4.22	2.89	3.49	2.80
320	3.96	4.80	3.36	3.95	3.22
400	4.45	5.30	3.75	4.35	3.60

NOTE. With one constraint in a span and the load applied to the bottom flange the values of ψ are determined from column 2.

TABLE 6 **Values of Factor η_0**

Grade of alloy	η_0	Grade of alloy	η_0
АМц-М	5.16	АВ-Т1 and АМг61	1.48
АМг-М and АД31-Т	3.87	Д1-Т	1.34
АВ-Т and АД31-Т1	2.33	Д16-Т	1
АМг6-М	1.94	В92 (plates)	1.55
АД33-Т1	1.63	В92 (shapes)	1.05

3. *Data for Analysis of Eccentrically Loaded Elements*

TABLE 7 **Factors φ_{ec} for Solid-Web Elements of Steel 3 and 4 ($R = 2.1$ t/cm²)**

λ	Values of factor φ_{ec} for equivalent eccentricity m_1 equal to																								
	0.1	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0	12.0	14.0	17.0	20.0
10	967	920	847	781	721	667	618	574	535	468	414	370	333	303	285	256	235	220	205	182	162	147	123	106	089	075
20	959	887	800	729	673	623	577	536	501	439	390	349	315	288	263	243	225	210	196	174	157	141	120	102	085	072
30	942	868	773	699	641	592	550	511	478	420	373	335	303	277	254	234	218	203	191	169	152	138	117	100	084	071
40	920	846	743	668	608	560	520	484	453	399	355	320	290	265	243	226	210	196	184	164	148	135	114	098	083	070
50	890	820	711	634	574	528	490	456	427	377	338	304	277	253	234	216	201	189	177	159	143	130	111	096	081	069
60	860	788	674	598	540	495	459	428	402	355	319	289	263	241	224	207	193	182	171	153	138	126	107	094	079	068
70	810	749	634	560	505	463	429	401	377	334	301	273	249	230	213	198	185	174	164	147	134	122	104	091	077	066
80	750	701	591	521	471	432	400	374	353	314	283	258	236	218	203	189	177	167	157	142	129	118	101	089	075	065
90	690	648	546	483	436	401	372	348	329	294	266	243	224	207	192	180	169	160	151	136	124	114	098	087	073	063
100	600	590	500	444	403	371	345	324	305	275	250	229	211	197	183	172	161	153	144	131	120	110	095	084	071	062
110	520	520	456	407	371	342	320	301	284	257	234	216	200	186	173	163	154	146	138	126	115	106	092	081	069	060
120	450	450	413	372	341	316	296	279	264	239	221	203	189	176	165	155	147	138	132	120	110	102	089	079	067	059
130	400	400	374	339	312	291	273	258	245	224	206	191	178	166	156	147	139	132	126	115	106	098	086	076	065	057
140	360	360	338	309	287	268	253	240	228	209	193	180	168	158	149	140	133	126	121	110	102	095	084	074	063	055
150	320	320	306	282	263	248	234	222	212	195	182	169	158	149	141	133	126	120	115	106	099	091	080	071	062	054
160	290	290	277	257	241	228	216	206	197	182	170	159	149	141	134	127	120	115	110	101	094	087	077	069	060	053
170	260	260	252	237	222	211	200	192	184	170	159	150	141	134	127	120	114	110	105	097	090	084	074	067	059	052
180	230	230	229	216	204	194	185	178	171	159	149	141	133	126	120	114	109	104	100	093	086	080	072	065	057	051
190	210	210	210	199	188	180	172	166	160	149	141	133	126	120	114	109	104	099	096	090	083	078	070	063	055	049
200	190	190	190	182	174	167	160	154	149	140	132	125	119	113	107	103	099	095	092	086	079	075	067	061	053	048

NOTES. 1. The values of φ_{ec} in the Table have been multiplied by 10^3.

2. For steel with a different value of the design strength R the values of the factor φ_{ec} are determined from the given table depending upon the relative slenderness ratio $\lambda_{rel} = \lambda \sqrt{\frac{R}{2.1}}$, but are taken not higher than the values of φ in Table 1 of Appendix II.

EXAMPLE. Find φ_{ec} for grade 10Г2С steel ($R = 2.9$ t/cm²) with $\lambda = 70$ and $m_1 = 1.25$.

Solution. We find $\lambda_{rel} = 70 \sqrt{\frac{2.9}{2.1}} = 82$.

From the table for $\lambda = 80$ and $m_1 = 1.25$ we obtain $\varphi_{ec} = 0.432$;

$\lambda = 90$ and $m_1 = 1.25$ we obtain $\varphi_{ec} = 0.401$.

By means of interpolation the value of φ_{ec} being sought is $\varphi_{ec} = 0.426$.

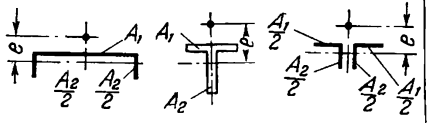
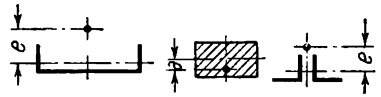
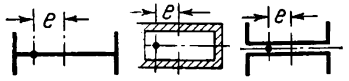
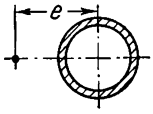
TABLE 8 **Factors φ_{ec} for Open-Web Elements of Steel 3 and 4 (R=2.1 t/cm²)**

λ_{rel}	Values of factor φ_{ec} for relative eccentricity m equal to																								
	0.1	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0	12.0	14.0	17.0	20.0
20	906	794	660	565	495	440	395	360	330	283	248	220	199	180	165	153	142	133	124	110	099	091	077	067	055	048
30	901	786	651	557	487	433	390	355	326	280	245	218	197	179	164	152	141	132	123	110	099	090	076	066	055	047
40	893	774	638	545	477	424	383	348	320	275	241	215	194	177	162	150	140	131	122	109	098	090	076	066	055	047
50	882	757	621	530	464	413	373	340	313	269	237	212	191	174	160	148	138	129	121	108	097	089	075	065	055	047
60	860	735	600	512	448	400	361	330	304	263	231	207	187	171	157	146	136	127	120	107	096	088	075	065	054	047
70	810	706	574	490	430	385	348	319	294	255	225	202	183	167	154	143	133	125	118	105	095	087	074	064	054	046
80	750	669	544	466	410	368	334	306	283	246	218	196	178	163	151	140	131	123	115	103	094	086	073	064	053	046
90	690	624	510	439	389	350	319	293	272	237	211	190	173	159	147	137	128	120	113	102	092	084	072	063	053	046
100	600	573	474	411	366	331	302	279	259	227	203	184	168	154	143	133	125	117	111	100	090	083	071	062	052	045
110	520	520	437	382	342	311	286	264	247	218	195	177	162	149	139	129	121	114	108	097	089	081	070	061	052	045
120	450	450	400	354	319	291	269	250	234	207	187	170	156	144	134	126	118	111	105	095	087	080	069	060	051	044
130	400	400	364	326	296	272	252	235	221	197	178	163	150	139	130	122	114	108	102	093	085	078	067	059	050	044
140	360	360	331	299	274	253	236	221	208	187	170	160	144	134	125	118	111	105	100	090	083	076	066	058	050	043
150	320	320	301	275	253	236	221	208	196	177	162	149	138	129	121	113	107	102	097	088	081	075	065	057	049	043
160	290	290	274	252	234	219	206	195	185	168	154	142	132	123	116	109	103	098	094	085	078	073	063	056	048	042
170	260	260	249	231	216	203	192	182	173	158	146	135	126	118	111	105	100	095	091	083	076	071	062	055	047	041
180	230	230	227	213	200	189	179	171	163	149	138	129	120	113	107	101	098	092	087	080	074	069	060	054	046	041
190	210	210	208	196	185	176	167	160	153	141	131	122	115	108	102	097	093	088	084	078	072	067	059	053	045	040
200	190	190	190	180	171	163	156	149	143	133	124	116	110	104	098	093	089	085	082	075	070	065	058	052	045	039

NOTES. 1. The values of φ_{ec} in the Table have been multiplied by 10^3.

2. For steel with a different value of the design strength R the values of the factor φ_{ec} are determined from the given table depending upon the relative slenderness ratio $\lambda_{red}=\lambda_{rel}\sqrt{\frac{R}{2.1}}$, but are not taken higher than the values of φ in Table I of Appendix II.

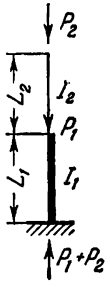
TABLE 9 **Section Shape Coefficients η for Computing Equivalent Eccentricity $m_1 = \eta m$**

Sketch of section	$20 \leqslant \lambda \leqslant 150$	$\lambda > 150$
	$0.775+0.0015\,\lambda$	1.0
	$1.30+0.5\sqrt{m}$ *	$1.3+0.5\sqrt{m}$ *
	1.0	1.0
	$1.45-0.003\,\lambda$	1.0
	$1.3-0.002\,\lambda$	1.0

* The formulae are valid with $\frac{A_1}{A_2} \leqslant 1$

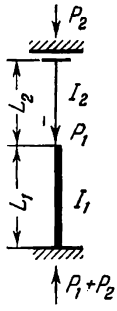
4. Data for Determining Effective Lengths of Stepped Columns

TABLE 10 Length Coefficients k_1 for Columns with Top End Free

Sketch	c_1 / c_2	0	0.2	0.4	0.6	0.8	1.0	1.4	1.8	2.5	10.0	20.0
	0	2.0	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
	0.2	2.0	2.02	2.04	2.06	2.07	2.09	2.12	2.15	2.21	2.76	3.38
	0.4	2.0	2.08	2.13	2.21	2.28	2.35	2.48	2.60	2.80		
	0.6	2.0	2.20	2.36	2.52	2.66	2.80	3.05	3.28			
	0.8	2.0	2.42	2.70	2.96	3.17	3.36	3.74				
	1.0	2.0	2.73	3.13	3.44	3.74	4.00					
	1.5	3.0	3.77	4.35	4.86							
	2.0	4.0	4.90	5.67								
	2.5	5.0	6.08	7.00								
	3.0	6.0	7.25									

NOTE. $c_1 = \frac{I_2 L_1}{I_1 L_2}$ and $c_2 = \frac{L_2}{L_1}\sqrt{\frac{F_2 I_1}{F_1 I_2}}$

TABLE 11 Length Coefficients k_1 for Columns with Top End Fixed Against Rotation

Sketch	c_1 / c_2	0	0.2	0.4	0.6	0.8	1.0	1.4	1.8	2.5	10.0	20.0
	0	2.0	1.86	1.76	1.67	1.60	1.55	1.46	1.40	1.32	1.10	1.05
	0.2	2.0	1.87	1.76	1.68	1.62	1.56	1.48	1.41	1.33	1.11	
	0.4	2.0	1.88	1.77	1.72	1.66	1.61	1.53	1.48	1.40		
	0.6	2.0	1.91	1.83	1.77	1.72	1.69	1.63	1.59			
	0.8	2.0	1.94	1.90	1.87	1.85	1.82	1.79				
	1.0	2.0	2.0	2.00	2.0	2.0	2.0					
	1.5	2.0	2.25	2.38	2.48							
	2.0	2.0	2.66	2.91								
	2.5	2.5	3.17	3.50								
	3.0	3.0	3.70	4.12								

NOTE. $c_1 = \frac{I_2 L_1}{I_1 L_2}$ and $c_2 = \frac{L_2}{L_1}\sqrt{\frac{F_2 I_1}{F_1 I_2}}$

TABLE 12 **Coefficients k_{12} and k_{11} for Columns with Pivoted Upper End Coefficients k_{12}**

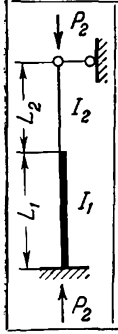
Sketch	$\beta=\frac{I_2}{I_1}$	Values of coefficient k_{12} for ratio $n=L_2/L_1$ equal to														
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
	0.04	1.02	1.84	2.25	2.59	2.85	3.08	3.24	3.42	3.70	4.00	4.55	5.25	5.80	6.55	7.20
	0.06	0.91	1.47	1.93	2.26	2.57	2.74	2.90	3.05	3.24	3.45	3.88	4.43	4.90	5.43	5.94
	0.08	0.86	1.31	1.73	2.05	2.31	2.49	2.68	2.85	3.00	3.14	3.53	3.93	4.37	4.85	5.28
	0.10	0.83	1.21	1.57	1.95	2.14	2.33	2.46	2.60	2.76	2.91	3.28	3.61	4.03	4.43	4.85
	0.20	0.79	0.98	1.23	1.46	1.67	1.85	2.02	2.15	2.28	2.40	2.67	2.88	3.11	3.42	3.71
	0.30	0.78	0.90	1.09	1.27	1.44	1.60	1.74	1.86	1.98	2.11	2.35	2.51	2.76	2.99	3.25
	0.40	0.78	0.88	1.02	1.17	1.32	1.45	1.58	1.69	1.81	1.92	2.14	2.31	2.51	2.68	2.88
	0.50	0.78	0.86	0.99	1.10	1.22	1.35	1.47	1.57	1.67	1.76	1.96	2.15	2.34	2.50	2.76
	1.00	0.78	0.85	0.92	0.99	1.06	1.13	1.20	1.27	1.34	1.41	1.54	1.68	1.82	1.97	2.10

TABLE 13 **Coefficients k_{11}**

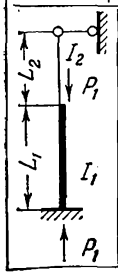
Sketch	$\beta=\frac{I_2}{I_1}$	Values of coefficient k_{11} for ratio $n=L_2/L_1$ equal to														
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
	0.04	0.67	0.67	0.83	1.25	1.43	1.55	1.65	1.70	1.75	1.78	1.84	1.87	1.88	1.90	1.92
	0.06	0.67	0.67	0.81	1.07	1.27	1.41	1.51	1.60	1.64	1.70	1.78	1.82	1.84	1.87	1.88
	0.08	0.67	0.67	0.75	0.98	1.19	1.32	1.43	1.51	1.58	1.63	1.72	1.77	1.81	1.82	1.84
	0.10	0.67	0.67	0.73	0.93	1.11	1.25	1.36	1.45	1.52	1.57	1.66	1.72	1.77	1.80	1.82
	0.20	0.67	0.67	0.69	0.75	0.89	1.02	1.12	1.21	1.29	1.36	1.46	1.54	1.60	1.65	1.69
	0.30	0.67	0.67	0.67	0.71	0.80	0.90	0.99	1.08	1.15	1.22	1.33	1.41	1.48	1.54	1.59
	0.40	0.67	0.67	0.67	0.69	0.75	0.84	0.92	1.00	1.07	1.13	1.24	1.33	1.40	1.47	1.51
	0.50	0.67	0.67	0.67	0.69	0.73	0.81	0.87	0.94	1.01	1.07	1.17	1.26	1.33	1.39	1.44
	1.00	0.67	0.67	0.67	0.68	0.71	0.74	0.78	0.82	0.87	0.91	0.99	1.07	1.13	1.19	1.24

TABLE 14 **Coefficients k_{11} and k_{12} for Columns with Fixed Upper End**
Coefficients k_{12}

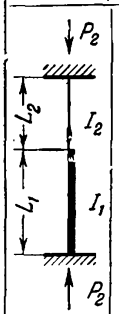
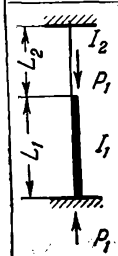
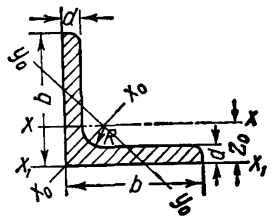
Sketch	$\beta = \frac{I_2}{I_1}$	Values of coefficient k_{12} for ratio $n = L_2/L_1$ equal to														
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
	0.04	0.78	1.02	1.53	1.73	2.01	2.21	2.38	2.54	2.65	2.85	3.24	3.70	4.20	4.76	5.23
	0.06	0.70	0.86	1.23	1.47	1.73	1.93	2.08	2.23	2.38	2.49	2.81	3.17	3.50	3.92	4.30
	0.08	0.68	0.79	1.05	1.31	1.54	1.74	1.91	2.05	2.20	2.31	2.55	2.80	3.11	3.45	3.73
	0.10	0.67	0.76	1.00	1.20	1.42	1.61	1.78	1.92	2.04	2.20	2.40	2.60	2.86	3.18	3.41
	0.20	0.64	0.70	0.79	0.93	1.07	1.23	1.41	1.50	1.60	1.72	1.92	2.11	2.28	2.45	2.64
	0.30	0.62	0.68	0.74	0.85	0.95	1.06	1.18	1.28	1.39	1.48	1.67	1.82	1.96	2.12	2.20
	0.40	0.60	0.66	0.71	0.78	0.87	0.99	1.07	1.16	1.26	1.34	1.50	1.65	1.79	1.94	2.08
	0.50	0.59	0.65	0.70	0.77	0.82	0.93	0.99	1.08	1.17	1.23	1.39	1.53	1.66	1.79	1.92
	1.00	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.10	1.20	1.30	1.40	1.50

TABLE 15 **Coefficients k_{11}**

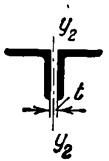
Sketch	$\beta = \frac{I_2}{I_1}$	Values of coefficient k_{11} for ratio $n = L_2/L_1$ equal to														
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
	0.04	0.66	0.68	0.75	0.94	1.08	1.24	1.37	1.47	1.55	1.64	1.72	1.78	1.81	1.85	1.89
	0.06	0.65	0.67	0.68	0.76	0.94	1.10	1.25	1.35	1.44	1.50	1.61	1.69	1.74	1.79	1.82
	0.08	0.64	0.66	0.67	0.68	0.84	1.00	1.12	1.25	1.34	1.41	1.53	1.62	1.68	1.75	1.79
	0.10	0.64	0.65	0.65	0.65	0.78	0.92	1.05	1.15	1.25	1.33	1.45	1.55	1.62	1.68	1.71
	0.20	0.62	0.64	0.65	0.65	0.66	0.73	0.83	0.92	1.01	1.09	1.23	1.33	1.41	1.48	1.54
	0.30	0.60	0.63	0.64	0.65	0.66	0.67	0.73	0.81	0.89	0.94	1.09	1.20	1.28	1.35	1.41
	0.40	0.58	0.63	0.63	0.64	0.64	0.66	0.68	0.75	0.82	0.88	1.01	1.10	1.19	1.26	1.32
	0.50	0.57	0.61	0.63	0.64	0.64	0.65	0.68	0.72	0.77	0.83	0.94	1.04	1.12	1.19	1.25
	1.00	0.55	0.58	0.60	0.61	0.62	0.63	0.65	0.67	0.70	0.73	0.80	0.88	0.93	1.01	1.05

1. *Standard Rolled Steel Shapes*

TABLE 1 **Equal Leg Rolled Steel Angles (Extract from GOST 8509-57)**



Symbols:
b = leg size
d = leg thickness
I = moment of inertia
r = radius of gyration



Dimensions of angle, mm		R, mm	Sectional area A, cm²	Weight per linear metre, kg	Distance to centre of gravity z_0, cm	Axis x-x		Axis x_1-x_1	Axis x_0-x_0		Axis y_0-y_0		Radii of gyration r_{y_2} for two angles with t equal to, mm			
b	d					I_x cm⁴	r_x cm	I_{x_1} cm⁴	I_{x_0} cm⁴	r_{x_0} cm	I_{y_0} cm⁴	r_{y_0} cm	8	10	12	14
45	4	5	3.48	2.73	1.26	6.63	1.38	12.1	10.5	1.74	2.74	0.89	2.16	2.24	2.32	2.40
	5		4.29	3.37	1.30	8.03	1.37	15.3	12.7	1.72	3.33	0.88	2.18	2.26	2.34	2.42
50	4	5.5	3.89	3.05	1.38	9.21	1.54	16.6	14.6	1.94	3.80	0.99	2.35	2.43	2.51	2.59
	5		4.80	3.77	1.42	11.2	1.53	20.9	17.8	1.92	4.63	0.98	2.38	2.45	2.53	2.61
56	4	6	4.38	3.44	1.52	13.1	1.73	23.3	20.8	2.18	5.41	1.11	2.58	2.66	2.73	2.81
	5		5.41	4.25	1.57	16.0	1.72	29.2	25.4	2.16	6.59	1.10	2.61	2.72	2.77	2.85

63	4	7	4.96	3.90	1.69	18.9	1.95	33.1	29.9	2.45	7.81	1.25	2.86	2.93	3.01	3.09
	5		6.13	4.81	1.74	23.1	1.94	41.5	36.6	2.44	9.52	1.25	2.89	2.96	3.04	3.12
	6		7.28	5.72	1.78	27.1	1.93	50.0	42.9	2.43	11.2	1.24	2.90	2.99	3.06	3.14
70	4.5	8	6.20	4.87	1.88	29.0	2.16	51.0	46.0	2.72	12.0	1.39	3.14	3.21	3.29	3.37
	5		6.86	5.38	1.90	31.9	2.16	56.7	50.7	2.72	13.2	1.39	3.16	3.23	3.30	3.38
	6		8.15	6.39	1.94	37.6	2.15	68.4	59.6	2.71	15.5	1.38	3.18	3.25	3.33	3.40
	7		9.42	7.39	1.99	43.0	2.14	80.1	68.2	2.69	17.8	1.37	3.20	3.28	3.36	3.44
	8		10.7	8.37	2.02	48.2	2.13	91.9	76.4	2.68	20.0	1.37	3.22	3.29	3.37	3.45
75	5	9	7.39	5.80	2.02	39.5	2.31	69.6	62.6	2.91	16.4	1.49	3.35	3.42	3.49	3.57
	6		8.78	6.89	2.06	46.6	2.30	83.9	73.9	2.90	19.3	1.48	3.30	3.44	3.52	3.60
	7		10.1	7.96	2.10	53.3	2.29	98.3	84.6	2.89	22.1	1.48	3.40	3.47	3.54	3.62
	8		11.5	9.02	2.15	59.8	2.28	113	94.9	2.87	24.8	1.47	3.43	3.50	3.57	3.65
	9		12.8	10.1	2.18	66.1	2.27	127	105	2.86	27.5	1.46	3.44	3.51	3.59	3.67
80	5.5	9	8.63	6.78	2.17	52.7	2.47	93.2	83.6	3.11	21.8	1.59	3.57	3.64	3.71	3.79
	6		9.38	7.36	2.19	57.0	2.47	102	90.4	3.11	23.5	1.58	3.58	3.65	3.72	3.80
	7		10.8	8.51	2.23	65.3	2.45	119	104	3.09	27.0	1.58	3.60	3.67	3.75	3.82
	8		12.3	9.65	2.27	73.4	2.44	137	116	3.08	30.3	1.57	3.62	3.69	3.77	3.84
90	6	10	10.6	8.33	2.43	82.1	2.78	145	130	3.50	34.0	1.79	3.96	4.04	4.11	4.19
	7		12.3	9.64	2.47	94.3	2.77	169	150	3.49	38.9	1.78	3.99	4.06	4.13	4.21
	8		13.9	10.9	2.51	106	2.76	194	168	3.48	43.8	1.77	4.01	4.08	4.16	4.23
	9		15.6	12.2	2.55	118	2.75	219	186	3.46	48.6	1.77	4.04	4.11	4.18	4.26
100	6.5	12	12.8	10.1	2.68	122	3.09	214	193	3.88	50.7	1.99	4.36	4.43	4.50	4.57
	7		13.8	10.8	2.71	131	3.08	231	207	3.88	54.2	1.98	4.38	4.45	4.52	4.59
	8		15.6	12.2	2.75	147	3.07	265	233	3.87	60.9	1.98	4.40	4.47	4.54	4.62

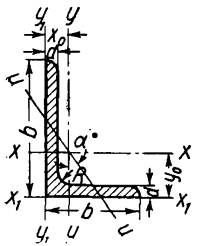
Table 1, continued

Dimensions of angle, mm		R, mm	Sectional area A, cm²	Weight per linear metre kg	Distance to centre of gravity z_0, cm	Axis x-x		Axis x_1-x_1	Axis x_0-x_0		Axis y_0-y_0		Radii of gyration r_{y_2} for two angles with t equal to, mm			
b	d					I_x cm⁴	r_x cm	I_{x_1} cm⁴	I_{x_0} cm⁴	r_{x_0} cm	I_{y_0} cm⁴	r_{y_0} cm	8	10	12	14
100	10	12	19.2	15.1	2.83	179	3.05	333	284	3.84	74.1	1.96	4.44	4.52	4.59	4.66
	12		22.8	17.9	2.91	209	3.03	402	331	3.81	86.9	1.95	4.48	4.56	4.63	4.71
	14		26.3	20.6	2.99	237	3.00	472	375	3.78	99.3	1.94	4.53	4.60	4.68	4.76
	16		29.7	23.3	3.06	264	2.98	542	416	3.74	112	1.94	4.57	4.64	4.72	4.80
110	7	12	15.2	11.9	2.96	176	3.40	308	279	4.29	72.7	2.19	4.78	4.85	4.92	5.00
	8		17.2	13.5	3.00	198	3.39	353	315	4.28	81.8	2.18	4.80	4.87	4.95	5.02
125	8	14	19.7	15.5	3.36	294	3.87	516	467	4.87	122	2.49	5.39	5.46	5.53	5.60
	9		22.0	17.3	3.40	327	3.86	582	520	4.86	135	2.48	5.41	5.48	5.56	5.63
	10		24.3	19.1	3.45	360	3.85	649	571	4.84	149	2.47	5.44	5.52	5.58	5.66
	12		28.9	22.7	3.53	422	3.82	782	670	4.82	174	2.46	5.48	5.55	5.62	5.70
	14		33.4	26.2	3.61	482	3.80	916	764	4.78	200	2.45	5.52	5.60	5.67	5.75
	16		37.8	29.6	3.68	539	3.78	1,051	853	4.75	224	2.44	5.56	5.63	5.72	5.78
140	9	14	24.7	19.4	3.78	466	4.34	818	739	5.47	192	2.79	6.02	6.10	6.16	6.24
	10		27.3	21.5	3.82	512	4.33	911	814	5.46	211	2.78	6.05	6.12	6.19	6.26
	12		32.5	25.5	3.90	602	4.31	1,097	957	5.43	248	2.76	6.08	6.15	6.25	6.30
160	10	16	31.4	24.7	4.30	774	4.96	1,356	1,229	6.25	319	3.19	6.84	6.91	6.97	7.05
	11		34.4	27.0	4.35	844	4.95	1,494	1,341	6.24	348	3.18	6.86	6.93	7.00	7.07
	12		37.4	29.4	4.39	913	4.94	1,633	1,450	6.23	376	3.17	6.88	6.95	7.02	7.09

160	14	16	43.3	34.0	4.47	1,046	4.92	1,911	1,662	6.20	431	3.16	6.91	6.98	7.05	7.13
	16		49.1	38.5	4.55	1,175	4.89	2,191	1,866	6.17	485	3.14	6.95	7.03	7.10	7.18
	18		54.8	43.0	4.63	1,299	4.87	2,472	2,061	6.13	537	3.13	7.00	7.07	7.14	7.22
	20		60.4	47.4	4.70	1,419	4.85	2,756	2,248	6.10	589	3.12	7.04	7.11	7.18	7.26
180	11	16	38.8	30.5	4.85	1,216	5.60	2,128	1,933	7.06	500	3.59	7.67	7.74	7.81	7.82
	12		42.2	33.1	4.89	1,317	5.59	2,324	2,093	7.04	540	3.58	7.69	7.76	7.83	7.84
200	12	18	47.1	37.0	5.37	1,823	6.22	3,182	2,896	7.84	749	3.99	8.48	8.55	8.62	8.69
	13		50.9	39.9	5.42	1,961	6.21	3,452	3,116	7.83	805	3.98	8.50	8.58	8.64	8.71
	14		54.6	42.8	5.46	2,097	6.20	3,722	3,333	7.81	861	3.97	8.52	8.60	8.66	8.73
	16		62.0	48.7	5.54	2,363	6.17	4,264	3,755	7.78	970	3.96	8.56	8.64	8.70	8.77
	20		76.5	60.1	5.70	2,871	6.12	5,355	4,560	7.72	1,182	3.93	8.65	8.72	8.79	8.86
	25		94.3	74.0	5.89	3,466	6.06	6,733	5,494	7.63	1,438	3.91	8.74	8.81	8.88	8.95
	30		111.5	87.6	6.07	4,020	6.00	8,130	6,351	7.55	1,688	3.89	8.83	8.90	8.97	9.05
220	14	21	60.4	47.4	5.93	2,814	6.83	4,941	4,470	8.60	1,159	4.38	9.31	9.37	9.45	9.52
	16		68.6	53.8	6.02	3,175	6.81	5,661	5,045	8.58	1,306	4.36	9.35	9.42	9.49	9.56
250	16	24	78.4	61.5	6.75	4,717	7.76	8,286	7,492	9.78	1,942	4.98	10.55	10.62	10.68	10.75
	18		87.7	68.9	6.83	5,247	7.73	9,342	8,337	9.75	2,158	4.96	10.59	10.65	10.72	10.80
	20		97.0	76.1	6.91	5,765	7.71	10,401	9,160	9.72	2,370	4.94	10.62	10.69	10.76	10.83
	22		106.1	83.3	7.00	6,270	7.69	11,464	9,961	9.69	2,579	4.93	10.67	10.74	10.81	10.88
	25		119.7	94.0	7.11	7,006	7.65	13,064	11,125	9.64	2,887	4.91	10.72	10.79	10.86	10.93
	28		133.1	104.5	7.23	7,717	7.61	14,674	12,244	9.59	3,190	4.89	10.78	10.85	10.92	10.99
	30		142.0	111.4	7.31	8,177	7.59	15,753	12,965	9.56	3,389	4.89	10.82	10.89	10.96	11.03

TABLE 2 **Unequal Leg Rolled Steel Angles (Extract from GOST 8510-57)**

Symbols:



B = size of large leg
b = size of small leg
d = leg thickness
I = moment of inertia
r = radius of gyration

Dimensions of angle, mm				Sectional area A, cm²	Weight per linear metre, kg	Distance to centre of gravity		Axis x-x		Axis y-y		Axis x_1-x_1	Axis y_1-y_1	Axis u-u		Radii of gyration r_y for two angles with t equal to, mm				Radii of gyration r_y for two angles with t equal to, mm			
b	b	d	R, mm			y_0, cm	x_0, cm	I_x, cm⁴	r_x, cm	I_y, cm⁴	r_y, cm	I_{x_1}, cm⁴	I_{y_1}, cm⁴	$I_{u\,min}$, cm⁴	$r_{u\,min}$, cm	8	10	12	14	8	10	12	14
56	36	4	6.0	3.58	2.81	1.82	0.84	11.4	1.78	3.70	1.02	23.2	6.25	2.19	0.78	1.60	1.68	1.76	1.84	2.85	2.93	3.01	3.09
		5		4.41	3.46	1.86	0.88	13.8	1.77	4.48	1.01	29.2	7.91	2.66	0.78	1.63	1.71	1.79	1.87	2.87	2.95	3.03	3.11
63	40	4	7.0	4.04	3.17	2.03	0.87	16.3	2.01	5.16	1.13	33.0	8.51	3.07	0.87	1.73	1.81	1.89	1.96	3.15	3.23	3.31	3.39
		5		4.98	3.91	2.08	0.86	19.9	2.00	6.26	1.12	41.4	10.8	3.73	0.86	1.75	1.83	1.91	1.99	3.19	3.26	3.34	3.42
		6		5.90	4.63	2.12	0.99	23.3	1.99	7.28	1.11	49.9	13.1	4.36	0.86	1.78	1.89	1.94	2.02	3.21	3.29	3.36	3.45
		8		7.68	6.03	2.20	1.07	29.6	1.96	9.15	1.09	66.9	17.9	5.58	0.85	1.83	1.91	1.99	2.08	3.26	3.34	3.42	3.50
70	45	4.5	7.5	5.07	3.98	2.25	1.03	25.3	2.23	8.25	1.28	51.0	13.6	4.88	0.98	1.92	1.99	2.07	2.15	3.47	3.54	3.62	3.70
		5		5.59	4.39	2.28	1.05	27.8	2.23	9.05	1.27	56.7	15.2	5.34	0.98	1.93	2.01	2.08	2.17	3.49	3.56	3.64	3.72

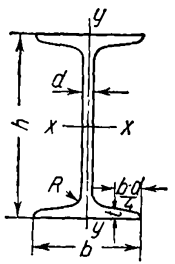
75	50	5	8.0	6.11	4.79	2.39	1.17	34.8	2.39	12.5	1.43	69.7	20.8	7.24	1.09	2.13	2.21	2.28	2.36	3.67	3.75	3.83	3.90
		6		7.25	5.69	2.44	1.21	40.9	2.38	14.6	1.42	83.9	25.2	8.48	1.08	2.15	2.22	2.30	2.38	3.70	3.78	3.86	3.94
		8		9.47	7.43	2.52	1.29	52.4	2.35	18.5	1.40	112	34.2	10.9	1.07	2.19	2.27	2.37	2.43	3.75	3.83	3.91	3.98
80	50	5	8.0	6.36	4.99	2.60	1.13	41.6	2.56	12.7	1.41	84.6	20.8	7.58	1.09	2.08	2.16	2.23	2.30	3.94	4.02	4.11	4.19
		6		7.55	5.92	2.65	1.17	49.0	2.55	14.8	1.40	102.0	25.2	8.88	1.08	2.10	2.18	2.26	2.34	3.97	4.05	4.13	4.21
90	56	5.5	9	7.86	6.17	2.92	1.26	65.3	2.88	19.7	1.58	132	32.2	11.8	1.22	2.29	2.36	2.44	2.52	4.40	4.47	4.55	4.63
		6		8.54	6.70	2.95	1.28	70.6	2.88	21.2	1.58	145	35.2	12.7	1.22	2.30	2.38	2.45	2.53	4.42	4.49	4.57	4.65
		8		11.18	8.77	3.04	1.36	90.9	2.85	27.1	1.56	194	47.8	16.3	1.21	2.35	2.43	2.51	2.58	4.47	4.52	4.62	4.70
100	63	6	10	9.59	7.53	3.23	1.42	98.3	3.20	30.6	1.79	198	49.9	18.2	1.38	2.55	2.62	2.70	2.77	4.84	4.92	4.99	5.07
		7		11.1	8.70	3.28	1.46	113	3.19	35.0	1.78	232	58.7	20.8	1.37	2.57	2.64	2.72	2.78	4.87	4.95	5.02	5.10
		8		12.6	9.87	3.32	1.50	127	3.18	39.2	1.77	266	67.6	23.4	1.36	2.59	2.66	2.74	2.82	4.89	4.97	5.04	5.12
		10		15.5	12.1	3.40	1.58	154	3.15	47.1	1.75	333	85.8	28.3	1.35	2.64	2.71	2.79	2.87	4.94	5.01	5.09	5.17
110	70	6.5	10	11.4	8.98	3.55	1.58	142	3.53	45.6	2.0	286	74.3	26.9	1.53	2.81	2.88	2.96	3.03	5.30	5.37	5.44	5.52
		7		12.3	9.64	3.57	1.60	152	3.52	48.7	1.99	309	80.3	28.8	1.53	2.82	2.89	2.97	3.04	5.31	5.38	5.45	5.53
		8		13.9	10.9	3.61	1.64	172	3.51	54.6	1.98	353	92.3	32.3	1.52	2.84	2.92	2.99	3.07	5.33	5.41	5.49	5.56
125	80	7	11	14.1	11.0	4.01	1.80	227	4.01	73.7	2.29	452	119	43.4	1.76	3.17	3.24	3.31	3.39	5.96	6.04	6.11	6.19
		8		16.0	12.5	4.05	1.84	256	4.00	83.0	2.28	518	137	48.8	1.75	3.19	3.27	3.34	3.41	5.98	6.06	6.13	6.21
		10		19.7	15.5	4.14	1.92	312	3.98	100	2.26	649	173	59.3	1.74	3.23	3.31	3.37	3.46	6.04	6.11	6.19	6.27
		12		23.4	18.3	4.22	2.00	365	3.95	117	2.24	781	210	69.5	1.72	3.28	3.35	3.43	3.51	6.08	6.15	6.23	6.31

Table 2, concluded

Dimensions of angle, mm			R, mm	Sectional area A, cm²	Weight per linear metre, kg	Distance to centre of gravity		Axis x-x		Axis y-y		Axis x_1-x_1	Axis y_1-y_1	Axis u-u		Radii of gyration r_y for two angles with t equal to, mm				Radii of gyration r_y for two angles with t equal to, mm			
B	d	b				y_0 cm	x_0 cm	I_x cm⁴	r_x cm	I_y cm⁴	r_y cm	I_{x_1} cm⁴	I_{y_1} cm⁴	$I_{u,min}$ cm⁴	$r_{u,min}$ cm	8	10	12	14	8	10	12	14
140	90	8	12	18.0	14.1	4.49	2.03	364	4.49	120	2.58	727	194	70.3	1.98	3.55	3.61	3.69	3.76	6.64	6.72	6.79	6.86
		10		22.2	17.5	4.58	2.12	444	4.47	146	2.56	911	245	85.5	1.96	3.60	3.67	3.74	3.82	6.69	6.77	6.84	6.92
160	100	9	13	22.9	18.0	5.19	2.23	606	5.15	186	2.85	1,221	300	110	2.20	3.87	3.95	4.02	4.09	7.60	7.67	7.75	7.82
		10		25.3	19.8	5.23	2.28	667	5.13	204	2.84	1,359	335	121	2.19	3.90	3.97	4.04	4.12	7.62	7.69	7.77	7.84
		12		30.0	23.6	5.32	2.36	784	5.11	239	2.82	1,634	405	142	2.18	3.95	4.02	4.09	4.16	7.67	7.75	7.82	7.90
		14		34.7	27.3	5.40	2.43	897	5.08	272	2.80	1,910	477	162	2.16	3.98	4.05	4.13	4.20	7.71	7.78	7.86	7.94
180	110	10	14	28.3	22.2	5.88	2.44	952	5.80	276	3.12	1,933	444	165	2.42	4.22	4.29	4.36	4.43	8.55	8.62	8.69	8.77
		12		33.7	26.4	5.97	2.52	1,123	5.77	324	3.10	2,324	537	194	2.40	4.26	4.33	4.40	4.47	8.59	8.67	8.75	8.82
200	125	11	14	34.9	27.4	6.50	2.79	1,449	6.45	446	3.58	2,920	718	264	2.75	4.79	4.86	4.93	5.00	9.44	9.51	9.59	9.66
		12		37.9	29.7	6.54	2.83	1,568	6.43	482	3.57	3,189	786	285	2.74	4.81	4.88	4.95	5.02	9.46	9.54	9.62	9.68
		14		43.9	34.4	6.62	2.91	1,801	6.41	551	3.54	3,726	922	327	2.73	4.85	4.92	4.99	5.06	9.50	9.58	9.65	9.73
		16		49.8	39.1	6.71	2.99	2,026	6.38	617	3.52	4,264	1,061	367	2.72	4.89	4.95	5.03	5.10	9.55	9.63	9.70	9.78
250	160	12	18	48.3	37.9	7.97	3.53	3,147	8.07	1,032	4.62	6,212	1,634	604	3.54	6.07	6.13	6.20	6.27	11.62	11.71	11.77	11.85
		16		63.6	49.9	8.14	3.69	4,091	8.02	1,333	4.58	8,308	2,200	781	3.50	6.14	6.21	6.27	6.34	11.73	11.78	11.86	11.94
		18		71.1	55.8	8.23	3.77	4,545	7.99	1,475	4.56	9,358	2,487	866	3.49	6.18	6.21	6.31	6.38	11.76	11.84	11.91	11.98
		20		78.5	61.7	8.31	3.85	4,987	7.97	1,613	4.53	10,410	2,776	949	3.48	6.20	6.28	6.35	6.42	11.81	11.88	11.95	12.03

TABLE 3 **Rolled Steel Shapes — I Beams (Extract from GOST 8239-56*)**

Symbols:



h = depth of beam
b = width of flange
d = thickness of web
t = mean thickness of flange
R = radius of fillet

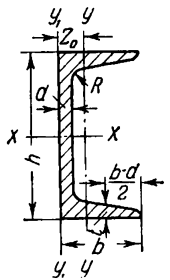
I = moment of inertia
W = section modulus
S = statical moment of half-section
r = radius of gyration

Section No	Weight per linear metre	Dimensions					Sectional area A	Reference data for axes						
								x-x				y-y		
		h	b	d	t	R		I_x	W_x	r_x	S_x	I_y	W_y	r_y
	kg	mm					cm²	cm⁴	cm³	cm	cm³	cm⁴	cm³	cm
10	9.46	100	55	4.5	7.2	7	12.0	198	39.7	4.06	23.0	17.9	6.46	1.22
12	11.5	120	64	4.8	7.3	7.5	14.7	350	58.4	4.88	33.7	27.9	8.72	1.38
14	13.7	140	73	4.9	7.5	8	17.4	572	81.7	5.73	46.8	41.9	11.5	1.55
16	15.9	160	81	5.0	7.8	8.5	20.2	873	109	6.57	62.3	58.6	14.5	1.70
18	18.4	180	90	5.1	8.1	9	23.4	1,290	143	7.42	81.4	82.6	18.4	1.88
18a	19.9	180	100	5.1	8.3	9	25.4	1,430	159	7.51	89.8	114	22.8	2.12
20	21.0	200	100	5.2	8.4	9.5	26.8	1,840	184	8.28	104	115	23.1	2.07
20a	22.7	200	110	5.2	8.6	9.5	28.9	2,030	203	8.37	114	155	28.2	2.32

Table 3, concluded

Section No.	Weight per linear metre	Dimensions					Sectional area A	Reference data or axes						
								x-x				y-y		
		h	b	d	t	R		I_x	W_x	r_x	S_x	I_y	W_y	r_y
	kg	mm					cm²	cm⁴	cm³	cm	cm³	cm⁴	cm³	cm
22	24.0	220	110	5.4	8.7	10	30.6	2,550	232	9.13	131	157	28.6	2.27
22a	25.8	220	120	5.4	8.9	10	32.8	2,790	254	9.12	143	206	34.3	2.50
24	27.3	240	115	5.6	9.5	10.5	34.8	3.460	289	9.97	163	198	34.5	2.37
24a	29.4	240	125	5.6	9.8	10.5	37.5	3,800	317	10.1	178	260	41.6	2.63
27	31.5	270	125	6.0	9.8	11	40.2	5,010	371	11.2	210	260	41.5	2.54
27a	33.9	270	135	6.0	10.2	11	43.2	5,500	407	11.3	229	337	50.0	2.80
30	36.5	300	135	6.5	10.2	12	46.5	7,080	472	12.3	268	337	49.9	2.69
30a	39.2	300	145	6.5	10.7	12	49.9	7,780	518	12.5	292	436	60.1	2.95
33	42.2	330	140	7.0	11.2	13	53.8	9,840	597	13.5	339	419	59.9	2.79
36	48.6	360	145	7.5	12.3	14	61.9	13,380	743	14.7	423	516	71.1	2.89
40	56.1	400	155	8.0	13.0	15	71.4	18,930	947	16.3	540	666	85.9	3.05
45	65.2	450	160	8.6	14.2	16	83.0	27,450	1,220	18.2	699	807	101	3.12
50	76.8	500	170	9.5	15.2	17	97.8	39,290	1,570	20.0	905	1,040	122	3.26
55	89.8	550	180	10.3	16.5	18	114	55,150	2,000	22.0	1,150	1,350	150	3.44
60	104	600	190	11.1	17.8	20	132	75,450	2,510	23.9	1,450	1,720	181	3.60
65	120	650	200	12.0	19.2	22	153	101,400	3,120	25.8	1,800	2,170	217	3.77
70	138	700	210	13.0	20.8	24	176	134,600	3,840	27.7	2,230	2,780	260	3.94
70a	158	700	210	15.0	24.0	24	202	152,700	4,360	27.5	2,550	3,240	309	4.01
70b	184	700	210	17.5	28.2	24	234	175,370	5,010	27.4	2,940	3,910	373	4.09

TABLE 4 **Rolled Steel Shapes — Channels (Extract from GOST 8240-56*)**



Symbols:

h = depth of channel
b = width of flange
d = thickness of web
t = mean thickness of flange
h = radius of fillet

I = moment of inertia
W = section modulus
r = radius of gyration
S = statical moment of half-section
z_0 = distance from axis y-y to outer side of web

Section No	Weight per linear metre	Dimensions					Sectional area A	Reference data for axes							
								x-x				y-y			
		h	b	d	t	R		I_x	W_x	r_x	S_x	I_y	W_y	r_y	z_0
	kg	mm					cm²	cm⁴	cm³	cm	cm³	cm⁴	cm³	cm	cm
5	4.84	50	32	4.4	7.0	6	6.16	22.8	9.10	1.92	5.59	5.61	2.75	0.954	1.16
6.5	5.90	65	36	4.4	7.2	6	7.51	48.6	15.0	2.54	9.00	8.70	3.68	1.08	1.24
8	7.05	80	40	4.5	7.4	6.5	8.98	89.4	22.4	3.16	13.3	12.8	4.75	1.19	1.31
10	8.59	100	46	4.5	7.6	7	10.9	174	34.8	3.99	20.4	20.4	6.46	1.37	1.44
12	10.4	120	52	4.8	7.8	7.5	13.3	304	50.6	4.78	29.6	31.2	8.52	1.53	1.54
14	12.3	140	58	4.9	8.1	8	15.6	491	70.2	5.60	40.8	45.4	11.0	1.70	1.67
14a	13.3	140	62	4.9	8.7	8	17.0	545	77.8	5.66	45.1	57.5	13.3	1.84	1.87
16	14.2	160	64	5.0	8.4	8.5	18.1	747	93.4	6.42	54.1	63.3	13.8	1.87	1.80

Table 4, concluded

Section No.	Weight per linear metre	Dimensions					Sectional area A	Reference data for axes							
								x-x				y-y			
		h	b	d	t	R		I_x	W_x	r_x	S_x	I_y	W_y	r_y	z_0
	kg	mm					cm^2	cm^4	cm^3	cm	cm^3	cm^4	cm^3	cm	cm
16a	15.3	160	68	5.0	9.0	8.5	19.5	823	103	6.49	59.4	78.8	16.4	2.01	2.00
18	16.3	180	70	5.1	8.7	9	20.7	1,090	121	7.24	68.8	86.0	17.0	2.04	1.94
18a	17.4	180	74	5.1	9.3	9	22.2	1,190	132	7.32	76.1	105	20.7	2.18	2.13
20	18.4	200	76	5.2	9.0	9.5	23.4	1,520	152	8.07	87.8	113	20.5	2.20	2.07
20a	19.8	200	80	5.2	9.7	9.5	25.2	1,670	167	8.15	95.9	139	24.2	2.35	2.28
22	21.0	220	82	5.4	9.5	10	26.7	2,110	192	8.89	110	151	25.1	2.37	2.21
22a	22.6	220	87	5.4	10.2	10	28.8	2,330	212	8.99	121	187	30.0	2.55	2.46
24	24.0	240	90	5.6	10.0	10.5	30.6	2,900	242	9.73	139	208	31.6	2.60	2.42
24a	25.8	240	95	5.6	10.7	10.5	32.9	3,180	265	9.84	151	254	37.2	2.78	2.67
27	27.7	270	95	6.0	10.5	11	35.2	4,160	308	10.9	178	262	37.3	2.73	2.47
30	31.8	300	100	6.5	11.0	12	40.5	5,810	387	12.0	224	327	43.6	2.84	2.52
33	36.5	330	105	7.0	11.7	13	46.5	7,980	484	13.1	281	410	51.8	2.97	2.59
36	41.9	360	110	7.5	12.6	14	53.4	10,820	601	14.2	350	513	61.7	3.10	2.68
40	48.3	400	115	8.0	13.5	15	61.5	15,220	761	15.7	444	642	73.4	3.23	2.75

2. *Data on Arrangement of Rivet Gauge Lines on Rolled Shapes*

TABLE 5 **Arrangement of Gauge Lines on Angles**

Angles with one row of rivets				Angles with two rows of rivets, with staggered and straight hole pattern				
b	e_1	z_0	d	b	e_1	e_2	z_0	d
mm				mm				
45	25	15	14	125	50	40	40	20-23
50 and 56	30	15	17	140	50	50	40	20-26
63	35	20	17-20	160	60	60	45	23-26
70	40	20	20	180	70	70	50	23-26
75	45	25	20-23	200	80	80	60	23-26
80	45	25	20-23	220	80	80	70	23-26
90	50	30	20-23	250	80	80	80	23-26
100	55	30	20-23					
110	60	35	20-23					

SYMBOLS: b = size of leg
e_1 = distance from back of angle to gauge line (Figs. 5-4 and 5-5)
e_2 = transverse pitch with staggered and straight pattern of holes
z_0 = distance from back of angle to gauge line for centering the elements of welded trusses (Fig. 7-21c)
d = recommended diameter of rivet hole.

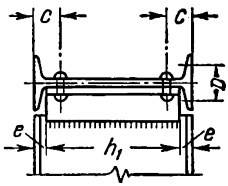


TABLE 6 **Arrangement of Gauge Lines on I Sections**

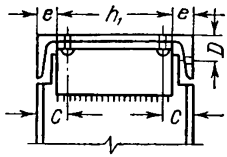
Section No.	Flange		d_{max}	Web			
	a	t		e	h_1	c	d
	mm						
10	32	7.2	10	15	70	36	14
12	38	7.3	12	16	88	37	14
14	44	7.5	12	17	107	42	17

Table 6, concluded

Section No	Flange		d_{max}	Web			
	a	t		e	h_1	c	d
	mm						
16	48	7.7	14	18	124	48	20
18	50	8	14	19	142	50	20
18a	54	8.2	17	19	142	50	20
20	54	8.2	17	20	161	50	20
20a	58	8.3	17	20	160	50	20
22	58	8.6	17	21	178	55	23
22a	62	8.8	17	21	178	55	23
24	60	9.5	20	22	196	58	23
24a	66	9.8	20	23	194	58	23
27	66	9.8	20	24	223	58	23
27a	72	10.2	20	24	222	58	23
30	72	10.2	20	25	250	60	23
30a	74	10.7	23	25	250	60	23
33	74	11.2	23	27	276	62	23
36	76	12.3	23	29	302	64	23
40	82	13.0	23	31	338	66	23
45	86	14.2	23	33	384	68	23
50	90	15.2	26	35	430	70	23
55	96	16.5	26	38	474	73	23
60	100	17.8	29	41	518	76	23
65	106	19.2	29	45	560	80	23
70	112	20.8	29	48	604	83	23
70a	114	24.0	29	51	598	86	23
70b	114	28.2	29	55	590	90	23

SYMBOLS: e = distance from flange to beginning of fillet, $c = e + 1.5\,d$
t = thickness of flange at gauge line
d_{max} = maximum diameter of hole
d = recommended diameter of hole

TABLE 7 **Arrangement of Gauge Lines on Channels**



Section No	Flange			Web				
	a	*t*	d_{max}	*e*	h_1	*c*	c_1	*d*
	mm							
5	20	7.0	10	14	22	—	—	—
6.5	25	7.4	12	15	35	—	—	—
8	25	7.4	14	16	48	—	25	12
10	30	7.5	14	17	66	35	26	12
12	30	7.7	17	17	86	38	29	14
14	45	8.0	17	18	104	44	33	17
14a	35	8.5	17	19	102	44	34	17
16	35	8.3	20	19	122	50	36	20
16a	35	8.8	20	20	120	50	37	20
18	40	8.7	20	21	138	50	38	20
18a	40	9.2	20	21	138	50	38	20
20	50	9.0	23	22	156	52	39	20
20a	50	9.6	23	22	156	52	39	20
22	50	9.6	23	23	174	54	40	20
22a	50	10.2	23	24	172	54	41	20
24	50	10.0	26	24	192	60	44	23
24a	50	10.7	26	25	190	60	45	23

Table 7, concluded

Section No.	Flange			Web				
	a	t	d_{max}	e	h_1	c	c_1	d
	mm							
27	60	10.5	26	25	220	60	45	23
30	60	11.0	29	27	246	62	47	23
33	60	11.7	29	29	272	65	49	23
36	60	12.6	29	31	298	66	51	23
40	60	13.5	29	33	354	66	53	23

SYMBOLS: e=distance from flange to beginning of fillet $c=e+1.5d$; $c_1=e+0.5D$
t=thickness of flange at gauge line
d_{max}=maximum diameter of hole
d=recommended diameter of hole
D=diameter of rivet head.

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